MAGNETISM

Magnetic field, Magnetic induction

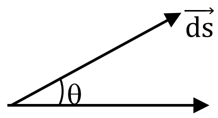
The region of space around a magnet, current carrying conductor or a moving charge in which magnetic effect can be experienced is called **magnetic field**.

A magnetic field is characterized by its field lines. The magnetic field may be geometrically represented by the lines of magnetic induction in the same way as the electric field is represented by the electric field lines.

If the field is strong, these field lines are more closely spaced. Therefore the number of magnetic field lines through a surface is a measure of the strength of the magnetic field there. The magnetic field strength is called **magnetic induction**. It is denoted by **B**. The strength of the magnetic field is quantitatively expressed by the vector quantity, magnetic induction (Magnetic flux density), **B**.

i.e., the amount of field lines normal to a surface determines the strength of the magnetic field. This is defined as **magnetic flux** $\boldsymbol{\phi_B}$. $\phi_B = \mathbf{B}.ds\cos\theta$

$\phi_B = \oint \mathbf{B}.ds\ \cos\theta$; $\phi_B = \oint \mathbf{B}.ds$



Magnetic induction (magnetic flux density) at a point is defined as the flux (field lines) passing through unit area around the point. The unit of magnetic induction is tesla (T) or weber/m^2 (Wb/m^2)

The direction of magnetic field is given by the direction along which the north pole of a compass needle tends to point and the tangent to the field lines gives the direction of magnetic field at that point.

Magnetizing field

When current passes through a conductor, magnetic field is generated. When a magnetic field generated by current passes through a magnetic material, the material itself contributes an internal magnetic field called magnetizing field (**H**). Its SI unit is ampere/metre (A/m).

Both **B** and **H** ,

$\mathbf{H} = \frac{\mathbf{B}}{\mu_m}$ where μ_m is the absolute permeability of the medium.

$\mathbf{H} = \frac{\mathbf{B}}{\mu_o} - \mathbf{M}$

$\mathbf{B} = \mu_o(\mathbf{H} + \mathbf{M})$ where μ_o is the permeability of free space. Its value is $\mu_0 = 4\pi \times 10^{-7}$ H/m.

And **M** is the magnetization produced inside the material.

Intensity Of Magnetisation (M): When a material is placed in a magnetic field, it gets magnetised.

The magnetic moment per unit volume of the material is called intensity of magnetisation. It is the extent to which a specimen is magnetised.

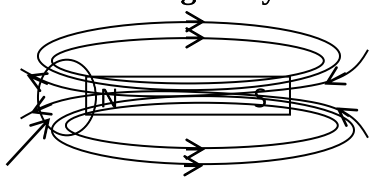
Magnetization M = $\frac{magnetic\ dipole\ moments\ of\ the\ material}{volume\ of\ the\ material}$

Gauss's law for magnetic flux density

Gauss's law for magnetic flux density states that the net magnetic flux through any closed surface is zero. The mathematical statement of this law is $\boldsymbol{\phi_B} = \oint \mathbf{B}\,\boldsymbol{ds} = \mathbf{0}$

Here the number of lines entering the Gaussian surface is equal to the lines leaving the surface.

Hence the net magnetic flux through any closed surface is zero.



Gaussian Surface

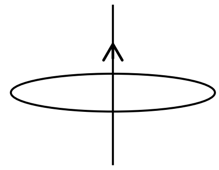
From this equation, it is clear that the magnetic field lines always form closed .loop. i.e., they never have end points. This shows that we cannot isolate a north pole or south pole from a magnet.

i.e., the significance Gauss's law in magnetism is monopole does not exist in magnets. Magnetic flux is scalar quantity and its unit is weber (Wb).

Ampere's circuital law

It states that the line integral of the magnetic field around any closed path around a current carrying conductor is equal to μ_0 times the current.

i.e., $\oint \mathbf{B}.\boldsymbol{dl} = \mu_0 \mathbf{I}$ where I is the current passing through



the closed path (Amperian loop).

Application

Magnetic field at a distance of 'r' from a current carrying conductor

Let us consider a long straight conductor carrying current 'I' with direction perpendicular to the plane of the paper and away from it.

$\oint \mathbf{B}.\boldsymbol{dl} = \mu_0 \mathbf{I}$; $\oint \mathbf{B}\boldsymbol{dl} \cos\theta = \mu_0 \mathbf{I}$;

$\oint \mathbf{B}\boldsymbol{dl} \cos 0 = \mu_0 \mathbf{I}$ or $\mathbf{B} \oint \mathbf{d}\boldsymbol{l} = \mu_0 \mathbf{I}$

Since $\oint \mathbf{dl} = 2\pi r$, $\mathbf{B} \times 2\pi r = \mu_0 \mathbf{I}$

Hence $\mathbf{B} = \frac{\mu_0 \mathbf{I}}{2\pi r}$

Faraday's law in terms of EMF produced by changing magnetic flux

(i) Whenever the magnetic flux linked with a closed circuit changes, an induced *emf* is set up in the circuit whose magnitude at any instant is proportional to the state of change of magnetic flux linked with the circuit. $\varepsilon \propto \frac{d\phi_m}{dt}$

(ii) The direction of induced e.m.f is such that it opposes the change in flux that produces it. This law is known as Lenz's law. $\varepsilon = -\frac{d\phi_m}{dt}$

Let $d\phi_m$ be the change in magnetic flux in time interval dt.

Instead of a single loop, if it is a tightly wound coil of N turns, emf will induce in each turn and total induced emf will induce in each turn and total induced emf will be $\varepsilon = -N\frac{d\phi_m}{dt}$

Magnetic Permeability(μ)

The magnetic permeability of μ of a material is defined as the ratio of resultant magnetic field **B** inside the material to the magnetizing field **H**.

$\mu = \frac{B}{H}$. Its unit is henry/metre (H/m)

The permeability of free space or the permeability constant (μ_0) is a measure of the resistance encountered when forming a magnetic field in vacuum. Its valued is $\mu_0 = 4\pi\times10^{-7}$H/m

Relative permeability (μ_r) is the ratio of absolute permeability of a material to permeability of free space or vacuum (μ_0). $\mu_r = \frac{\mu}{\mu_0}$.

It is a dimensionless quantity.

Also, Relative Permeability is defined as the ratio of magnetic induction in a material to that in air or vacuum under the same magnetizing field H. $\mu_r = \frac{B}{B_0}$

Magnetic Susceptibility(χ)

The ratio of magnetization M to the magnetizing field H is called magnetic susceptibility.

It indicates the degree of magnetization of a material in response to an applied magnetic field. It is denoted by the letter χ. $\chi = \frac{M}{H}$

It is a dimensionless quantity

The key difference between magnetic permeability and susceptibility is that magnetic permeability describes the ability of a material to support the formation of a magnetic field inside itself whereas susceptibility describes whether a material is attracted to a magnetic field or is repelled from it.

Relation Connecting B, H and M

The magnetic induction or magnetic flux density B inside a material is proportional to the magnetizing field H.

Therefore $B = \mu H$.

In terms of magnetization(M) and magnetizing field H, the magnetic induction (B) inside a material is, $B = \mu_0(H+M)$

$B = \mu_0 H + \mu_0 M$

Dividing throughout by H, $\frac{B}{H} = \mu_0 + \mu_0 \frac{M}{H}$

$\mu = \mu_0 + \mu_0\chi$; $\mu = \mu_0(1 + \chi)$

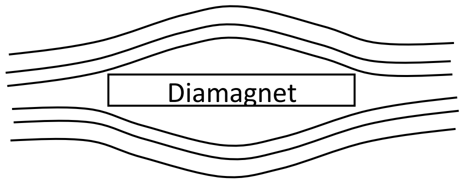
$\mu_r = \frac{\mu}{\mu_0} = 1 + \chi$;

$\boldsymbol{\chi = \mu_r - 1}$ where χ is susceptibility and μ_r is relative permeability.

Classification of magnetic materials-para, dia and ferromagnetic materials

(i) Diamagnetic Substances

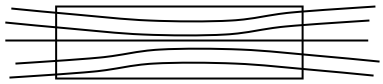
* *The substance which when placed in external magnetizing field get magnetized feeble in a direction opposite to the magnetizing field are called diamagnetic substance.* ie, The individual atoms or molecules have no net magnetic dipole moment in the presence of an external magnetic field.
* Examples: Gold, copper, bismuth, lead, quartz, air, hydrogen, water, alcohol, sodium chloride etc.
* They try to expel the magnetic field lines when placed in an external field and the resultant field within the material is reduced.



* They are repelled by a magnet.
* They tend to move from a region of strong magnetizing field to a region of weak field when placed in non uniform field.
* Diamagnetic materials have relative permeability less than one. $\mu_r < 1$
* Susceptibility is negative and small. χ is –ve and small.
* Susceptibility is independent of temperature and the substance does not obey Curie law.
* Do not exhibit the phenomenon of Hysteresis.

(ii) Paramagnetic Substance

* *The substance which when placed in external magnetizing field get magnetized feebly in the direction of the magnetizing field are called paramagnetic substance.* i.e., The individual atoms or molecules have a net magnetic dipole moment in the presence of an external field.
* Examples: Platinum, aluminium, lithium, magnesium, chromium, copper chloride etc.
* They try to concentrate the magnetic field lines within them when placed in an external magnetic field and the resultant field within the material is enhanced.

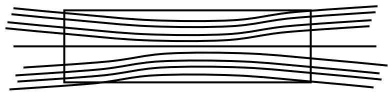


* They are weakly attracted by a magnet.
* They tend to move from a region of weak magnetic field to a region of strong field when placed in non uniform field.
* Relative permeability is slightly greater than unity. $\mu_r > 1$.
* Susceptibility is small and positive. χ is + ve
* Susceptibility varies inversely with temperature and the substance obeys **Curie Law**.

  Curie's Law: Magnetic susceptibility is inversely proportional to temperature. $\chi \propto \frac{1}{T}$
* Do not exhibit the phenomenon of Hysteresis.

(iii) Ferromagnetic Substance

* *The substances, which when placed in external magnetizing field, get magnetized very strongly in the direction of magnetizing field.* The individual atoms or molecules have a net magnetic dipole moment and and are in the direction of magnetizing field H.
* Eg: Iron, Nickel, Cobalt, Steel, Alnico etc. Ferromagnetism is not found in liquids and gases.
* They are strongly attracted by a magnet.



* The magnetic field lines are highly concentrated within them when placed in an external field and the resultant field is strongly enhanced.
* They tend to move from a region of weak magnetic field to a region of strong magnetic field when placed in a nonuniform field.
* Relative permeability is greater than unity. $\mu_r > 1$ and is very high.
* Susceptibility is large and positive. χ is + ve.
* Susceptibility varies inversely with temperature. The ferromagnetism decreases with the increase in temperature. Above Curie temperature, ferromagnetic substance is converted to paramagnetic substance.(Curie temperature or curie point is the temperature above which the ferromagnetic material loses ferromagnetism and turns to paramagnetic substance)
* Relative permeability is greater than unity. $\mu_r < 1$ and is very high.
* Exhibit the phenomenon of Hysteresis.

ELECTROMAGNETIC THEORY

Scalar function and scalar field

If every point (x,y,z) in a region of space R corresponds to a scalar quantity f, then the scalar quantity is written as $f(x,y,z)$ and is known as scalar function. The region of space over which the function is defined is known as scalar field.

For example, consider the temperature of a room with room heater. Since the temperature of the room varies from point to point, temperature T is a function of position coordinates and can be written as $T(x,y,z)$. Here T is the scalar function and the space defined by scalar function is known as scalar field (temperature field).

Vector function and vector field

If every point (x,y,z) in a region of space R corresponds to a vector quantity f, then the vector quantity is written as $f(x,y,z)$ and is known as vector function. The region of space over which the function is defined is known as vector field.

For example, consider the wind velocity in a room. It changes from point to point. Here wind velocity $v(x,y,z)$ is a function of position coordinates and hence is a vector function. The space defined by the vector function is known as vector field (velocity field).

Vector partial differential operator (known as del ∇)

The vector partial differential operator determines how much the vector changes with respect to x, y, z axis.

$$\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$$

Gradient

If $\phi(x,y,z)$ is a scalar function, the product of 'del' (∇) with ϕ is known as gradient of the scalar function ϕ and is abbreviated as gradϕ. Though ϕ is a scalar function, gradϕ is a vector function.

i.e., $\text{grad}\phi = \nabla\phi = \mathbf{i}\frac{\partial\phi}{\partial x} + \mathbf{j}\frac{\partial\phi}{\partial y} + \mathbf{k}\frac{\partial\phi}{\partial z}$

Significance of gradient: The gradient of a scalar at any point in a scalar field is a vector whose magnitude is equal to the maximum rate of increase of scalar function (ϕ) at that point. Its direction is along the normal to the surface at that point.

Example: If the scalar function ϕ represent temperature, then gradϕ represents the temperature gradient which is the rate of change of temperature w.r.t position coordinates and its direction gives the direction in which this change is maximum.

Divergence

The dot product of vector differential operator 'del' with a vector function is known as the divergence of the vector function and is abbreviated as 'div' of the function.

Let **A** (x,y,z) be a vector function.

$\mathbf{A} = \mathbf{i}A_x + \mathbf{j}A_y + \mathbf{k}A_z$ and $\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$

Divergence of A is div $\mathbf{A} = \nabla.\mathbf{A} = \left(\mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}\right).\left(\mathbf{i}A_x + \mathbf{j}A_y + \mathbf{k}A_z\right)$

$$\nabla.\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}.$$

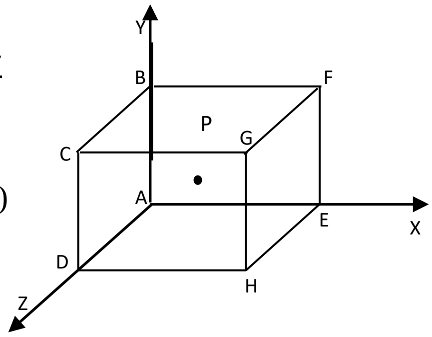
Just as dot product of two vector quantities is a scalar quantity, **∇. A** is a scalar quantity.

Physical significance of divergence

The divergence of a vector function at a point is the net flux per unit volume per second evaluated at that point.

To illustrate the physical significance of divergence, consider a fluid in motion.

Consider a parallelopiped ABCDEFGH with sides Δx, Δy, Δz parallel to x,y,z axis respectively. Let v(x,y,z) be the velocity at point P at the centre of the parallelopiped.



The x- component of velocity at point P is v_x.

The x- component of velocity at the centre of the face EFGH is $v_x + \frac{\partial v_x}{\partial x}\frac{\Delta x}{2}$

(Since the centre of parallelopiped is $\frac{\Delta x}{2}$ units away from the centre of face of parallelopiped, we multiply by $\frac{\Delta x}{2}$ with $\frac{\partial v_x}{\partial x}$ to obtain change in velocity.)

The x- component of velocity at the centre of the face ABCD is

$v_x - \frac{\partial v_x}{\partial x}\frac{\Delta x}{2}$

Net velocity of the flow along x-axis is

$$v_x + \frac{\partial v_x}{\partial x}\frac{\Delta x}{2} - \left(v_x - \frac{\partial v_x}{\partial x}\frac{\Delta x}{2}\right) = 2\frac{\partial v_x}{\partial x}\frac{\Delta x}{2} = \frac{\partial v_x}{\partial x}\Delta x$$

Therefore, Net volume of fluid per unit time along x-direction is $\left(\frac{\partial v_x}{\partial x}\Delta x\right)\Delta y\Delta z$

Net volume of fluid per unit time along y-direction is $\left(\frac{\partial v_y}{\partial y}\Delta y\right)\Delta x\Delta z$

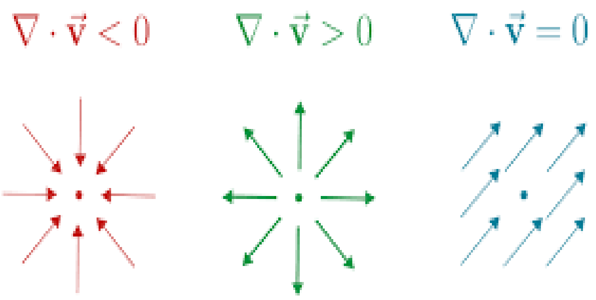
Net volume of fluid per unit time along z-direction is $\left(\frac{\partial v_z}{\partial z}\Delta z\right)\Delta x\Delta y$

The net outward flow in unit time through all the faces is $\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right)\Delta x\Delta y\Delta z$

Therefore the net outward flow per unit volume in unit time $= \frac{\left(\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}\right)\Delta x\Delta y\Delta z}{\Delta x\Delta y\Delta z}$

$$= \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) = \nabla . \mathbf{v}$$

Thus, the **physical significance of divergence** of a vector function at a point is the net outflow per unit volume in unit time at that point.



Divergence may be positive, negative or zero.

(a) If divergence exists at a point and is positive, it means that the fluid expands and consequently, the density decreases at the point. That means there is a source of fluid at that point.

(b) If divergence exists at a point and is negative, there is a net amount of fluid flowing towards the point which means there is a sink into which fluid flows.

(c) If divergence is zero, then we can infer that the amount of fluid flowing into the point is same as the amount flowing out of the point. Any vector whose divergence is zero is called **solenoidal vector**.

For electric and magnetic fluxes, the existence of divergence will mean the presence of sources or sinks in the field . For example, the electric field **E** is due to charges in a region. If div **E** = 0, in an electric field, it means, there are no charges anywhere in the electric field.

Curl

The cross product of vector differential operator 'del' with a vector function is known as the curl of the vector function. The curl of a vector is a vector.

Let **A** (x,y,z) be a vector function.

$\mathrm{A} = \mathbf{i}A_x + \mathbf{j}A_y + \mathbf{k}A_z$ and $\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$

$$\text{Curl } \mathbf{A} = \nabla \times \mathbf{A} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \mathbf{A}_x & \mathbf{A}_y & \mathbf{A}_z \end{vmatrix} = \mathbf{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \mathbf{j}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \mathbf{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

Physical significance of curl

Curl is a measure of how much a vector field circulates or rotates about a given point. When the flow is anti-clockwise, curl is considered to be positive and when it is clock-wise, curl is negative.

When curl **v** (**v** is a velocity vector) is non zero, the vector field must have circulation.

When curl **v** = 0 in some region, there will be no ciruculation or rotation at all in that region. The vector field **v** is then called **irrotational (**$\nabla \times A = 0$**)** in that region.

(Rotational vector – Any vector whose curl is not zero and Irrotational vector – any vector whose curl is zero.)

To illustrate of physical significance of curl, consider flow of water in a river. Let us assume the width of the river as X-axis, the flow of water as Y-axis and the depth of the river as Z-axis. Consider a floating stick flowing down in the stream.

The velocity of different liquid layers will increase as we proceed across the river.

The change in velocity of the particle along x-axis is $\left(v_y + \frac{\partial v_y}{\partial x}\right) - v_y = \frac{\partial v_y}{\partial x}$

The change in velocity of the particle along y-axis is $\left(v_x + \frac{\partial v_x}{\partial y}\right) - v_x = \frac{\partial v_x}{\partial y}$

This will rotate the stick about z axis, in the clock wise direction. So the rotating effect of the stick about +ve z-axis is $\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$

Similarly, the rotational effect of stick about y-axis is $\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)$

The rotational effect of stick about x-axis is $\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)$

Curl **v** gives the vector sum of all three components of rotation of the velocity field.

Some important vector identities

i.	div (gradϕ) = $\nabla^2\phi$	$\nabla.(\nabla\phi) = \nabla^2\phi$
ii.	curl (gradϕ) = 0	$\nabla\times(\nabla\phi) = 0$
iii.	div (curl A) = 0	$\nabla.(\nabla\times A) = 0$
iv	curl (curl A) = grad(diva) – del^{2}A	$\nabla\times(\nabla\times A) = \nabla(\nabla. A) - \nabla^2 A$

Line Integrals

The integration of a vector along a curve in a vector field is called **line integral**.

Let **F**(x,y,z) is a vector function in a vector field and C is a curve in the field connecting two points. The line integral of **F**(x,y,z) along the path C is given by $\int \mathbf{F}.\mathbf{d}\boldsymbol{l}$.

Physical significance of line integral:

(i) If **F**(x,y,z) is a force vector and $\mathbf{d}\boldsymbol{l}$ is a small displacement along the path of a particle, then

$\int_A^B \mathbf{F}.\mathbf{d}\boldsymbol{l}$ = work done in displacing the particle from A to B.

(ii) If **E**(x,y,z) is the intensity of electric field and $\mathbf{d}\boldsymbol{l}$ is the displacement of the unit charge, then

$\int_A^B \mathbf{E}.\,\mathbf{d}\boldsymbol{l}$ = work done in displacing the charge from A to B (potential difference between A and B).

Surface Integrals

Let **F**(*x*,*y*,*z*) is a vector function and a surface **S** is the vector field. Let this surface **S** is divided into finite number of sub surfaces represented as a vector $\mathbf{d}\boldsymbol{s}$. Its magnitude is its area and direction is that of the normal to the $\mathbf{d}\boldsymbol{s}$. Then $\int_s \mathbf{F}.\,\mathbf{d}\boldsymbol{s}$

Physical Significance:

The surface integral of a vector field gives the total amount of flux passing through the surface.

(i) If **F** represents the velocity of a fluid particle, then the total outward flux of **F** across a closed surface **S** is $\oint_S \mathbf{F}.\,\mathbf{d}\boldsymbol{s}$

(ii) If **B** is the magnetic induction vector and **S** the surface through which magnetic flux passes, then the $\oint_S \mathbf{B}.\,\mathbf{d}\boldsymbol{s}$ is the total amount of magnetic flux passing through the surface.

Volume Integral

Let **F**(*x*,*y*,*z*) is a vector function and a surface **S** is the enclosing region. Let this region **S** is divided into finite number of sub regions represented as a vector $\mathbf{d}\boldsymbol{v}$ which is the volume of any region enclosing any point. Hence $\int_v \mathbf{F}.\,\boldsymbol{dv}$ is defined as the volume integration.

Physical Significance:

The significance of volume integral is to calculate flux densities.

Gauss Divergence Theorem (surface integral → volume integral)

The surface integral of a vector function **A** taken over a closed surface S is equal to the volume integral of the divergence of the vector function taken over the volume V bounded by the surface.

$\int_s \mathbf{A}.\,\boldsymbol{ds} = \int_v \mathrm{div}\mathbf{A}\boldsymbol{dv}$ i.e., $\int_s \mathbf{A}.\,\boldsymbol{ds} = \int_v (\nabla.\,\mathbf{A})\boldsymbol{dv}$

Stoke's Theorem (surface integral → line integral)

The surface integral of the curl of a vector function **A** taken over a surface is equal to the line integral of the vector function taken over the boundary of the surface.

$\int_s \mathbf{curl\ A}\boldsymbol{ds} = \int_c \mathbf{A}.\,\boldsymbol{dl}$ i.e., $\int_s (\nabla \mathrm{x}\, \mathbf{A})\boldsymbol{ds} = \int_c \mathbf{A}.\,\boldsymbol{dl}$

Equation of continuity

Equation of continuity says that the net outward flow of current must be equal to the rate of decrease of charge within that volume.

Let us consider a volume V bounded by a surface S. The net charge enclosed is q . If a net current I flows across the surface out of this region, which is represented as I = $\int_s \mathrm{J}.\,ds$ → (1)

By principle of conservation of charge, I = $\int_s \mathrm{J}.\,ds = -\frac{\partial q}{\partial t}$ → (2)

Since $\rho = \frac{q}{v}$ where v is the volume and ρ is the volume charge density, then q = $\int_v \rho.\,\boldsymbol{dv}$ → (3)

Putting (3) in (2), we have, $\int_s \mathrm{J}.\,ds = -\frac{\partial}{\partial t}\int_v \rho.\,\boldsymbol{dv}$ or $\int_s \mathrm{J}.\,ds = \int_v -\frac{\partial \rho}{\partial t}\,\boldsymbol{dv}$

From Gauss Divergence theorem, $\int_s \mathrm{J}.\,ds = \int_v \nabla .\,\mathrm{J}\,dv$

Hence $\int_v \nabla .\,\mathrm{J}\,dv = \int_v -\frac{\partial \rho}{\partial t}\,\boldsymbol{dv}$

Hence the equation of continuity is given as $\boldsymbol{\nabla} .\,\mathbf{J} = -\frac{\partial \rho}{\partial t}$

Maxwell's Equations

(i) Maxwell's First Equation $\left(\textbf{div E} = \frac{\rho}{\varepsilon_0} \ \textbf{ or } \nabla.\mathbf{D} = \rho\right)$

According to **Gauss theorem in electrostatics**, total electric flux passing normally through a closed surface is equal to $\frac{1}{\varepsilon_0}$ times the total charge enclosed within the surface.

i.e., $\int_S E.\,ds = \frac{q}{\varepsilon_0}$ → (1)

If ρ is the volume charge density, then $q = \int_v \rho.\,\boldsymbol{dv}$ → (2) (since $\rho = \frac{q}{v}$ where v is the volume)

Hence, $\int_s E.\,ds = \frac{1}{\varepsilon_0} \times \int_v \rho.\,\boldsymbol{dv}$

By Gauss Divergence Theorem, $\int_S E.\,ds = \int_v (\nabla.\mathbf{E})\boldsymbol{dv}$

Therefore, $\int_v (\nabla.\mathbf{E})\boldsymbol{dv} = \frac{1}{\varepsilon_0} \times \int_v \rho.\,\boldsymbol{dv}$ → (3)

Hence $\nabla.\mathbf{E} = \frac{\rho}{\varepsilon_0}$ → (4) or $\nabla.\boldsymbol{\varepsilon_0}\mathbf{E} = \boldsymbol{\rho}$ → (5)

Since $\varepsilon_0\mathbf{E} = \mathbf{D}$ (electric flux density),

We have $\nabla.\mathbf{D} = \rho$ → (6) . This is Maxwell first equation.

(ii) Maxwell's Second Equation $(\textbf{div B} = \mathbf{0} \textbf{ or } \nabla.\mathbf{B} = \mathbf{0})$

According to **Gauss theorem in magnetism**, number of magnetic lines of force entering the surface is equal to the number of magnetic lines of force leaving the surface. Hence the magnetic flux in that field is zero. i.e., $\int_S \mathbf{B}.\boldsymbol{ds} = 0$ → (1)

By Gauss Divergence Theorem, $\int_S \mathbf{B}.\boldsymbol{ds} = \int_v (\nabla.\mathbf{B})\boldsymbol{dv}$

Since $\int_S \mathbf{B}.\boldsymbol{ds} = 0$, we have $\int_v (\nabla.\mathbf{B})\boldsymbol{dv} = 0$ → (2)

Hence we have, $\nabla.\mathbf{B} = \mathbf{0}$ → (3) . This is Maxwell second equation.

(iii) Maxwell's Third Equation $\left(\textbf{curl E} = -\frac{\partial \mathbf{B}}{\partial \mathbf{t}} \textbf{ or } \nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial \mathbf{t}}\right)$

According to **Faraday's law of electromagnetic induction**, whenever there is a change in magnetic flux, emf is induced which is proportional to negative of rate of change of magnetic flux.

Induced emf $\varepsilon = -\frac{\partial \phi}{\partial \mathbf{t}}$. → (1)

We have, $\phi = \int_S \mathbf{B}.\boldsymbol{ds}$. → (2)

Therefore $\varepsilon = -\frac{\partial}{\partial t}\int_S \mathbf{B}.\boldsymbol{ds}$ → (3)

The line integral of induced electric field intensity **E** around the closed path gives **induced emf,**

$\boldsymbol{\varepsilon} = \int_c \mathbf{E}.\boldsymbol{dl}$ → (4)

Then $\int_c \mathbf{E}.\boldsymbol{dl} = -\frac{\partial}{\partial t}\int_S \mathbf{B}.\boldsymbol{ds}$ → (5)

By Stoke's law, $\int_s (\nabla \text{x } \mathbf{E})\boldsymbol{ds} = \int_c \mathbf{E}.\boldsymbol{dl}$

Hence, $\int_s (\nabla \text{x } \mathbf{E})\boldsymbol{ds} = -\frac{\partial}{\partial t}\int_S \mathbf{B}.\boldsymbol{ds}$ → (6)

Or $\nabla \text{x } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ or Curl $\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ → (7)

This is Maxwell third equation.

Significance: The emf around a closed path is equal to negative rate of change of magnetic flux linked with the path.

(iv) Maxwell's Fourth Equation (curl H = J + $\frac{\partial \mathbf{D}}{\partial t}$ or $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$)

According to Ampere's circuital theorem, the line integral of the magnetic field around any closed path around a current carrying conductor is equal to μ_0 times the current. i.e., $\oint \mathbf{B}.\boldsymbol{dl} = \mu_0 \mathbf{I}$ → (1)

$\mathbf{B} = \mu_0 \mathbf{H}$. Hence $\int_c \mu_0 \mathbf{H} . \boldsymbol{dl} = \mu_0 \mathbf{I}$ → (2)

$\int_c \mathbf{H} . \boldsymbol{dl} = \mathbf{I}$ → (3)

But $\mathbf{I} = \int_s \mathbf{J}. ds$ → (4)

$\int_c \mathbf{H} . \boldsymbol{dl} = \int_s \mathbf{J}. ds$ → (5)

By Stoke's law, $\int_c \mathbf{H}.\boldsymbol{dl} = \int_s (\nabla \times \mathbf{H}) \boldsymbol{ds}$

Hence $\int_s (\nabla \times \mathbf{H}) \boldsymbol{ds} = \int_s \mathbf{J}. ds$ → (6)

Therefore, $(\nabla \times \mathbf{H}) = \mathbf{J}$ → (7) This is Ampere's law.

From vector analysis, the divergence of the curl of any vector is equal to zero. i.e. , div (curl **A**) = 0 i.e., div (curl **H**) = div J or div **J** = 0 (equation of continuity for steady currents)

This is true only for steady currents only and is not applicable for time varying currents (unsteady current).

Maxwell modified this equation by considering the time varying electric fields also. This time varying electric field produces a current called **displacement current ($\mathbf{J_D}$)**.

Therefore, the equation (7) becomes, $(\nabla \times \mathbf{H}) = \mathbf{J} + \mathbf{J_D}$ → (8)

Taking divergence of (8) on both sides, $\nabla . \nabla \times \mathbf{H} = \nabla . (\mathbf{J} + \mathbf{J_D})$

We know, $\nabla . \nabla \times \mathbf{H} = 0$, then $\nabla . (\mathbf{J} + \mathbf{J_D}) = 0$

i.e., $\nabla . \mathbf{J} = - \nabla . \mathbf{J_D}$ → (9)

By equation of continuity, for time varying electric field, $\nabla . \mathbf{J} = -\frac{\partial \rho}{\partial t}$ → (10)

Then $\nabla . \mathbf{J_D} = \frac{\partial \rho}{\partial t}$ → (11)

By Maxwell's first equation, $\nabla . \mathbf{D} = \rho$. Hence $\nabla . \mathbf{J_D} = \frac{\partial}{\partial t}(\nabla . \mathbf{D})$ i.e, $\nabla . \mathbf{J_D} = \nabla . \frac{\partial \mathbf{D}}{\partial t}$

i.e., $\mathbf{J_D} = \frac{\partial \mathbf{D}}{\partial t}$ → (12) Here J_D is the displacement current density.

Therefore, Maxwell's fourth equation can be written as

$(\nabla \times \mathbf{H}) = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$ → (14) (Here $D = \varepsilon_0 \mathbf{E}$)

Significance: The magnetomotive force round a closed path is equal to the conduction current plus displacement current through any surface bounded by the path.

Differential form of Maxwell Equation		Integral form of Maxwell Equation
1. div **D** = ρ	$\nabla \cdot \mathbf{D} = \rho$	1. $\int_s \mathbf{D}. ds = \int_v \rho. dv$
2. div **B** = 0	$\nabla \cdot \mathbf{B} = 0$	2. $\int_s \mathbf{B}. ds = 0$
3. Curl **E** = $-\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	3. $\int_c \mathbf{E}. dl = \int_s -\frac{\partial \mathbf{B}}{\partial t}. ds$
4. Curl **H** = **J** + $\frac{\partial \mathbf{D}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$	4. $\int_c \mathbf{H}. dl = \int_s \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\right). ds$

CONSTITUTIVE EQUATIONS

1. $\mathbf{D} = \varepsilon\, \mathbf{E}$
2. $\mathbf{B} = \mu\, \mathbf{H}$
3. $\mathbf{J} = \sigma\, \mathbf{E}$

where ε, μ and σ are permittivity, permeability and the conductivity of the medium. In free space ε and μ are replaced by ε_0 and μ_0 respectively.

Comparison of Displacement Current with Conduction current

Conduction current	Displacement current
Conduction current is the electric current that flows through a conductor because of an applied potential difference.	Displacement current is an apparent current that is produced by time varying electric field that is set up in a dielectric in a capacitor.
It obeys Ohm's law V = IR	It does not obey Ohm's law.
Conduction current density is represented by $\mathbf{J} = \frac{\mathbf{I}}{A} = \sigma\mathbf{E}$	Displacement current density is represented by $\mathbf{J_D} = \varepsilon \frac{d\mathbf{E}}{dt}$

Electromagnetic Waves in Free Space

Maxwell predicted that accelerated charges generate electric and magnetic disturbances that can travel indefinitely through space.

Electromagnetic waves are waves that are created as a result of vibrations between an electric field and a magnetic field. EM waves include radiowaves, micro waves IR waves, visible rays, UV rays, X-rays and gamma rays. EM waves propagate in free space with the speed of light. They requires no medium for propagation. EM consists of electric field and magnetic field which are mutually perpendicular and are perpendicular to the direction of propagation.

Velocity Of Electromagnetic Waves In Free Space

In free space ε and μ are replaced by ε_0 and μ_0 respectively. Since there are no free charges, ρ = 0; and conduction current J = 0

Maxwell's equation for free spaces can be written as

div $\mathbf{D} = 0$ → (1)

div $\mathbf{B} = 0$ → (2)

curl $\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ → (3)

curl $\mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$ → (4)

Taking curl on both sides of (3), $\nabla \times (\nabla \times \mathbf{E}) = \nabla \times \left(-\frac{\partial \mathbf{B}}{\partial t}\right)$

Hence $\nabla \times (\nabla \times \mathbf{E}) = -\frac{\partial}{\partial t}(\nabla \times \mathbf{B})$

$$\nabla \times (\nabla \times \mathbf{E}) = -\frac{\partial}{\partial t}\left(\nabla \times \mu_0 \mathbf{H}\right)$$

$$= -\mu_0 \frac{\partial}{\partial t}(\nabla \times \mathbf{H}) = -\mu_0 \frac{\partial}{\partial t}\left(\frac{\partial \mathbf{D}}{\partial t}\right) = -\mu_0 \frac{\partial^2 \varepsilon_0 \mathbf{E}}{\partial t^2} = -\mu_0\varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla \times (\nabla \times \mathbf{E}) = -\mu_0\varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad \rightarrow (5)$$

But from vector analysis, $\nabla \times (\nabla \times \mathbf{E}) = \nabla(\nabla\cdot \mathbf{E}) - \nabla^2\mathbf{E}$

From Maxwell's first equation, for free charges div $\mathbf{D} = 0$ or $\nabla\cdot \mathbf{E} = 0$

Then $\nabla \times (\nabla \times \mathbf{E}) = -\nabla^2\mathbf{E}$

From (5), we have $\nabla^2\mathbf{E} = \mu_0\varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$ → (6)

Similarly from Maxwell's fourth equation, we get, $\nabla^2\mathbf{H} = \mu_0\varepsilon_0 \frac{\partial^2 \mathbf{H}}{\partial t^2}$ → (7)

We know that differential equation of wave motion is $\nabla^2 \Psi = \frac{1}{v^2}\frac{\partial^2\Psi}{\partial t^2}$ where V is the velocity of the wave.

Comparing eqn (6) or (7) with differential eqn of wave motion, we get,

$\frac{1}{v^2} = \varepsilon_0\, \mu_0$ and hence velocity $v = \frac{1}{\sqrt{\varepsilon_0\, \mu_0}}$

Thus **E** and **H** propagates as waves with velocity $v = \frac{1}{\sqrt{\varepsilon_0\, \mu_0}}$ in free space.

Putting the values for permittivity of free space $\varepsilon_0 = 8.85\times10^{-12}$ C^2/Nm^2 and permeability at free space, $\mu_0 = 4\pi\times10^{-7}$H/m , we have, $v = \frac{1}{\sqrt{8.85\times10-12\times4\pi\times10-7}} = 2.9974\times10^8$ m/s

This is same as the experimentally determined value of the velocity of light. This coincidence led Maxwell to assume that light itself is an electromagnetic phenomenon.

Poynting's vector and energy flow

When electromagnetic waves are propagated through space, there is an accompanying transfer of energy from the source to the receiving points.

The energy flow of the electromagnetic field can be calculated using the Poynting vector **S**.

S = E × **H** and this vector is in the direction of propagation (i.e., perpendicular to **E** and **H**)and accounts for radiation pressure.SI unit of pointing vector is watt/m^2.

Poynting's Theorem states that the vector **S = E** × **H** at any point gives the rate of energy flow outward through unit area of the surface in a direction normal to the surface..