INTERFERENCE

Principle of superposition

When two or more light wave travels through a medium, the resultant displacement at any point is the vector sum of the displacement due to each wave. This is the principle of superposition of waves which forms the basis of interference.

Let y_1 be the displacement due to one wave at any instant and let y_2 be the displacement of the other wave at the same instant. Then resultant displacement is $y = y_1 \pm y_2$

Interference of Light

When two or more light waves of the same frequency and amplitude with a constant phase difference travel in the same direction, superimpose upon each other, the resultant intensity of light is not distributed uniformly in space. This non uniform distribution of the light intensity due to the superposition of light waves is called interference.

At some points, the intensity is a maximum and the interference at these points is called constructive interference. Here, the crest of one wave falls on the crest of another wave and trough of one wave falls on the trough of another wave.

At some points, the intensity is minimum and the interference at these points is called destructive interference. Here the crest of one wave falls on the trough of the other and the trough of one wave falls on the crest of another wave.

So when two or more light waves interfere, we get alternate dark and bright bands of equal width. These bands are called interference fringes. This phenomenon of interference is in accordance with the law of conservation of energy.

Condition for constructive interference: Path difference $\Delta = n\lambda$ where n = 0, 1,2, 3,……

Condition for destructive interference: Path difference $\Delta = (2n+1)\frac{\lambda}{2}$ where n = 0, 1,2, 3,……

Conditions for sustained interference pattern

By sustained interference, we mean that the nature and order of interference at a point of the medium should remain unchanged with time. The two conditions for this to happen are

(i) The interfering waves must be coherent.

(ii) The two sources must have zero phase difference or must have constant phase difference.

(iii) The path difference between the waves must be less than or equal to coherence length.

Coherent sources

Two waves are said to be coherent,

(i) If they emit light waves with a constant phase difference between them

(ii) If they have same frequency (or wavelength) and same amplitude.

(iii) Eg: Two slits illuminated by a monochromatic source.

Incoherent Sources

Two or more sources of light which do not emit light waves with a constant phase difference are called incoherent sources. Two independent sources can never be mutually coherent (are incoherent). A source of light consists of a large number of atoms and the emission of light pulses from various atoms is random and hence there is no constant phase relationship between two pulses.

Coherence

Coherence is an important property associated with the idea of interference. Coherence means that two or more electromagnetic waves are in a fixed and predictable phase relationship to each other. In general, the phase between two electromagnetic waves can vary from point to point or change from time to time. Thus there are two independent concepts of coherence namely spatial coherence and temporal coherence.

Temporal coherence: If two waves have same propagation characteristics at two different instant of time, they are said to be temporally coherent.

Spatial coherence: If two waves have identical propagation characteristics at different points in space, the waves are said to be spatially coherent.

Coherence length : The average length of a wave packet from a source of light is coherence length.

Coherence length $l = \frac{\lambda^2}{\Delta\lambda}$ where λ is the mean wavelength of the light and Δλ is the wavelength spread.

Coherence time: The time taken by a wave train of length l_c to pass a given point is called coherence time.

$\tau = \frac{l_c}{c}$

Phase difference and path difference

Phase difference (ϕ) = $\frac{2\pi}{\lambda}$ × path difference (x)

Types of interference

It is of two types.

1. Interference produced by the **division of wavefront**. Here, incident wavefront is divided into two parts by making use of principle of reflection or refraction. Then the two parts are made to reunite to produce interference fringes. Eg: in Young's double slit experiment.
2. Interference produced by the **division of amplitude**. The amplitude (intensity) of the incident light is divided into two parts either by partial reflection or refraction. These light waves with divided amplitude reinforce after travelling different distances and produce interference. Example: Newton's rings.

Optical path

The distance travelled by light through a medium is optical path.

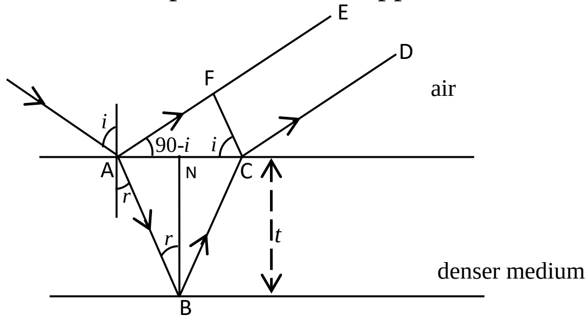
Optical path = geometrical path × μ where μ is refractive index. For air, μ = 1

Interference in thin films – Reflected system

We see beautiful colours on oil films or soap bubbles. This is due to interference phenomenon involving multiple reflections. Here the method involved is interference by division of amplitude. The light reflected from the upper and lower surfaces of a thin film interferes and interference patterns are produced.

Interference due to reflected light

Consider a thin film of thickness 't' and refractive index μ. Let light from a monochromatic source be incident on the surface of the film. Reflections take place from the upper and lower surface of the film.



Optical path difference between two beams is (AB + BC) in film - AF in air

Since AB = BC, optical path = 2AB × μ − AF

Optical Path (Δ) = 2μAB − AF → (1)

Considering Δ ABN, $\cos r = \frac{BN}{AB} = \frac{t}{AB}$

So, $AB = \frac{t}{\cos r}$ → (2) and $AN = t\ \tan r$ → (3)

In Δ AFC, $AF = AC \sin i = 2AN \sin i$

Sub eqn (3), $AF = 2t\ \tan r \sin i$

By Snell's law, $\mu = \frac{\sin i}{\sin r}$; $\sin i = \mu \times \sin r$

Hence $AF = 2t\ \tan r \times \mu\ \sin r$

$$AF = 2t \frac{\sin r}{\cos r} \times \mu\ \sin r$$

$$= 2\mu t \times \frac{\sin^2 r}{\cos r} \qquad \rightarrow (4)$$

Substituting eqn (2) and (4) in eqn (1), we get

$$\text{optical path } (\Delta) = \frac{2\mu t}{\cos r} - 2\mu t \times \frac{\sin^2 r}{\cos r} = \frac{2\mu t}{\cos r}(1 - \sin^2 r) = \frac{2\mu t}{\cos r}(\cos^2 r)$$

Optical Path (Δ) = $2\mu t \cos r$. This is known as **Cosine's law**.

When light is reflected from the surface of an optically denser medium, like the air film interface, then **the reflected rays undergoes a path change of $\frac{\lambda}{2}$**. (This is in the case of reflection only.)

Then the actual path difference is $\Delta = 2\mu t \cos r + \frac{\lambda}{2}$

Condition for constructive interference

$2\mu t\cos r + \frac{\lambda}{2} = n\lambda$ where n = 1,2,3…….

i.e., $\mathbf{2\mu t\cos r = (2n - 1)\frac{\lambda}{2}}$

Condition for destructive interference

$2\mu t\cos r + \frac{\lambda}{2} = (2n + 1)\frac{\lambda}{2}$ where n = 1,2,3…….

i.e., $\mathbf{2\mu t\cos r = n\lambda}$

Applications of cosine law are colours of thin films, nonreflecting film or antireflection coating, interference filter, interference produced by a wedge shaped thin films, Newton's ring

Colour of thin films

When a beam of white light is incident normally on a thin film, different colours are seen. Cosine law provides an explanation for this. The condition of destructive interference is $2\mu t\cos r = n\lambda$. The portion of the film which satisfies this condition will not include the colour of that wavelength. The mixture of the remaining colours will be seen at that point. For example, if blue colour satisfies this condition, blue colour will be absent and a combination of other colours will be seen at that region.

⇒ If the thickness of the film varies continuously in the case of oil film, the colour of the film continuously changes.

⇒ If we observe the film from different positions, the colour of the film will be varying as the angle of refraction *r* changes.

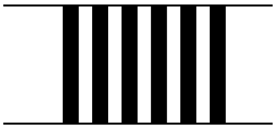
⇒ If the thickness of the film *t* is very small (t = 0), then destructive interference occur due to the additional path difference of $\frac{\lambda}{2}$ and film appears dark.

⇒ If '*t*' is very large, almost all the colours will undergo constructive interference and white light is produced. **Only a thin film of a few wavelengths thick, alone will show different colours**.

⇒ The condition of brightness and darkness in the reflected system and transmitted system are opposite to each other.

Air Wedge

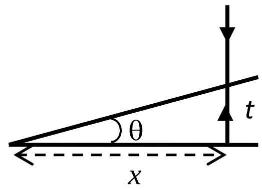
A wedge shaped film is constructed using two glass plates with one end placed in contact and the other end separated by a thin paper or wire of thickness '*t*'. A wedge shaped air film is formed between them. When the film is illuminated normally by monochromatic light, interference occurs between the rays reflected form the top and bottom surfaces of the film. As a result, a large number of equidistant parallel dark and bright bands are observed.



The spacing between two consecutive dark or bright fringes is called band width.

Interference produced by wedge shaped films

When the film is illuminated normally by monochromatic light, interference occurs between the rays reflected form the top and bottom surfaces of the film.



Let 't' be the thickness of film at a distance x from the edge.

We have path difference between two rays $\Delta = 2\mu t \cos r$

For a normal incidence, $r = 0$ (i.e., cos 0 = 1) ,

Then path difference between the rays is $\Delta = 2\mu t$ → (1)

From figure, $t = x \tan\theta$ → (2)

Putting (2) in (1), we get $\Delta = 2\mu \times x \tan\theta$

The condition for destructive interference in thin film is $2\mu\, x \tan\theta = n\lambda$ where n = 0,1,2,3 ……

or $$x = \frac{n\lambda}{2\mu \tan\theta} \quad \rightarrow (3)$$

If x_1 is the distance of the n^{th} dark band from the edge and x_2 is the $(n+m)^{th}$ dark band,

then $x_1 = \frac{n\lambda}{2\mu \tan\theta}$ and $x_2 = \frac{(n+m)\lambda}{2\mu \tan\theta}$

Then band width $$\beta = \frac{x_2 - x_1}{m} = \frac{\frac{(n+m)\lambda}{2\mu \tan\theta} - \frac{n\lambda}{2\mu \tan\theta}}{m} = \frac{m\lambda}{2\mu\, m \tan\theta}$$

Band width $$\beta = \frac{\lambda}{2\mu \tan\theta} \quad \rightarrow (4)$$

Since for air film, $\mu = 1$, $$\boldsymbol{\beta = \frac{\lambda}{2 \tan\theta}} \quad \rightarrow (5)$$

Since angle of wedge (θ) is too small, then $\tan\theta = \theta$, then $$\boldsymbol{\beta = \frac{\lambda}{2\,\theta}} \quad \rightarrow (6)$$

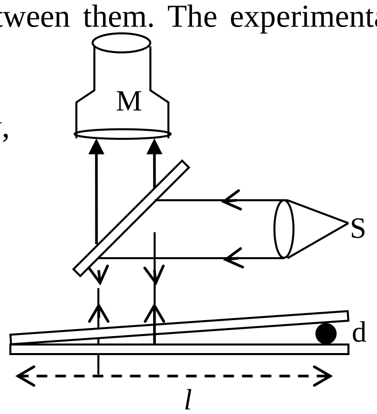
The same relation holds good for bright bands also. Since the locus of all points having same thickness of the film being a straight line, we get straight line fringes.

Applications

Diameter of a thin wire

A thin wire is wound near one end of a plane glass plate. It is kept over another plane glass plate so that they touch along one edge and a wedge shaped air film is formed between them. The experimental arrangement for determining the diameter of the wire is as shown in figure.

When a parallel beam of monochromatic light is made to fall normally, we can observe a large number of straight parallel alternate bright and dark fringes using a microscope.



Let 'd' be the diameter of the wire and 'l' be the length of the wedge, then,

$$\tan\theta = \frac{d}{l} \quad \rightarrow (1)$$

Band width $\beta = \frac{\lambda}{2\tan\theta} = \frac{\lambda}{2\,{}^{d}/_{l}} = \frac{l\,\lambda}{2\,d}$ → (2)

Diameter of thin wire, $\mathbf{d} = \frac{\boldsymbol{l}\,\boldsymbol{\lambda}}{\mathbf{2}\,\boldsymbol{\beta}}$ → (3)

Testing of planeness of surfaces

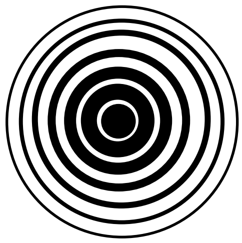
Air wedge experiment can be used to test the optical planeness of surfaces. The surface to be tested is used as one of the plates in air wedge experiment. The other will be a standard plane surface. If the surface is optically plane, then straight fringes of equal thickness are observed. If the fringes are non uniform and distorted, the given surface is not optically plane.

A surface is said to be optically plane, if it is flat up to $\frac{1}{10}$th of the wavelength of the light used. With the air wedge experiment, a flatness of a surface up to $\frac{1}{10}$th of the wavelength of the light used can be determined.

Newton's Rings

When a plano convex lens is placed on a plane glass plate, with its convex surface touching the plate, an air film of gradually increasing thickness is formed between the two. Light reflected from the top and bottom surfaces of the air film between lens and the glass plate interferes to produce interference pattern. The thickness of the film is zero at the point of contact at O and increases radially outwards.

The thickness of the air film will be constant over a circle and the pattern consists of concentric bright and dark rings. These rings are called **Newtons rings**. Newton's rings consist of a set of alternate dark and bright rings with central spot dark.

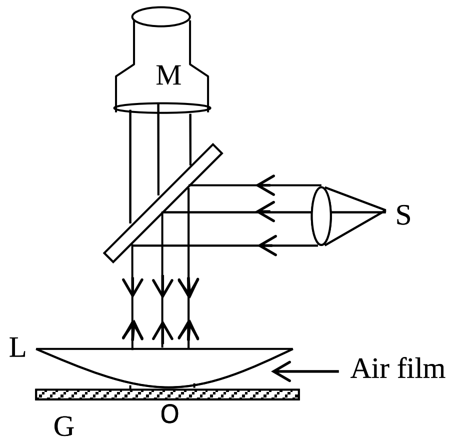


From the interference pattern, it can be seen that the centre spot is dark.

At the centre, i.e., at the point of contact, the thickness of the air film is zero. But the beam of light gets reflected from the upper surface of the glass plate G and undergoes an additional path difference of $\frac{\lambda}{2}$. This results in destructive interference. Hence the **central spot is dark**.

Experimental arrangement for Newton's rings:

Light from a monochromatic source S is allowed to fall on a glass plate kept at 45° to the incident beam. This beam is reflected normally on to the planoconvex lens L placed on a glass plate G.

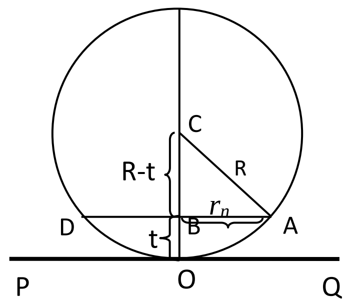


Light rays reflected from top and bottom surface of the air film interferes. Circular bright and dark fringes can be observed by looking through a travelling microscope focused on to the system.

The locus of points having the same thickness of the air film falls on a circle. Therefore fringes take the form of concentric rings.

Determination of radius of n^th ring

Let DOA is the lens placed on the glass plate PQ. DOA is a part of the spherical surface with centre C. Let R be the radius of curvature of the plano convex lens and r_n be the radius of the nth ring. Let λ be the wavelength of the light used.



The condition for minimum intensity in thin interference film is

$2\mu t\cos r = n\lambda \quad \rightarrow (1)$

Since light is falling normally $r = 0$, and $\cos r = 1$ and for air film $\mu = 1$

So equation (1) becomes $2t = n\lambda \quad \rightarrow (2)$

In figure, from ΔABC,

$$R^2 = r_n^2 + (R - t)^2 = r_n^2 + R^2 - 2Rt + t^2$$

Since t is very small compared to R, $R^2 = r_n^2 + R^2 - 2Rt$

Or $r_n^2 = 2Rt$ i.e., $t = \frac{r_n^2}{2R} \quad \rightarrow (3)$

Substituting (3) in (2), $2 \times \frac{r_n^2}{2R} = n\lambda$

Or $\boldsymbol{r_n = \sqrt{nR\lambda}}$ where n = 0,1,2,3….. for dark ring.

Similarly, for bright ring, $r_n = \sqrt{\frac{(2n\pm1)R\lambda}{2}}$ where n is the order of the ring.

Determination of wavelength of monochromatic light used

Let D_n and D_{n+m} are diameters of n^{th} and $(n+m)^{th}$ dark rings respectively.

We have $r_n = \sqrt{nR\lambda}$ where R is the radius of curvature of the plano convex lens

Here, $D_n = 2r_n$ or $D_n^2 = 4r_n^2 = 4nR\lambda$

Likewise, $D_{(n+m)}^2 = 4(n+m)R\lambda$

Then $D_{(n+m)}^2 - D_n^2 = 4mR\lambda$

Wavelength of the monochromatic light used $\lambda = \frac{D_{(n+m)}^2 - D_n^2}{4mR}$

Determination of refractive index of a liquid

If a liquid of refractive index μ is introduced between the lens and the plate, path difference = 2μt. Let d and d_{n+m} are diameters of n^{th} and $(n+m)^{th}$ dark rings respectively.

The condition for minimum intensity in thin interference film is 2μt = $n\lambda$ and $t = \frac{r_n'^2}{2R}$

So, $2\mu\frac{r_n'^2}{2R} = n\lambda$ or $r_n'^2 = \frac{nR\lambda}{\mu}$

$d_n^2 = \frac{4nR\lambda}{\mu}$ and $d_{(n+m)}^2 = \frac{4(n+m)R\lambda}{\mu}$

Then $d_{(n+m)}^2 - d_n^2 = \frac{4mR\lambda}{\mu}$

Hence, $\frac{D_{(n+m)}^2 - D_n^2}{d_{(n+m)}^2 - Dd_n^2} = \frac{4mR\lambda}{\frac{4mR\lambda}{\mu}}$ or $\boldsymbol{\mu = \frac{D_{(n+m)}^2 - D_n^2}{d_{(n+m)}^2 - d_n^2}}$

Also, $\boldsymbol{\mu = \frac{r_m^2}{r_m'^2} = \frac{D_m^2}{d_m^2}}$. Here, r_m and r'_m represent the radius of same dark ring without and with liquid film.

Since μ for liquid is greater than 1, $r'_m < r_m$. Hence we conclude that the rings contract with introduction of a liquid.

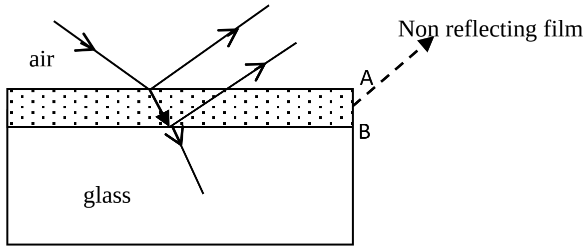
Newton's rings in white light

⇒ With white light, few colour rings appear near the point of contact. The radius of the rings depends on wavelength. So bright rings of the same order for several colours will be seen with different radii. A few coloured fringes are observed around the point of contact. Due to the overlapping of the higher order of fringes, the pattern disappears in the outer portion.

⇒ In transmitted light, there is a central spot which is bright and rings are less distinct.

Antireflection coating or nonreflecting films

One of the important applications of thin film interference lies in reducing the reflectivity of lens surfaces. In many optical instruments like telescope, range finders etc, the incident light has to undergo reflections from many surfaces. When the light reflects back from each surface, there occurs considerable loss in the intensity of the transmitted light. This loss can be reduced by antireflection coatings.



Anti reflection coating

⇒ Antireflection coating or non reflecting films are a type of coating applied to the surface of lenses, and other optical instruments to reduce reflection.

⇒ It is a transparent dielectric material whose refractive index lies between refractive index of air and that of glass.

It should be equal to square root of the refractive index of the glass used. Magnesium fluoride or cryolite is the most widely used material for antireflection coating. They are usually deposited by vacuum evaporation technique.

⇒ The incident light is reflected from the upper and lower surface of thin film with a phase change of π. If the thickness of the film is such that both the reflected rays are in opposite phase , they cancels each other and the intensity of transmitted light is increased.

DIFFRACTION OF LIGHT

When light falls on obstacles or small apertures whose size is comparable with the wavelength of light, there is a departure from straight lines propagation and the light bends round the corners of the obstacles or apertures and enters in the geometrical shadow. This bending of light is called diffraction of light.

Since the wavelength of sound waves is greater than that of light waves and is comparable with the size of the obstacles we come across in our daily life, the diffraction of sound waves are more evident in daily life than that of the light waves.

Difference between interference and diffraction

Interference	Diffraction
Interaction takes place between two wavefronts originating from two coherent sources.	Interaction takes place between secondary wavelets originating from different points of the same wavefront.
The intensity of all positions of maxima are of same intensity.	The intensity of all positions of maxima are of varying intensity.
Fringewidth in interference pattern are equal.	Fringewidth in diffraction pattern are not equal.
It is absolutely dark in the region of minimum intensity.	The minimum intensity regions are not perfectly dark in the case of interference pattern.

Types Of Diffraction

The diffraction phenomenon is divided into two categories.

(i) Fresnel diffraction

(ii) Fraunhofer diffraction

Fresnel diffraction	**Fraunhofer diffraction**
The source of light or the screen or both at finite distances from obstacle or aperture causing the diffraction.	The source of light and the screen are at infinite distances from the obstacle or aperture which causes the diffraction.
As the sources is at finite distance, the incident wavefront on the obstacle is either spherical or cylindrical.	As the source is at infinity, the waves reaching the obstacles are plane wavefronts.
Lenses are not used for collimation and focussing	Two convex lenses are used for collimation and focussing.

Plane Transmission Grating

An arrangement consisting of large number of parallel slits of equal width and separated from one another by equal opaque spaces is called diffraction grating or transmission grating.

It is constructed by ruling equidistant parallel lines with fine diamond point on an optically transparent sheet of material. The ruled lines are opaque region while the spaces in between the lines are transparent region. Such a grating is called a plane transmission grating. There will be about 6000 rulings per centimeter. So the width of a slit is of the order of wavelength of light.

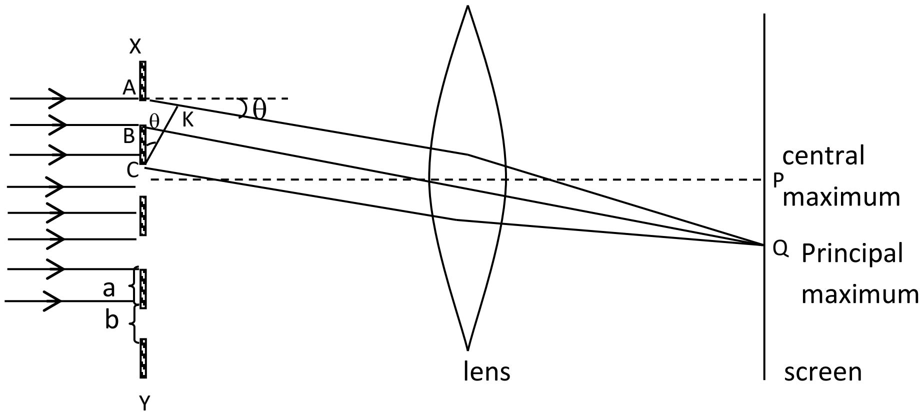
A grating can be made as follows. A thin layer of gelatin solution is poured over the surface of a ruled grating and is allowed to harden. When stripped from the grating, the gelatin film retains an impression of the ruling of the original grating. The film is then mounted between two plane glass plates to get a transmission grating.

Theory of diffraction grating

Consider a plane transmission grating XY. Here AB represents a slit of width *a* and BC represents an opaque portion of width *b*. The distance ($a+b$) is called **grating element**.

Let a plane wavefront be incident normally on grating. Each point on the slit will send out secondary wavelets in all directions.

- Most of the secondary wavelets proceeding from the slit will continue to travel in the direction of the incident light. When focused by a convex lens, they will give a line of maximum intensity on a screen placed at the focal plane of the lens. This is called the central maximum at P.



- Secondly we consider two diffracted waves from the adjacent slits with angle of diffraction is ‘θ’. Here AK is the path difference between the two waves.

since, $\sin\theta = \frac{AK}{AC}$

$AK = AC \sin\theta = (a+b)\sin\theta$

If $\mathbf{(a+b)\sin\theta = n\lambda}$ → (1) where n = 1,2,3,…..

These wavelets reinforce. In this case, all the wavelets of wavelength λ originating from the various corresponding points reinforce to give a principal maximum at Q at an angle θ with the incident direction.

Also by putting n = 1,2,3,….., we can obtain different angles of diffraction corresponding to the maxima of different orders.

Exactly similar maxima are obtained above P due to the light diffracted upwards.

Thus on either side of central maximum P, diffraction maxima of different orders are obtained symmetrically.

- Let N be the number of lines in one meter of grating,

then $N(a+b) = 1$ or $N = \frac{1}{(a+b)}$

So equation (1) becomes, $\frac{1}{N} \sin\theta = n\lambda$

$\mathbf{\sin\theta = N}\boldsymbol{n\lambda}$ where θ is angle of diffraction, λ is wavelength, n is the order of the spectrum, N is the number of lines per unit length.

This is known as **grating equation**.

For first order maxima, $n = 1$, $(a+b)\sin\theta = \lambda$

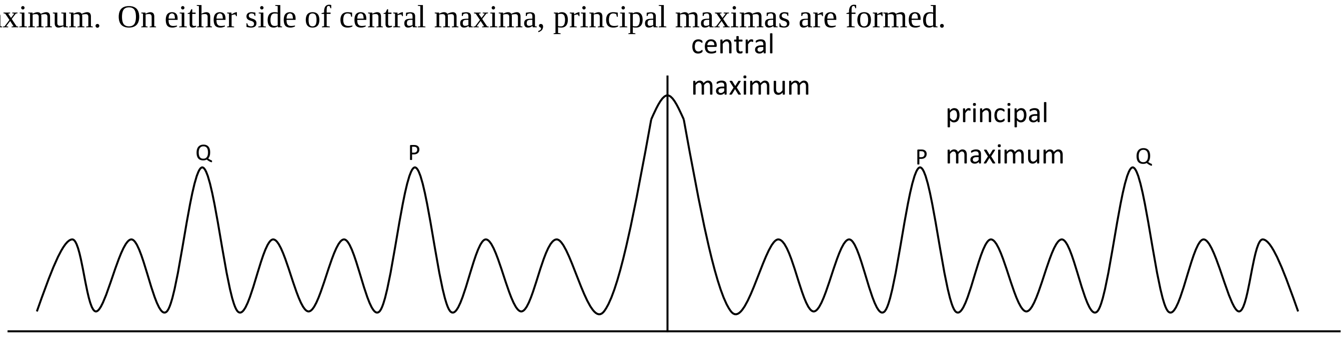
For second order maxima, $n = 2$, $(a+b)\sin\theta = 2\lambda$

If the path difference between the wave diffracted from the consecutive slit, $(a+b)\sin\theta = n\lambda$, then we get principal maximas on either side of central maxima. This law is called grating law.

The angle θ cannot exceed 90°. Hence, the number of orders possible is limited by the value of N.

On increasing the number of lines per cm, correspondingly the grating element (a+b) decreases and hence N increases. As a result, the angle of diffraction θ increases for a given order. This results in a less number of spectra separated by large angles or an increased dispersive power.

The variation of intensity with angular diffraction is given below. The intensity of central image is maximum. On either side of central maxima, principal maximas are formed.



Comparison of Prism Spectra and Grating Spectra

Prism Spectera	Grating Spectra
Prism spectrum is formed by dispersion of light.	Grating spectrum is due to diffraction
The prism forms only one spectrum.	The grating forms a number of spectra of different orders on each side of the central image.
In prism spectra, the angle of deviation is least for red and greatest for violet. Hence the spectral colours are in the order from red to violet.	In grating spectra, the angle of deviation is least for violet and greatest fro red and hence the spectral colours are in the order from violet to red.
The prism spectrum is bright, since all the incident light is distributed only in a single spectrum.	The grating spectra is fainter because most of the intensity goes to zero order maximum and rest is distributed among the spectra of different orders.
The resolving power of a prism is small and hence the prism spectrum does not show the fine structure of spectral lines.	The resolving power of the grating is large and hence the grating spectrum show the fine structure of spectral lines.
The spectral lines in the prism spectra are curved, being convex towards the red end.	The spectral lines in grating spectra are almost straight.

Resolving Power

When two point sources are very close together, the two diffraction patterns produced by each of them may overlap. Hence it may be difficult to distinguish them as separate. To see the two objects or two spectral lines which are very close together, optical instruments like telescopes, microscopes, prisms, grating etc are employed. The minimum distance between two points on the object so that their images are just seen as separate from each other is called the **limit of resolution** of the optical instrument. It can also be defined in terms of angle θ subtended at the objective of the instrument by the two points under consideration.

The ability of an instrument to discriminate wavelengths λ and $\lambda + d\lambda$ is expressed by the ratio $\frac{\lambda}{d\lambda}$ which is called the resolving power R of the instrument. $R = \frac{\lambda}{d\lambda}$

Rayleigh's criterion for resolution of grating

According to Rayleigh criterion, two spectral lines of wavelength λ and $\lambda+d\lambda$ can be identified as two, if the principal maximum due to one falls exactly on the first minimum of the other.

If they lie closer than this, they merge into one image and cannot be resolved.

The minimum angle that the two points on the object subtend at the optical centre of the lens so that they may be resolved is θ (limit of resolution). $\boldsymbol{\theta = \frac{1.22\lambda}{b}}$.

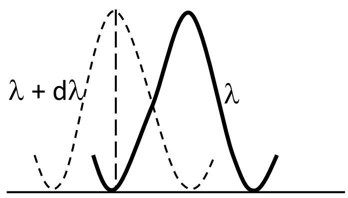
The resolving power is defined as inverse of the limit of resolution $R = \frac{1}{\theta} = \frac{\mathbf{b}}{\mathbf{1.22\lambda}}$ **.**

Resolving power of grating

The resolving power of a grating is the ability to show two neighboring spectral lines in a spectrum as separate.

If λ and $\lambda+d\lambda$ are the wavelengths of two neighboring spectral lines, then the resolving power is the ratio of λ and $d\lambda$. i.e., Resolving Power $= \frac{\lambda}{d\lambda}$

According to Rayleigh criterion, two spectral lines of wavelength λ and $\lambda+d\lambda$ will appear resolved if the principal maximum due to one falls exactly on the first minimum of the other.



Hence, if we are looking at the n^{th} order spectrum, grating equation for principal maximum for wavelength λ is $(a+b)\sin\theta = n\lambda$

Now for first minimum after the principal maximum for wavelength λ is

$(a+b)\sin\theta = n\lambda + \frac{\lambda}{N}$ $\rightarrow$ (1) where $\frac{\lambda}{N}$ is the extra path difference.

For n^{th} principal maximum for wavelength $(\lambda+d\lambda)$, $(a+b)\sin\theta = n(\lambda+d\lambda)$ $\rightarrow$ (2)

From (1) and (2), $n\lambda + \frac{\lambda}{N} = n(\lambda+d\lambda)$ $\rightarrow$ (2)

or $n\,d\lambda = \frac{\lambda}{N}$ where N is total number of lines on grating.

Hence, **Resolving power =** $\boldsymbol{\frac{\lambda}{d\lambda} = n\,N}$ where n is order of spectrum and N is total number of lines on grating. If N is large, resolving power of grating is large.

Dispersive power of grating

Dispersive power of grating is defined as the ratio of change in angle of diffraction to the change in wavelength. It is a measure of angular separation between the lines of a spectrum.

Dispersive power $= \boldsymbol{\frac{d\theta}{d\lambda} = \frac{n\,N}{\cos\theta}}$

From the condition of n^{th} order maxima, $(a+b)\sin\theta = n\lambda$

Differentiating w.r.t. λ, $(a+b)\cos\theta \times \frac{d\theta}{d\lambda} = n \times 1$

i.e., $\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} = \frac{n\,N}{\cos\theta}$. Here also as N increases (increasing the number of lines per cm of the grating), dispersive power also increases.

$\Rightarrow$ The dispersive power gives an idea about the angular separation between the two spectral lines whereas the resolving power tells about the ability of the instrument to show nearby spectral lines as separate ones.