ii) Extrinsic region :- Here the majority carrier concentration is constant while minority concentration is neglegable. The device operates satisfactory in this region. Because, majority carrier concentration remains constant. Hence, conductive remains constant with respect to temperature.

iii) Intrinsic region :- Here the extrinsic property of semiconductor is lost ie, extrinsic will be increases.

MODULE - 2

Carrier transport in a semiconductor - mobility and conductivity

Carrier transport in a semiconductor is mainly by two different mechanism

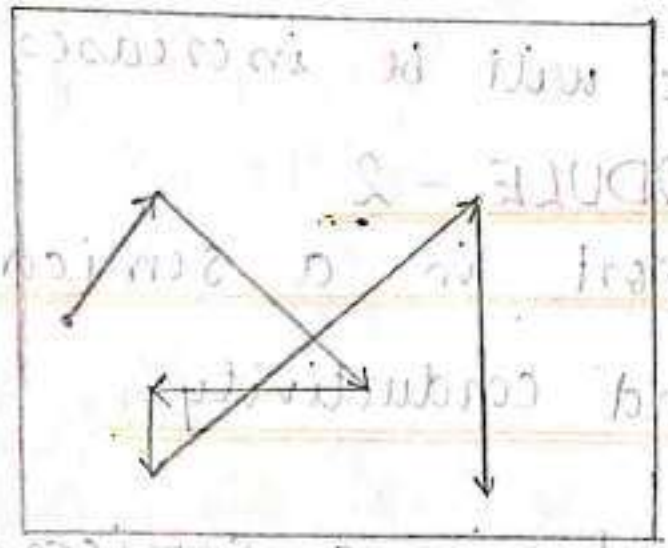
a) Drift

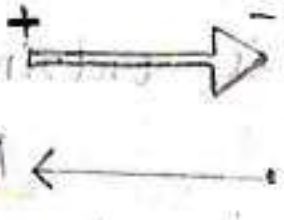
b) Diffusion

Drift current is due to the movement of charge under an electric field

Diffusion current is due to the movement of charge carrier from a region of high concentration to region of low concentration. ie, due to the difference in concentration.

a) Drift of carrier in an electric field





well directed drift velocity with an applied electron field

Random thermal motion of electron in a solid

⇒ The charge

→ The charge carriers in a solid state are in constant motion even at thermal equilibrium. At room temperature, the thermal motion of an individual electron may be visualized as random scattering from lattice vibration.

→ For a large no. of electrons considered then there will be no preffered direction of motion and no net current flow.

→ If electric field E be applied, the electron will experience a force.

$$f = -qE$$

Thus the group of electrons will have a net average velocity and it is called drift velocity (v_d)

$$v_d\ (cm/s) = \mu\ (cm^2/v\,sec)\ E\ (V/cm)$$

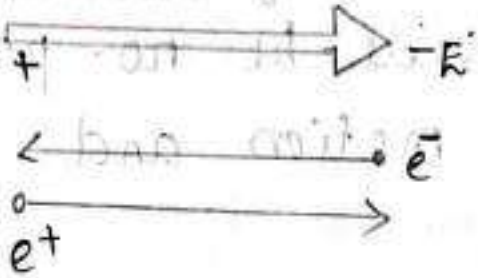
$$v_d = \mu E$$

where,

μ = mobility of charge carriers

E - applied electric field

Mobility (μ) - $cm^2/Vsec$



$$\mu = \frac{\text{drift velocity}}{\text{field intensity}} = \frac{V_d}{E}$$

→ It indicates how fast a charge carriers move from one directions to another.

$$\mu_n = \frac{-V_d}{E}, \quad \mu_p = \frac{V_d}{E}$$

Electron mobility — Hole mobility

Note :-

	Si	Ge
μ_n	1350	3800
μ_p	480	1800

Also $\mu_n \gg \mu_p$ always.

Note :- Normality

$$V_d < V$$

$V_d \rightarrow$ drift velocity

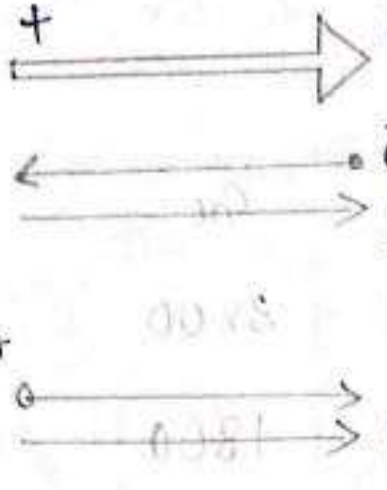
$V \rightarrow$ mean - thermal velocity

b). Current density and conductivity

$$J = \sigma E \quad (\text{Ohm's Law})$$

$J \rightarrow$ current density

$\sigma \rightarrow$ conductivity



$$J = J_n + J_p$$

$$(\text{drift}) = (\text{drift}) + (\text{drift})$$

$$\sigma = n_0 q \mu_n + p_0 q \mu_p$$

$$= \sigma_n + \sigma_p$$

$$= \left[q n_0 \mu_n + q p_0 \mu_p \right] E$$

J_n : electron

J_p : hole

Note :- con. of e^-, e^+

charge of e^-, e^+

mobility of e^-, e^+

J = current density (A/cm^2)

I = current density (A)

σ = conductivity ($\Omega^{-1} cm^{-1}$)

$I_{drift} = J_{drift} \times Area$

* Intrinsic conductivity (σ_i)

For an INTSC, $n_0 \times p_0 = n_i$

$$\therefore \sigma_i = \sigma_n + \sigma_p$$

$$\sigma_i = n_i q (\mu_n + \mu_p)$$

120
today

Drift of carrier in Electric field

The charge carriers in a solid are in constant motion at thermal equilibrium.

- If an electric field in the x direction ($\mathcal{E}_x$) is applied into the semiconductor each electron experiences a force

$$f = -q\mathcal{E}_x \quad \text{——} (1)$$

$-q$ → charge of electron

$\mathcal{E}_x$ → electric field in x direction

∴ group of electron will have a net average velocity in the x direction, This mean velocity is called drift velocity.

If P_x is the total momentum of the group of electron /cm3 the accelerating force due to electric field is given by

$$F = \frac{dP_x}{dt} = -nqE_x \quad \text{---} (2)$$

The force due to electric field is balanced in steady state by the decelerating force of the collision process.

The probability of collision ~~force~~ for an electron in any time interval ~~dt~~ is $\frac{dt}{t}$, therefore the differential change in momentum due to collision is

$$-dP_x = P_x \frac{dt}{t} \quad \text{---} (3)$$

$\bar{t}$ → mean time between ~~dt~~ collision

from eqn (3), mean free time,

$$\frac{dP_x}{dt} = -\frac{P_x}{t} \quad \text{---} (4)$$

at steady state condition sum of accelerating and decelerating effect must be zero.

ie,

from eqn ② and ④

$$\frac{-p_x}{\bar{t}} - nq\mathcal{E}_x = 0 \quad \text{——⑤}$$

$$\frac{-p_x}{\bar{t}} = nq\mathcal{E}_x$$

$$\langle p_x \rangle = \frac{p_x}{n} = -q\bar{t}\,\mathcal{E}_x \quad \text{——⑥}$$

ie, electrons have constant average velocity in the negative x direction

$$\text{ie, } \langle v_x \rangle = \frac{\langle p_x \rangle}{m_n^*} = \frac{-q\bar{t}\,\mathcal{E}_x}{m_n^*} \quad \text{——⑦}$$

$m_n^* \rightarrow$ effective mass

The current density due to electron drift is the no. of electrons crossing unit area per unit time multiplied by the charge of an electron.

ie, J_n (drift) $= -qn v_x$ —— (8)

$$= -qn \frac{-q\bar{t}\mathcal{E}_x}{m_n^*}$$

$$= \frac{nq^2\bar{t}\mathcal{E}_x}{m_n^*}$$

$$J_n \text{ drift} = \sigma_n \mathcal{E}_x \quad \text{—— (9)}$$

where $\sigma_n = \frac{nq^2\bar{t}}{m_n^*}$ conductivity.

The conductivity can also be written as

$$\sigma_n = q_n \mu_n \quad \text{—— (10)}$$

(total charge × mobility)

$$\mu_n = \frac{-q\bar{t}}{m_n^*}$$

(mobility)

conductivity of electron

$$\sigma_n = q_p \mu_p \quad \text{—— (10)}$$

∴ conductivity of holes

$$\sigma_p = q_p \mu_p \quad \text{—— (11)}$$

where $\mu_p \rightarrow$ hole mobility $= \frac{q\tau}{m_p^*}$

Sub. eqn (10) in (9) μ_n

ie, $J_n(\text{drift}) = q n \mu_n \varepsilon_x$

$n \rightarrow$ no. of electron

ie, electron drift current density

$|||^{ly}$ hole drift current density is given by

$$J_p(\text{drift}) = q p \mu_p \varepsilon_x$$

$\therefore$ Total drift current

$$J_{drift} = J_n(\text{drift}) + J_p(\text{drift})$$

$$= q n \mu_n \varepsilon_x + q p \mu_n \varepsilon_x$$

$$J_{drift} = q(n\mu_n + p\mu_p)\varepsilon_x \quad \text{—(13)}$$

* Calculate electron, hole and intrinsic carrier concentrations. Sketch band diagram

$N_c = 10^{19} \frac{1}{cm^3}$, $N_v = 5 \times 10^8 \frac{1}{cm^3}$

$E_g = 0.2eV$ $T = 627^\circ C = 900K$

$n = 10^{17} \frac{1}{cm^3}$

Ans) We know,

$$n = N_c \times e^{\frac{-E_c - E_f}{KT}}$$

$$\frac{n}{N_c} = e^{\frac{-E_c - E_f}{KT}}$$

$$e^{+E_c - E_f} = \frac{n}{N_c} \times -KT$$

Taking ln on both side

$$E_c - E_f = -KT \ln\left(\frac{n}{N_c}\right)$$

$$E_c - E_f = (8.08 \times 10^{-5}) \times 900 \cdot \ln\left(\frac{10^{17}}{10^{19}}\right)$$

$$E_c - E_f = -0.072\,eV \cdot \ln\left(\frac{10^{17}}{10^{19}}\right)$$

$$= \underline{0.33\,eV}$$

$$E_f - E_v = \left[(E_c - E_v) - (E_c - E_f)\right]$$

$$= \left[E_g - (E_c - E_f)\right]$$

$$= \left[2eV - 0.303\,eV\right]$$

$$= \underline{1.67\,eV}$$

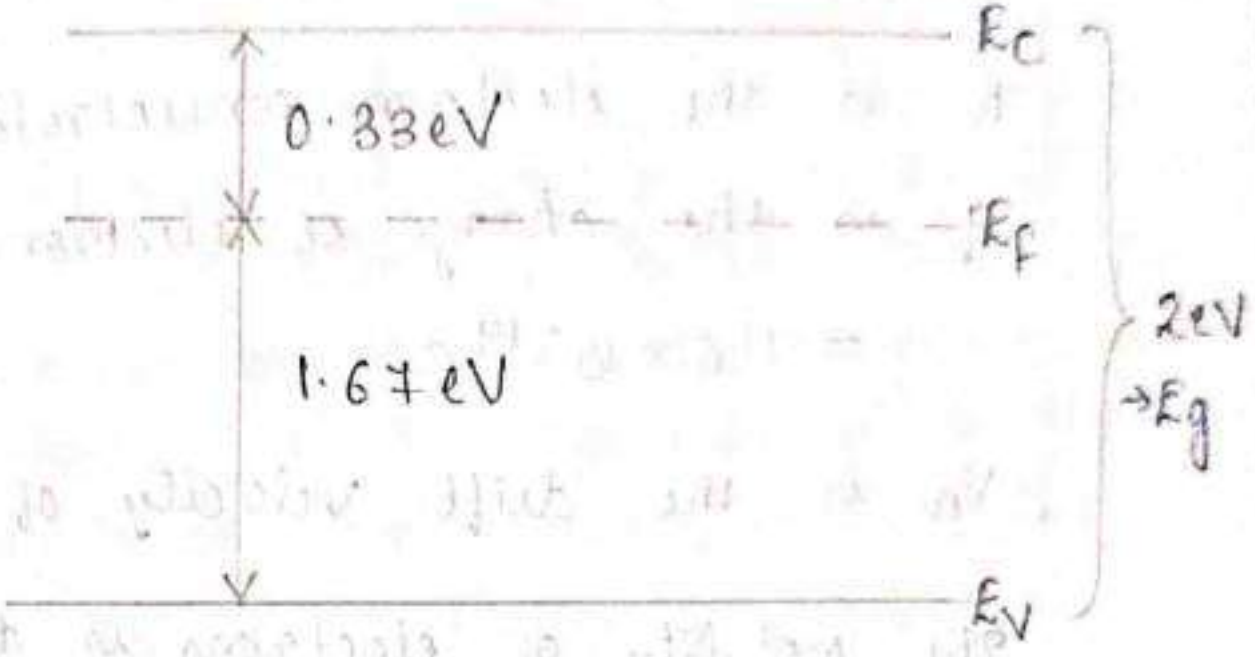
$$p = N_v \cdot e^{\frac{-E_f - E_v}{kT}}$$

$$= 5 \times 10^{18} \times e^{\frac{-1.67}{0.072}}$$

$$= \underline{4.2 \times 10^{9}\ \frac{1}{cm^3}}$$

$$n_i = \sqrt{n \cdot p} = \underline{2.04 \times 10^{13}\ \frac{1}{cm^3}}$$

Band diagram



1.09.20 Friday

Derivation of conductivity of semiconductor

By ohm's law, the electron drift current density may be expressed as

$$J_n = \sigma_n \mathcal{E}$$

where

σ_n represents conductivity due to electrons

$\mathcal{E}$ is the electric field.

By definition of current density,

$$J_n = -n q v_n$$

where

n is the electron concentration

q is the charge of electron.

$$= 1.6 \times 10^{-19} C$$

v_n is the drift velocity of electrons

The mobility of electrons is defined as

$$\mu_n = \frac{-v_n}{\mathcal{E}}$$

Sub. eqn

$$J_n = q n \mu_n \mathcal{E}$$

compare eqn

$$\sigma_n = q n \mu_n$$

Similarly

$$\sigma_p = \mu_p q p$$

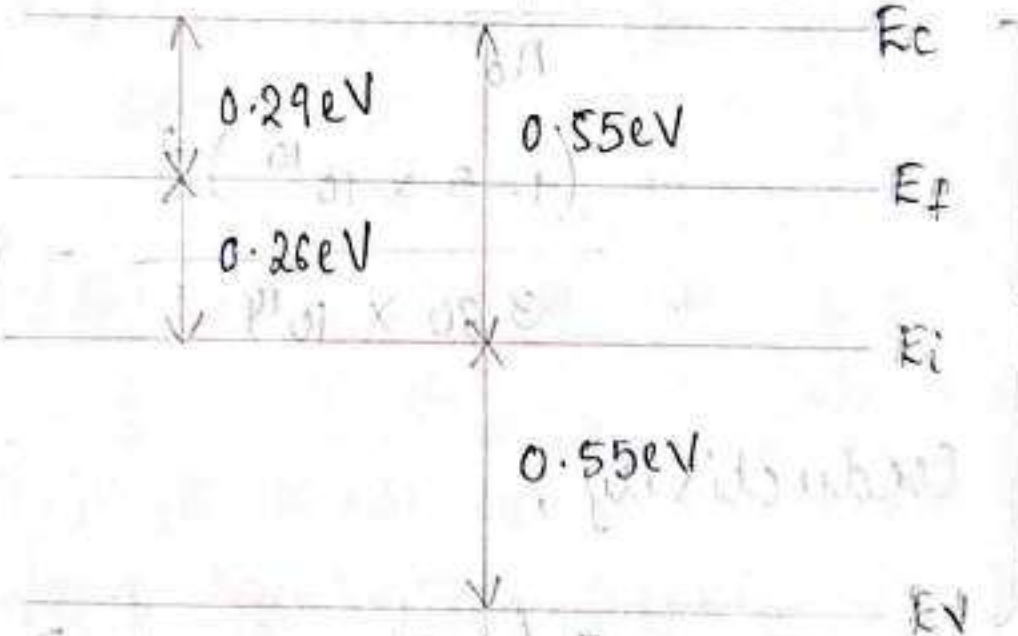
$$\therefore \sigma = \sigma_n + \sigma_p$$

$$= q(n\mu_n + p\mu_p)$$

---

* The Fermi level position in a silicon sample at 300K is 0.29 eV below E_c. Determine the carrier concentrations and conductivity of the specimen. Given that $n_i = 1.5 \times 10^{10}\ cm^{-3}$, $\mu_n = 1350\ cm^2/Vs$, $\mu_p = 480\ cm^2/Vs$ [E_g for Si = 1.1eV]

AND)



For silicon $E_g = 1.1eV$

$$\therefore E_f - E_i = \frac{E_g}{2} - (E_c - E_f)$$

$$= 0.55 - 0.29$$

$E_f - E_i = 0.26\,eV$

The Specimen is n-type as the Fermi level is above E_i.

$$n_0 = n_i e^{(E_f - E_i)/KT}$$

$$= 1.5 \times 10^{10} e^{0.26/0.072}$$

$$= 3.30 \times 10^{14}\ cm^{-3}$$

$$p_0 = \frac{n_i^2}{n_0}$$

$$= \frac{(1.5 \times 10^{10})^2}{3.30 \times 10^{14}} = 6.8 \times 10^5\ cm^{-3}$$

Conductivity,

$$\sigma = q(n_0 \mu_n + p_0 \mu_p)$$

$$= 1.6 \times 10^{-19} (3.30 \times 10^{14} \times 1350 + 6.8 \times 10^5 \times 480)$$

$$= 0.0712\ \Omega^{-1}$$

14/09/20
Monday

Excess carriers in semiconductors

Charge carriers in semiconductors which are in excess of their thermal equilibrium value are called excess carriers. Excess carriers can be generated by optical excitation, electron bombardment, injection across a p-n junction etc.

Photo generation

This is the process of generating ~~process~~ charge carriers (excess) by shining light on the semiconductor. If a photon of energy greater than band gap of semiconductor falls on it, the energy of photon is absorbed by an electron in the valence band and is excited to the conduction band. This results in the generation of an EHP ie, optically generated electron and hole concentration are equal. These carrier concentration are in excess of the thermal equilibrium value.

ie, $n = n_0 + \delta n$

$p = p_0 + \delta p$

$\delta n = \delta p$

where,

n is the total electron concentration

p is the total hole concentration

n_0 is the equilibrium electron conc

p_0 is the equilibrium hole conc.

δn is the excess electron concentration

δp is the excess hole concentration

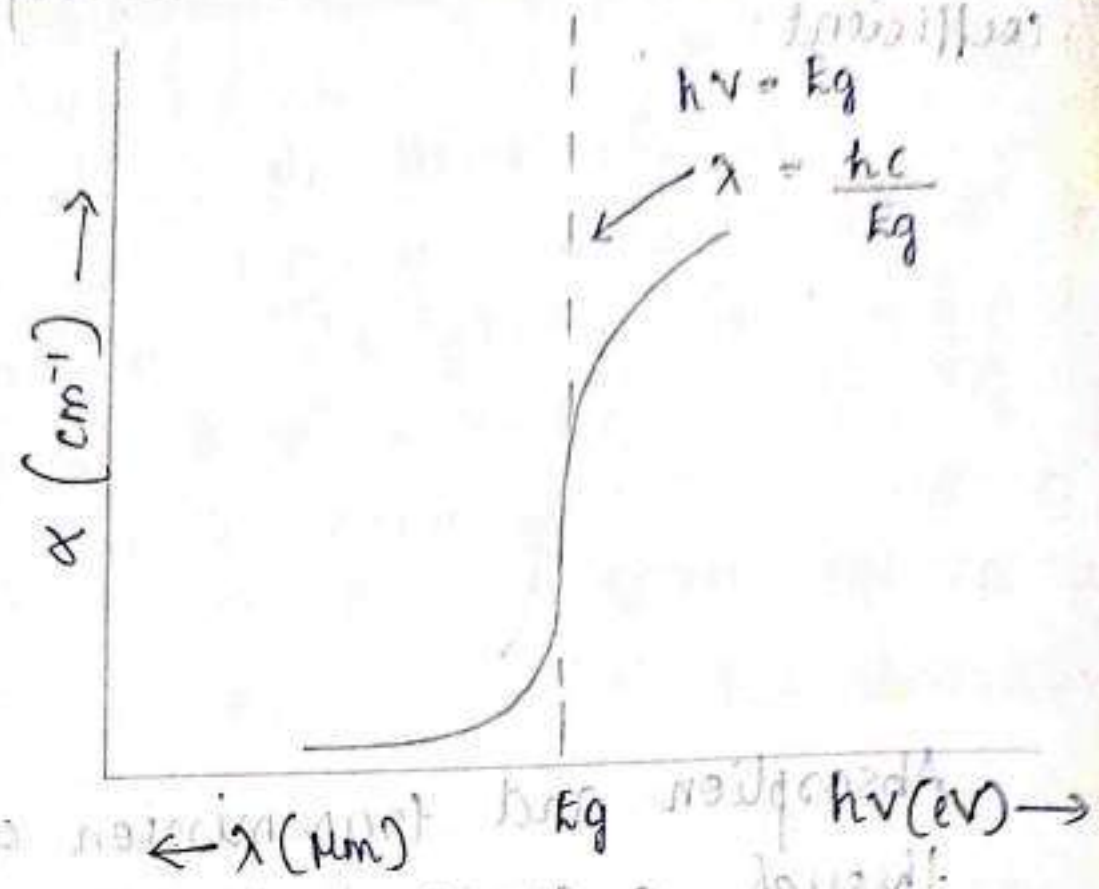
If light of intensity I_0 falls on one face of a sample of thickness t as shown in figure, then the amount of light transmitted through it is given by

$$I(t) = I_0 e^{-\alpha t}$$

where α is called the absorption coefficient.

Absorption and transmission of light through a semiconductor specimen of thickness t

The absorption coefficient of a material depends on the incident wave length or energy ($E = \frac{hc}{\lambda}$). The absorption coefficient as a function of incident proton energy is given in figure



If the incident photon has energy greater than the band gap energy, the velocity of the excited electron will be higher than the mean thermal velocity. The excess energy of electron is released to the lattice as heat during scattering events until its velocity becomes the thermal velocity at that temperature. Photogeneration and recombination process are illustrated in figure.

Photo conductivity :- The conductivity due to the photogenerated carriers is called photo conductivity

$$\sigma_{op} = q n_{op} \mu_n + q p_{op} \mu_p$$

$$= q \mu_n \delta n + q \delta p \mu_p$$

The photo conductivity as a function of photon energy will have a shape similar to that of the absorption coefficient

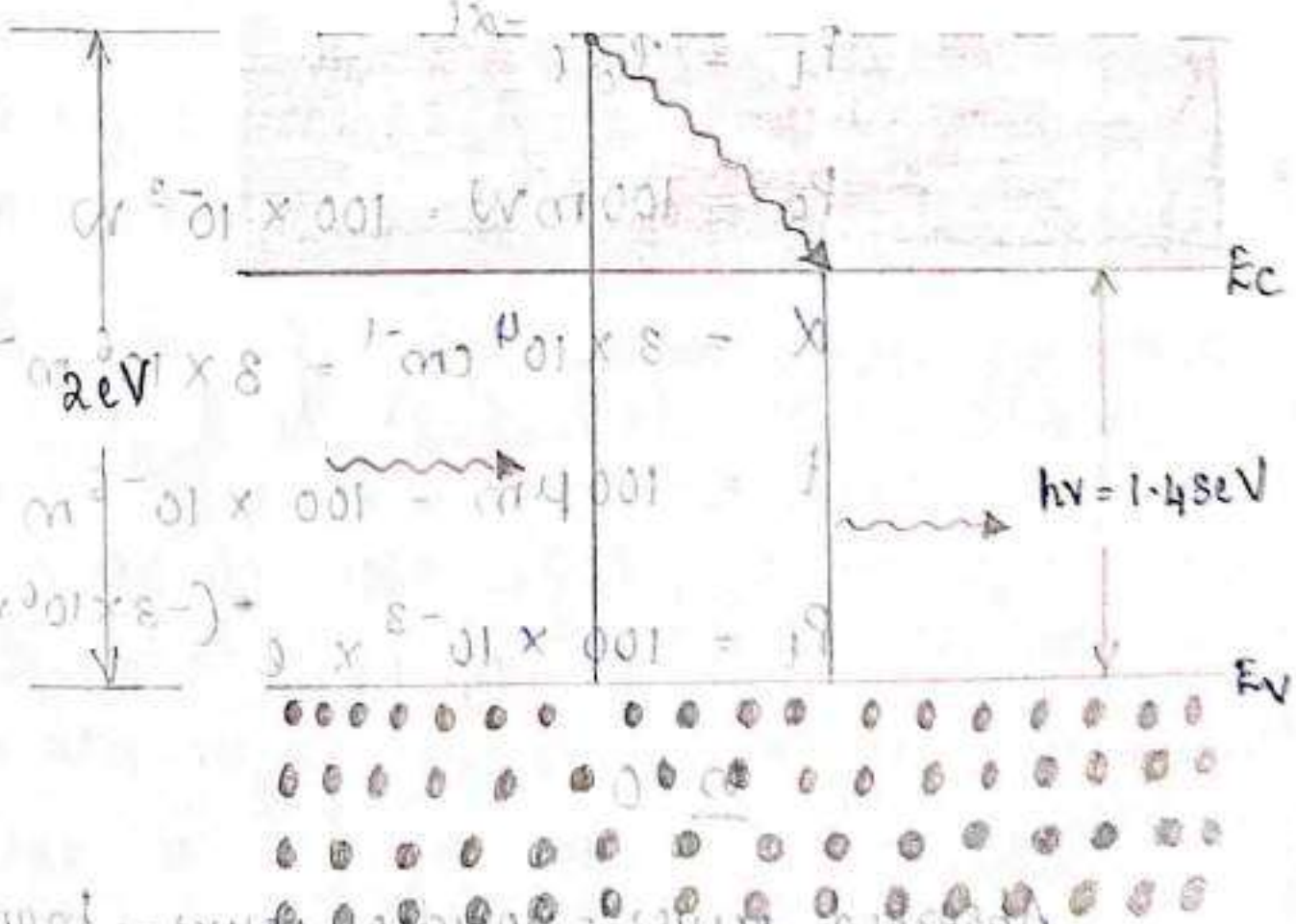


Photo excitation and radiative recombination

* A 100 mW laser beam with wavelength $\lambda = 6382\ \text{Å}$ is focussed onto a GaAs sample 100 μm thick. The absorption coefficient at this wavelength is $3 \times 10^4\ cm^{-1}$. Find the no. of photons emitted per second by radiative recombination in the GaAs specimen, assuming perfect quantum efficiency. What power is delivered to the sample as heat?

Ans) Transmitted power,

$$P_t = I_0 e^{-\alpha t}$$

$$P_0 = 100\ mW = 100 \times 10^{-3}\ W$$

$$\alpha = 3 \times 10^4\ cm^{-1} = 3 \times 10^6\ m^{-1}$$

$$t = 100\ \mu m = 100 \times 10^{-6}\ m$$

$$P_t = 100 \times 10^{-3} \times e^{(-3 \times 10^6 \times 100 \times 10^{-6})}$$

$$\approx 0$$

Absorbed power = Incident power − transmitted power

$= 100\,mW$

Band gap of GaAs $= 1.43\,eV$

Energy of incident photon $E = \frac{hc}{\lambda}$

$$E = \frac{1.24}{\lambda\,(\text{in } \mu m)}$$

$$E = \frac{1.24}{0.6328} = 1.96\,eV$$

Energy of emitted photon is equal to the band gap of GaAs. The difference b/w energies of absorbed and emitted photons will be released to the lattice as heat.

Therefore, the amount of power converted to heat

$$= \frac{1.96 - 1.43}{1.96} \times 100 \times 10^{-3}\,W$$

$$= 27\,mW$$

Power of emitted photon = 100 − 27

= 73mW

No. of ~~emitted~~ photons emitted per second

= ~~Power of transmitted~~ .

$$= \frac{\text{Power of emitted photons}}{\text{energy of emitted photons}}$$

$$= \frac{73 \times 10^{-3}}{1.43 \times 1.6 \times 10^{-19}}$$

$$= 3.19 \times 10^{17}$$

16/09/20
Wednesday

Low - level and High level injection

When the injected excess carrier concentration is small compared to the equilibrium majority carrier concentration, the injection of excess carriers is called low level injection. [ie, $\delta n, \delta p \ll n_0$ for n-type semiconductors and $\delta n, \delta p \ll p_0$ for p-type

Semiconductors]. In low level injection, the minority carrier concentration remains almost unchanged.

When the injected excess carrier concentration is comparable to or greater than the equilibrium majority carrier concentration, the injection is called high level ~~concentration~~ injection. In high level injection, the majority carrier concentration also change significantly. One-tenth of equilibrium majority carrier concentration is usually considered as the boundary between low-level and high level injection

Excess carrier lifetime [Transient decay of excess EHP] - direct recombination

In direct recombination, electron from the conduction band directly falls into a vacant state (hole) in the valence band and the difference in energy is released as photon. The probability of recombining a hole and electron is constant at any time

Thus the decay of excess carriers will have an exponential form.

The rate of decay at any time t is proportional to the product of no. of electrons and no. of holes remaining at time t. Therefore, the net rate of change of conduction band electron concentration $\left(\frac{dn(t)}{dt}\right)$ equals thermal generation rate $(\alpha_r n_i^2)$ minus the recombination rate $(\alpha_r n_p)$.

$$-\frac{dn(t)}{dt} = \alpha_r n(t) p(t) - \alpha_r n_i^2 \quad \text{---(1)}$$

Assume that excess EHP is created at $t = 0$ by a flash of light and let the initial excess electron and hole concentration be Δn and Δp respectively. Also, $\Delta n = \Delta p$. As electrons and holes recombine ~~be~~ ~~and~~ in pairs, the instantaneous value of carrier concentrations $\delta n(t)$ and $\delta p(t)$ are also equal.

$$\therefore n(t) = n_0 + \delta n(t)$$

$$p(t) = p_0 + \delta p(t)$$

and equation ① becomes

$$-\frac{d}{dt}\delta n(t) = \alpha_r \left[n_0 + \delta n(t)\right]\left[p_0 + \delta p(t)\right] - \alpha_r n_i^2 \quad \left(\because \frac{dn_0}{dt} = 0\right)$$

$$= \alpha_r \left[n_0 p_0 + \delta n(t)(p_0 + n_0) + \delta^2 n(t)\right] - \alpha_r n_i^2$$

$$= \alpha_r \left[(n_0 + p_0)\,\delta n(t) + \delta^2 n(t)\right] \quad \left(\because \alpha_r n_0 p_0 = \alpha_r n_i^2\right)$$

——— ②

For low-level injection $\delta^2 n(t)$ is negligible compared to other terms.

For p-type semiconductor

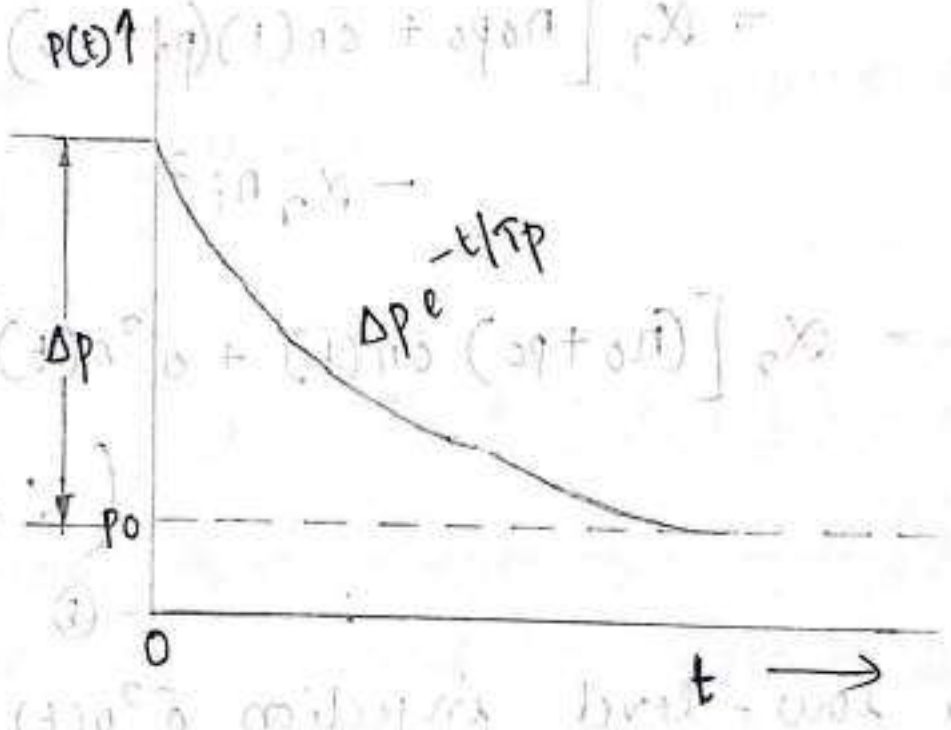
$$n_0 + p_0 \approx p_0$$

$$\therefore -\frac{d}{dt}\delta n(t) = \alpha_r p_0\, \delta n(t)$$

Solution to this equation is,

$$\delta n(t) = \Delta n\, e^{-\alpha_r p_o t} = \Delta n\, e^{-t/\tau_n}$$

Where $\tau_n = (\alpha_r p_o)^{-1}$ is called recombination lifetime. τ_n is also called minority carrier lifetime. Δn is the excess carrier concentration at $t = 0$



Exponential decay of excess hole in a p-type semiconductor

Similarly, the decay of excess holes in n-type semiconductor is given by,

$$\delta p(t) = \Delta p\, e^{-t/\tau_p}$$

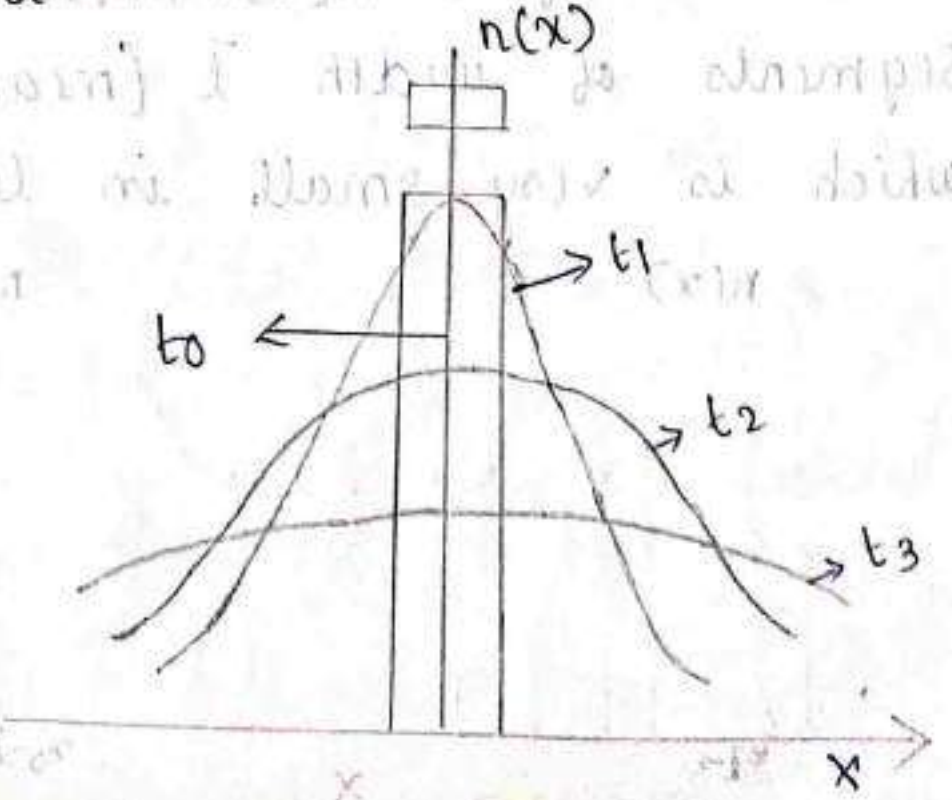
where $\tau_p = (\alpha_s n_0)^{-1}$ and Δp is δp at $t = 0$

On the case of direct recombination, the excess majority carriers decay at the same rate as the minority carriers.

8/09/20
Friday

Diffusion of charge carriers

When excess carriers are generated, non-uniformly in a semiconductor, the carrier concentration varies with position in the sample, thus charge carriers will move from region of high concentration to low concentration. This type of motion is called diffusion and the current due to diffusive motion of charge carriers is called diffusion current

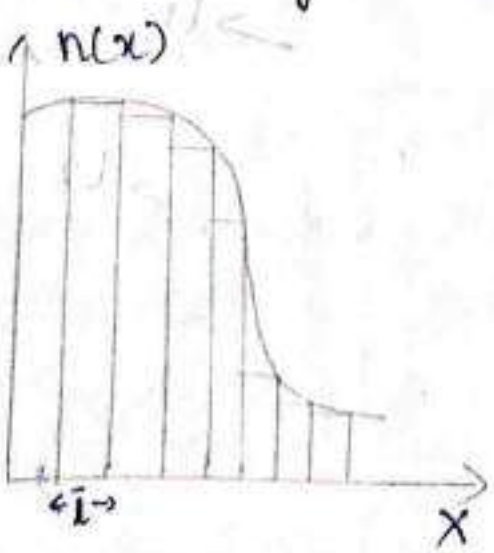


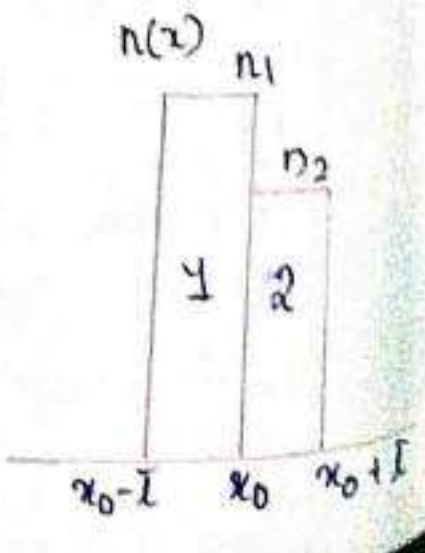
Here a pulse of excess electron injected at $x=0$ at time $t=0$ is spreadout in time. Initially excess electron concentration at $x=0$ but ds time releases relaxes this excess electron spread on and finally the carrier concentration becomes uniform through are and $n(x)$ becomes constant

Diffusion current

Derivation

Consider an arbitrary one dimensional distribution of $\bar{e}$ $n(x)$. Divide x in to segments of width $\bar{l}$ (mean freepath) which is very small in length.





The electrons in any segment have equal chances of movement towards, left or right since the movement is purely random.

Total no. of electron in seg 1 = $n_1 \bar{l} A$ —— ①

Total no. of electron in seg 2 = $n_2 \bar{l} A$ —— ②

∴ Net no. of $\bar{l}$ crossing x_0 from, left - right (from seg 1 - seg 2) in one mean free time = $\frac{1}{2}(n_1 \bar{l} A - n_2 \bar{l} A)$ —— ③

Then the electron flux density $\phi_{n(x)}$,

$$\phi_{n(x)} = \frac{\frac{1}{2}(n_1 \bar{l} A - n_2 \bar{l} A)}{\bar{t} \times A} \quad \text{—— ④}$$

$$= \frac{1}{2\bar{t}A} \bar{l} A (n_1 - n_2) \quad \text{—— ⑤}$$

If $\bar{l}$ is a small differential length, then $n_1 - n_2$ can be written as,

$$n_1 - n_2 = \frac{n(x) - n(x + \Delta x)}{\Delta x} \bar{l} \quad \text{—— ⑥}$$

$$= \frac{-d}{dx} n(x) \bar{l} \quad \text{——⑦}$$

where x is taken as the Δx & $\Delta x = \bar{l}$

Sub. ⑦ in ⑥

$$\phi_{n(x)} = \frac{1}{2\bar{t}A} \times \bar{l} A \left[\frac{-d}{dx} n(x) \cdot \bar{l} \right]$$

$$\phi_{n(x)} = \frac{-\bar{l}^2}{2\bar{t}} \frac{d}{dx} n(x) \quad \text{——⑧}$$

[flux density : $f\bar{e}$]

$$\phi_{n(x)} = -D_n \frac{d}{dx} n(x) \quad \text{——⑨}$$

Similarly

$$\phi_{p(x)} = -D_p \frac{d}{dx} p(x) \quad \text{——10}$$

$D_n = \frac{\bar{l}}{2\bar{t}} \bar{e}$ diffision constant in cm^2/s

$D_p \rightarrow$ hole diffusion constant

Diffusion current density

$$J_{n\,diff} = -q\,\phi_{n(x)} \quad \text{—— (11)}$$

$$J_{n\,diff} = -q\left(-D_n \frac{d}{dx} n(x)\right) \quad \text{—— (12)}$$

$$J_{n\,diff} = -q\,D_n \frac{d}{dx} n(x) \quad \text{—— (13)}$$

$$J_{p\,diff} = q\,D_p \frac{d}{dx} p(x) \quad \text{—— (14)}$$

Total electron current density of a semiconductor is given by

$$J_n = J_{n\,drift} + J_{n\,diff}$$

$$J_n(x) = q n \mu_n \mathcal{E}_x + q D_n \frac{d}{dx} n(x) \quad \text{—— (15)}$$

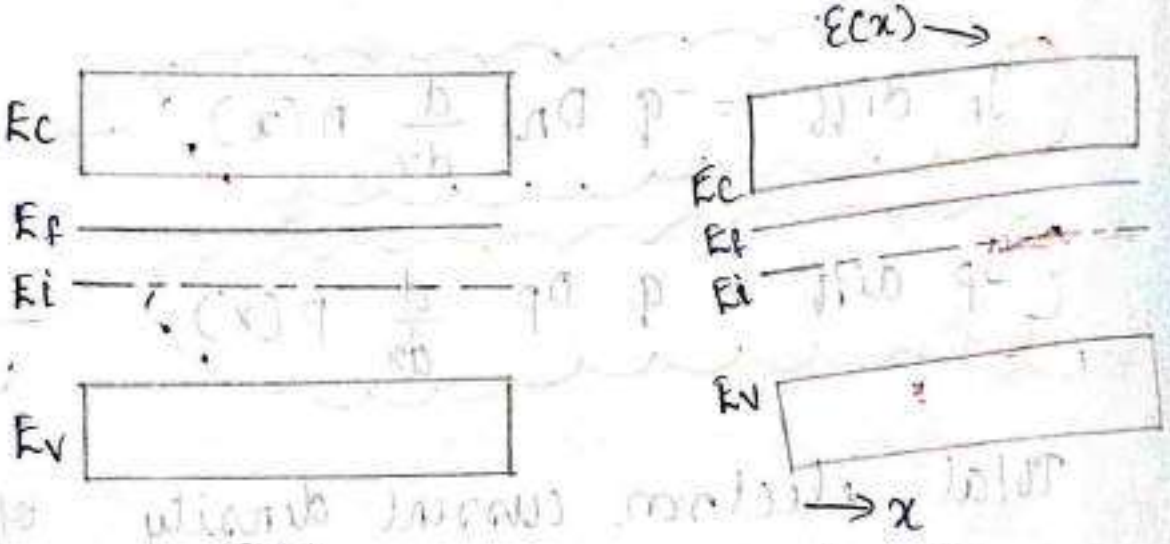
Total hole current density

$$J_{px} = q_p \mu_p \mathcal{E}_x - q D_p \frac{d}{dx} p(x) \quad \text{—— (16)}$$

23/09/20
Wednesday

Energy band diagram of a semiconductor with electric field

The energy band diagrams of a uniformly doped - n type semiconductor under equilibrium condition and with an electric field applied along the positive x direction are shown in figure.



Energy band diagram of a semiconductor (a) at equilibrium (b) in an electric field $\mathcal{E}_x$

The applied electric field is proportional to the gradient in electron energy as given by eqn ②. Therefore energy bands bend upward in the

direction of the electric field. Energy bands of a semiconductor will be flat when electric field in it is zero.

Einstein relations

Einstein relation states that the ratio of diffusion coefficient to mobility of electrons and holes are equal under thermal equilibrium and is equal to the volt equivalent of temperature (kT/q)

ie, $$\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = \frac{kT}{q} \quad \text{——— ①}$$

Electrostatic potential and potential energy of a particle of charge q are related as follows

$$\text{Electrostatic potential } V_{(x)} = \frac{E(x)}{q}$$

$$V(x) = \frac{E_x}{q} = \frac{\text{Potential energy}}{\text{charge}}$$

$$= \frac{E(x)}{-q} \quad \text{(for electrons)}$$

Also, electric field

$$\mathcal{E}(x) = \frac{-d\,V(x)}{dx}$$

$$= -\frac{d}{dx}\left(\frac{E_i}{-q}\right)$$

$$\left[\because \frac{d\,E(x)}{dx} = \frac{dE_c}{dx} = \frac{dE_i}{dx} = \frac{dE_v}{dx}\right]$$

(ie, All energies in the EB diagram bend equally in a given electric field)

$$\mathcal{E}(x) = \frac{1}{q}\frac{dE_i}{dx} \quad \text{——} ②$$

At equilibrium $J_n = J_p = 0$

ie, $J_p = q p_0(x) \mu_p \mathcal{E}(x) - q D_p \frac{dp_0(x)}{dx} = 0$

$$q\mu_p \cdot p_0(x) \cdot \mathcal{E}(x) = q D_p \cdot \frac{dp_0(x)}{dx}$$

$$\therefore \mathcal{E}(x) = \frac{D_p}{\mu_p} \cdot \frac{1}{p(x)} \cdot \frac{dp_0(x)}{dx} \quad \text{——} ③$$

$$p_0(x) = n_i e^{E_i(x) - E_f(x)/kT}$$

$$\frac{dp_0(x)}{dx} = n_i e^{E_i(x) - E_f(x)/kT} \times \frac{1}{kT}\left(\frac{dE_i}{dx} - \frac{dE_f}{dx}\right)$$

$$= \frac{P_0(x)}{KT}\left(\frac{dE_i}{dx} - \frac{dE_f}{dx}\right) \quad \text{——— (4)}$$

Sub (4) in (3)

$$\mathcal{E}(x) = \frac{D_p}{\mu_p} \times \frac{1}{P_0(x)} \cdot \frac{P_0(x)}{KT}\left(\frac{dE_i}{dx} - \frac{dE_f}{dx}\right)$$

under thermal equilibrium,

$$\frac{dE_f}{dx} = 0$$

$$\frac{dE_i}{dx} = q\mathcal{E}_x \quad (\text{by eqn (2)})$$

$$\therefore \mathcal{E}(x) = \frac{D_p}{\mu_p} \cdot \frac{1}{KT} \cdot q\mathcal{E}_x$$

$$\text{ie, } \frac{D_p}{\mu_p} = \frac{KT}{q}$$

$$\text{Similarly, } \frac{D_n}{\mu_n} = \frac{KT}{q}$$

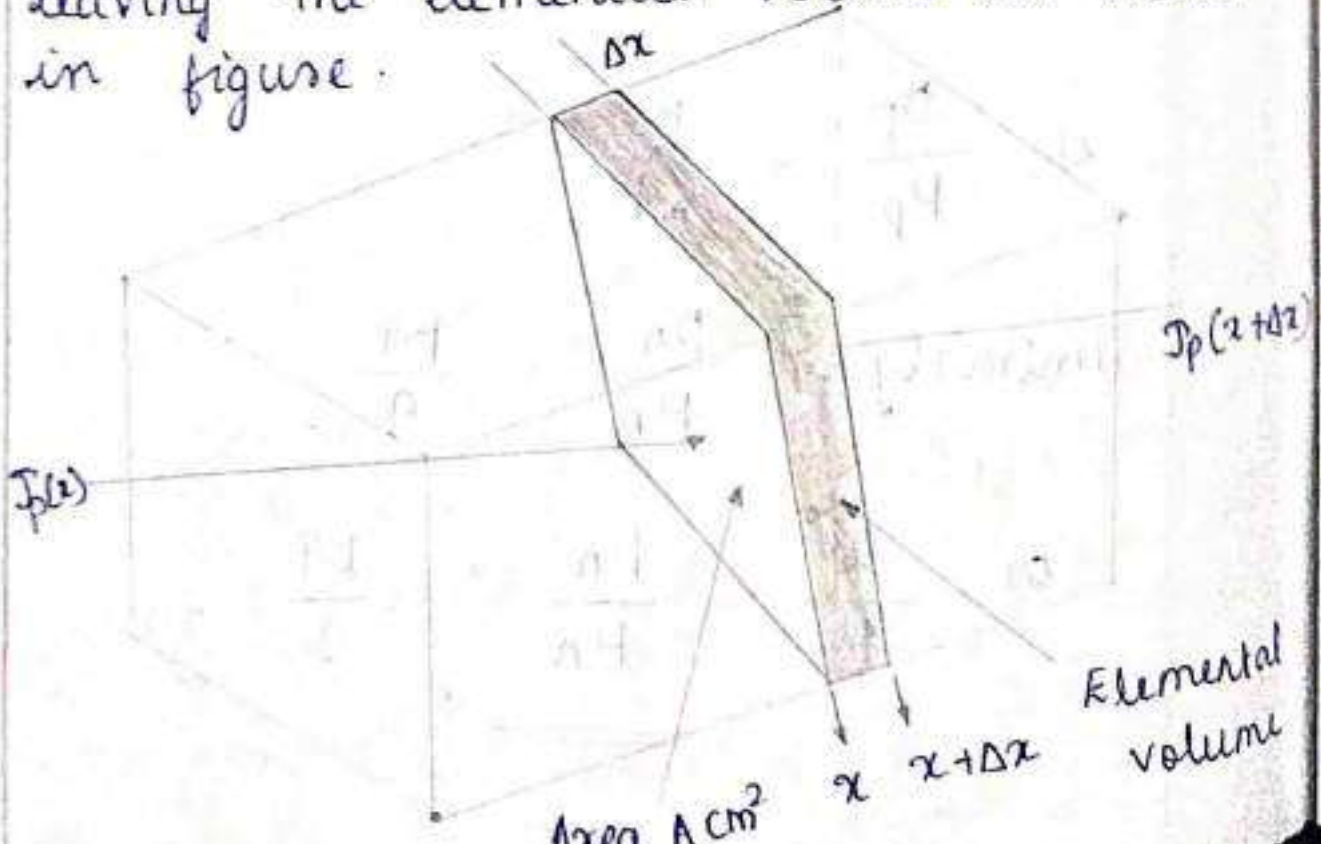
$$\text{or } \frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = \frac{KT}{q}$$

25/09/20
Friday

Continuity equation [Effect of diffusion and recombination]

Continuity equation is an important equation for the analysis of semiconductors. It can be used to determine the distribution of charge carriers in a semiconductor. It considers the time and space variation of charge carriers simultaneously. A simplified continuity equation in differential form is derived below.

Consider a semiconductor bar with area of cross-section A. Let $J_p(x)$ be the current density entering an elemental volume $\Delta x A$ and $J_p(x + \Delta x)$ be the current density leaving the elemental volume as shown in figure.



Current density entering and leaving elemental volume $\Delta x A$.

The rate of increase of hole concentration in the elemental volume $\left(\frac{\partial p}{\partial t}\right)$ is given by

$\frac{\partial p}{\partial t}$ = ~~rate~~ rate of hole built up

= (net flux/volume of hole in elemental volume - net recombination rate in it)

= (hole flux/unit volume entering elemental volume $\Delta x A$ - hole flux/unit volume leaving $\Delta x A$ - recombination rate)

$$J_p = q \phi_p$$

$$\phi_p = \frac{J_p}{q} = \text{flux/unit area}$$

$\therefore$ flux/unit volume $= \frac{J_p}{q \Delta x}$

ie, $$\frac{\partial p}{\partial t} = \frac{J_p(x)}{q \Delta x} - \frac{J_p(x + \Delta x)}{q \Delta x} - \frac{\partial p}{\tau_p}$$

$$= \frac{-1}{q} \frac{[J_p(x + \Delta x) - J_p(x)]}{\Delta x} - \frac{\partial p}{\tau_p}$$

$$= \frac{-1}{q} \frac{\partial J_p}{\partial x} - \frac{\partial p}{\tau_p} \quad \text{——— (1)}$$

The above equation is called continuity equation for holes

Similarly, the continuity equation for electron is given as

$$\frac{dn}{dt} = \frac{1}{q}\frac{dJ_n}{dx} - \frac{\delta n}{\tau_n} \quad \text{——(2)}$$

When the current is only due to diffusion

ie, $J_p = -q D_p \cdot \frac{dp}{dx} = -q D_p \frac{d}{dx}\delta p$

($\because p = p_0 + \delta p$ and $\frac{dp_0}{dx} = 0$ for uniformly doped semiconductor)

Similarly,

$$J_n = q D_n \cdot \frac{dn}{dx} = q D_n \frac{d}{dx}\delta n$$

Therefore, eqns ① and ② reduces to

$$\frac{\delta p}{dt} = D_p \frac{d^2\delta p}{dx^2} = \frac{\delta n}{L_n^2}$$

$$\frac{dp}{dt} = D_p \frac{d^2\delta p}{dx^2} - \frac{\delta p}{\tau_p} \quad \text{——(3)}$$

$$\frac{dn}{dt} = D_n \frac{d^2\delta n}{dx^2} - \frac{\delta n}{\tau_n} \quad \text{——(4)}$$

Eqns ③ and ④ are called diffusion eqns.

Steady-state carrier injection and diffusion length

Under steady-state conditions the time derivatives are zero and the diffusion equations reduces to

$$\frac{d^2\delta n}{dx^2} = \frac{\delta n}{D_n \tau_n} = \frac{\delta n}{L_n^2} \quad \text{——— ①}$$

$$\frac{d^2\delta p}{dx^2} = \frac{\delta p}{D_n \tau_p} = \frac{\delta p}{L_p^2} \quad \text{——— ②}$$

where, $L_n = \sqrt{D_n \tau_n}$ is called electron diffusion length and $L_p = \sqrt{D_p \tau_p}$ is hole diffusion length.

These equations are called steady-state diffusion equations. The solution of the steady-state diffusion for holes is of the form.

$$\delta p(x) = C_1 e^{x/L_p} + C_2 e^{-x/L_p} \quad \text{——— ③}$$

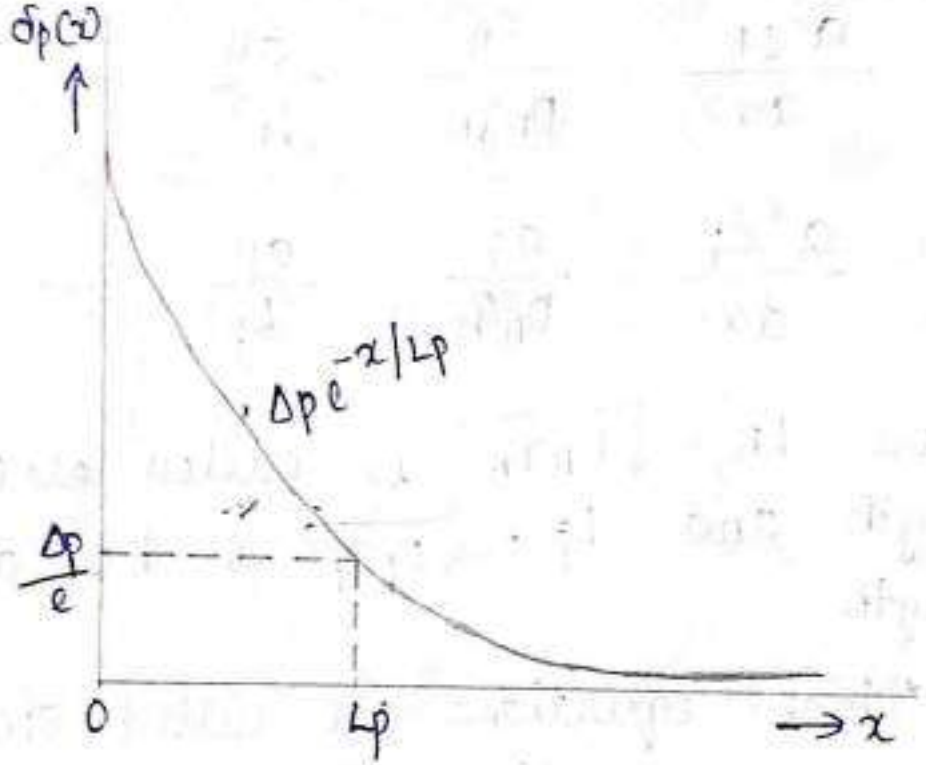
Consider an n-type semiconductor bar in which excess carriers are introduced at $x = 0$. Let $\delta p(x = 0) = \Delta p$. The excess carrier

concentration decays to zero at the end of the semiconductor bar ($x = \infty$)

At $x = 0$, $\delta p = \Delta p$; $C_1 + C_2 = \Delta p$

At $x \to \infty$, $\delta p = 0$; $\therefore C_1 = 0$

$\therefore C_2 = \Delta p$



Decay of excess holes and diffusion length

Thus the solution is,

$$\delta p(x) = \Delta p e^{-x/L_p} \quad \text{——— (4)}$$

The distribution of excess carrier given by eqn (4) is shown in figure

The injected excess carrier concentration decay exponentially to 1/e times its initial value at a diffusion length from the point of injection. It can be shown that L_p is the average distance a hole diffusion before it recombines.

01/10/20 Thursday

Gradient in Quasi-Fermi Levels

At equilibrium, gradient in Fermi level is zero. But a combination of drift and diffusion with a net current implies a gradient in quasi fermi levels. The total electron current is given by,

$$J_n(x) = q n(x) \mu_n \mathcal{E}_x + q D_n \cdot \frac{dn(x)}{dx} \quad \text{——①}$$

But,

$$\frac{dn(x)}{dx} = \frac{d}{dx} n_i e^{(F_n - E_i)/kT}$$

$$= \frac{n(x)}{kT} \left[\frac{dF_n}{dx} - \frac{dE_i}{dx} \right] \quad \text{——②}$$

Sub. eqns ② in ① and applying Einstein's relation,

$$J_n(x) = q n(x) \mu_n \mathcal{E}_x + \mu_n n(x) \frac{dF_n}{dx}$$

$$= q n(x) \mu_n \mathcal{E}_x + \mu_n n(x) \frac{dF_n}{dx}$$

$$- \mu_n n(x) q \mathcal{E}_x$$

$$= \mu_n n(x) \frac{dF_n}{dx}$$

$$= q \mu_n n(x) \cdot \frac{d}{dx}\left(\frac{F_n}{q}\right)$$

$$= \sigma_n(x) \cdot \frac{d}{dx}\left(\frac{F_n}{q}\right)$$