GRAPH THEORY AND COMBINATORICS

Module II

Euler graphs, Hamiltonian paths and circuits, Dirac's theorem for Hamiltonicity, Travelling salesman problem. Directed graphs – types of digraphs, Digraphs and binary relation

Module - II Theorems

Theorem 1:

A connected graph G is an Euler graph if and only if all vertices of G are of even degree

Theorem 2:

In a connected grap G with exactly 2K odd vertices there exist K edge-disjoint subgraphs such that they together contain all edges of G and that each is a unicursal graph.

Theorem 3:

In a complete graph with n vertices there are $(n-1)/2$ edge-disjoint hamiltonian circuits, if n is an odd number ≥ 3.

Theorem 4: (Dirac's Theorem for Hamiltonian)

A sufficient condition for a simple graph G to have a hamiltonian circuit is that the degree of every vertex in G be atleast $n/2$ where n in the number of vertices in G.

Theorem 5:

In digraph G the sum of all in-degrees is equal to the sum of all out-degrees, each sum being equal to the number of edges in G.

$$\sum_{i=1}^{n} d^{+}(v_i) = \sum_{i=1}^{n} d^{-}(v_i)$$

Euler graph: (Eulerian):

If some closed walk in a graph contains all the edges of the graph then the walk is called an Euler line and the graph is an Euler graph.

Euler line: A closed walk running through every edge of graph G exactly once.

Euler Trail: A trail that traverses every edges of G is called as Euler trail.

Euler Tour: A tour of G is a closed walk that traverses every edge of G atleast once

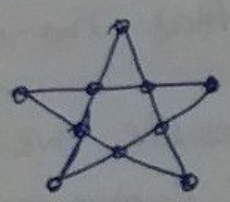
An Euler tour is a tour which traverse each edge exactly once.

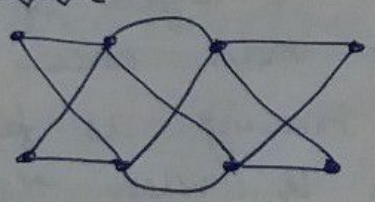
① Euler graph is always connected because it contains Euler line

② Euler graphs don't have isolated vertices.

Example:

Euler graph

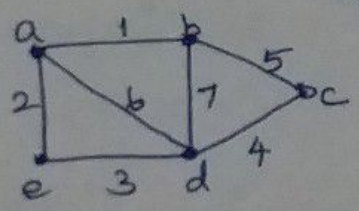




Note:

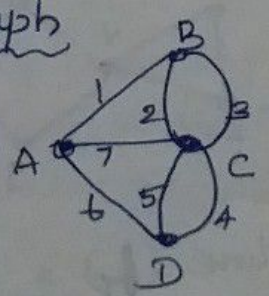
* A closed walk with all edges are included exactly one
* All vertices degree are even.

Non-Eulerian graph



a 2e 3d 4c 5b 1a 6d 7b

Starting & ending vertices are not same (Not a closed walk)



A1B2C5D4C7A

Closed walk, But edges 3 & are not covered.

Konigsberg Bridge problem

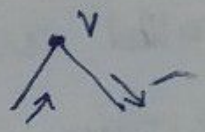
Theorem 1:

A given connected graph G is an Euler graph if and only if all vertices of G are even degree.

Proof:

G is an Euler graph therefore, It contains the Euler line.

We observe that every time the walk meets a vertex V twice



(ie) we enteeeds & exit from V

Therefore, We entee & exit in all vertices in the graph G including starting vertex & ending vertex

Thus, G is an Euler graph, the degree of every vertex is even.

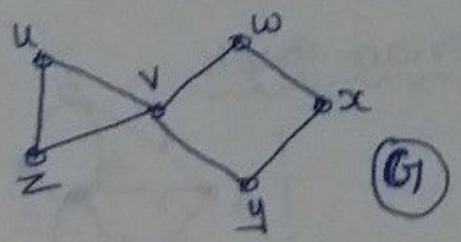
Proof with sufficient condition:

Conversely, Assume that all vertices of G are of even degree.

(1) Construct a walk starting at any vertex u & going through the edges of G such that no edge is traced more than once

(2) Continue tracing as far as possible. We shall eventually reach u when the tracing comes to an e

Ex:



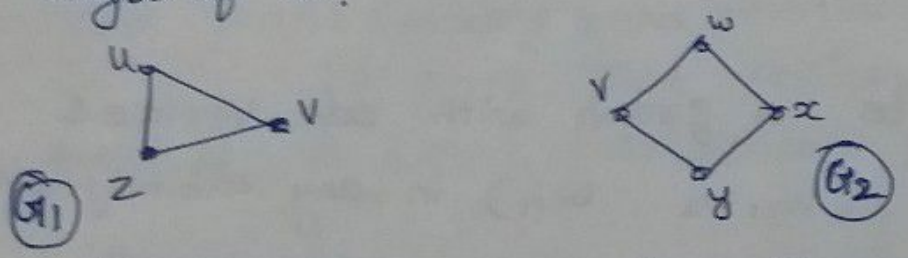
Euler line (h) = u-v-w-x-y-z-u

This closed walk (h) includes all the edges in G.

∴ G is an Euler graph

Suppose, A closed walk $(h_1) = u-v-z-u$ formed. It doesn't contain all the edges of G.

① Remove edges in h_1 from G and obtain a subgraph G_1. G_2 is formed with remaining edges of G.



$h_1 = u-v-z-u$ $\qquad$ $h_2 = v-w-x-y-v$

②. Moreover, G_1 must touch G_2 atleast one Vertex v because G is connected graph.

③ G_1 & G_2 are combined to form a new walk which starts and ends at vertex u.

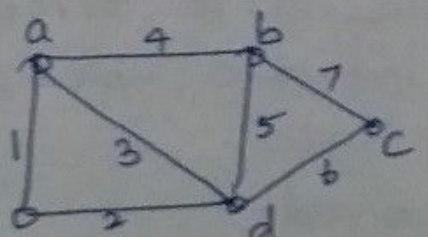
This process repeated until we obtain a closed walk that traverses all the edges of G. Thus G is Euler graph. Hence Theorem proved

Unicursal graph : (or) open Euler line

An open walk that includes all edges of a graph without retracing any edge is called unicursal graph or open Euler line.

Unicursal line → An openwalk by connecting all edges in G. A graph that has a unicursal line is called a unicursal graph.

Ex:



Unicursal line } : a1c2d3a4b5d6e7b

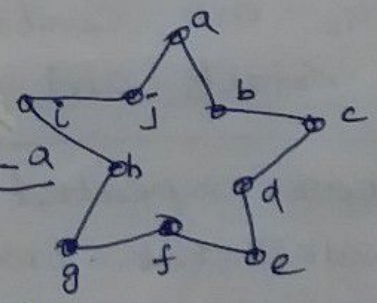
Theorem 2:

In a connected graph G with exactly 2K odd vertices, there exist K edge-disjoint subgraphs such that they together contain all edges of G and that each is unicursal graph.

Proof: Let G be a graph with odd vertices $(v_1, v_2 \ldots v_n)$ $(w_1, w_2 \ldots w_n)$ in any arbitary order.

Add the K edges to G between the vertex pairs (v_1, w_1) (v_2, w_2) ... (v_n, w_n) to form a new graph G'. Since every vertex of G' is of even degree, G' has an Euler line L

Euler line: (L) : a-b-c-d-e-f-g-h-i-j-a



If K edges removed from L then L will be split into K walks. each walk is a unicursal line

L = a // b-c-d-e-f-g-h-i-j-a
Remove this edge.

1 Unicursal line of L (L_1): b-c-d-e-f // g-h-i-j-a
Remove this edge

2 unicursal lines of L_1 (L_2): b-c-d-e-f
g-h-i-j-a

The first removal will leave a single unicursal line. the second removal will split that into two unicursal lines. and each successive removal will split a unicursal line into two unicursal lines until there are K unicursal lines

Hence, theorem proved.

Hamiltonian circuit (or) cycle:

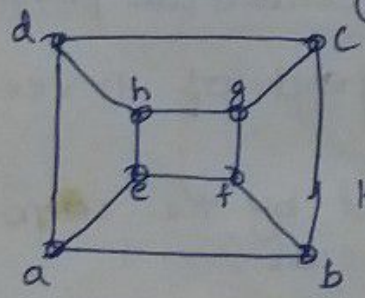
It is a closed walk, that traverses every vertex of G exactly once except starting & ending vertex.

Hamiltonian path: (starting & end vertices are not same)

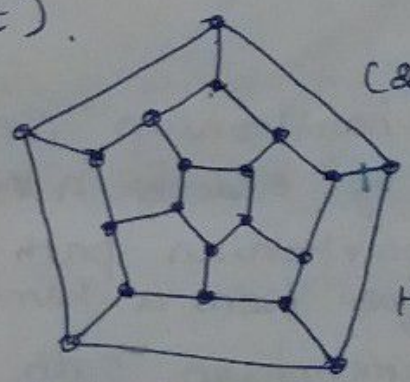
A path that contains every vertex of a graph G is called hamiltonian path of G.

Hamiltonian :

A graph is hamiltonian if it contains a hamiltonian cycle (circuit).



G_1 Hamiltonian



(20 edges)

G_2 Hamiltonian

Hamiltonian circuit of G_1 = a–b–f–g–c–d–h–e–a

Hamiltonian path of G_1 = a–b–f–g–c–d–h–e

= c–d–h–e–a–b–f–g...

.... etc.

Number of Hamiltonian circuits in a graph:

A graph may contain more than one hamiltonian circuit.

* The number of edge disjoint hamiltonian circuit in a complete graph ~~with~~ has $\boxed{\frac{n-1}{2} \text{ circuits}}$ where n is odd number

Theorem 3:

In a completed graph with n vertices there are $(n-1)/2$ edge disjoint hamiltonian circuits, If n is an odd number ≥ 3.

Proof:

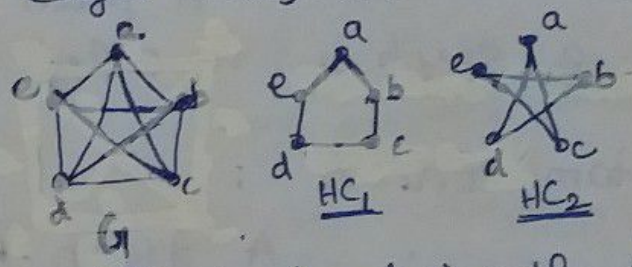
Consider a graph G of n vertices and has $\frac{n(n-1)}{2}$ edges. Consider a hamiltonian circuit in G consists of n edges.

∴ The number of edge disjoint hamiltonian circuits in G cannot exceed $(n-1)/2$

Hence, there are $(n-1)/2$ edge disjoint hamiltonian circuit when n is odd.

Hence theorem proved.

In G ⇒ $n = 5 \Rightarrow \frac{5(n-1)}{2} = \frac{5(4)}{2}$
$= 10$ edges.



$n = 5 = \frac{5-1}{2} = 2$ edge disjoint hamiltonian circuits are possible.

Note:

① A hamiltonian circuit in a graph of n vertices consists of exactly n edges.

② Hamiltonian path is obtained by the removal of one edge from a hamiltonian circuit.

③ Hamiltonian path is a subgraph of a hamiltonian circuit.

④ Every path that has a hamiltonian circuit also has a hamiltonian path.

⑤ In a connected graph with n vertices the length of the hamiltonian path is $n-1$

⑥ Hamiltonian circuit is analysed only in simple graphs. So, general graphs may be made as simple graph by removing self loops & parallel edges before looking for a hamiltonian circuit.

⑦ cycle C_n is a hamiltonian

⑧ Complete graph K_n is a hamiltonian.

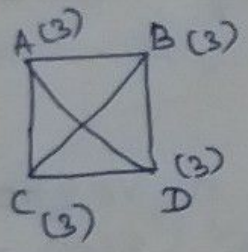
⑨ Complete bipartite graph $K_{m,n}$ is hamiltonian (if $m=n$)

⑩ Easy to construct hamiltonian circuit in a complete graph of n vertices.

Dirac's Theorem: (Introduction)

A graph with $n \geq 3$ vertices and every vertex having vertex degree $\geq n/2$ (half of vertices) is hamiltonian.

Ex:1:



All vertices degree ≥ 2 / Satisfies Dirac's Condition.

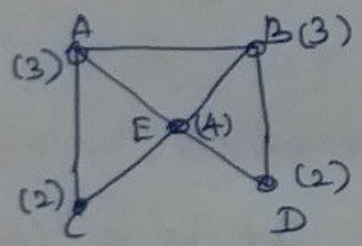
$n \geq 3$ vertices

$\frac{n}{2} = \frac{4}{2} = 2$

Hamiltonian circuit = ABDCA

(Ex:1 & Ex:2)
These graph satisfies the Dirac's Condition, so this graph is a hamiltonian.

Ex:2

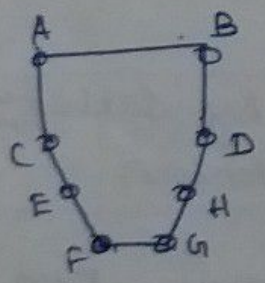


$n \geq 3 \rightarrow$ yes $n = 5$

$\frac{n}{2} = \frac{5}{2} =$ either 2 (or) 3 above.

Hamiltonian circuit = BDECAB.

Ex: 3



$n = 8 \rightarrow 8/2 = 4$

All degrees are $2 \leq 4$

Dirac's condition fails.

This graph doesn't satisfy the Dirac's Condition but it has hamiltonian circuit.

Conclusion: If a graph satisfies the Dirac's condition then that will be a hamiltonian.

However, An absence of this condition will not cause the graph cannot be a hamiltonian. But, the presence of the condition is sufficient for the graph to be a hamiltonian.

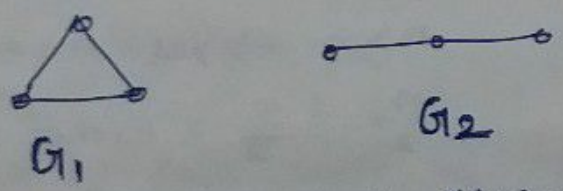
Dirac's theorem 4:

A sufficient condition for a simple graph G to have a hamiltonian circuit is that the degree of every vertex in G be atleast $n/2$ where n is the number of vertices in G.

Proof:

Let G be a graph with n vertices, $n \geq 3$ and degree of each vertex is δ, $\delta(v) \geq n/2$, where, "v" is a vertex.

Assume that, $n=3$ then $\delta(v) = 3/2 = 1.5 \therefore 1$ (or) 2

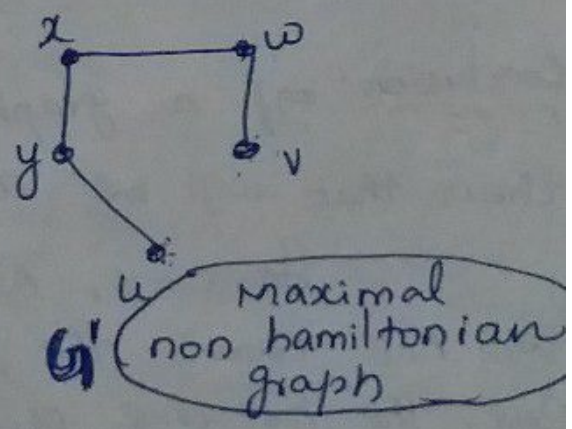


G_1 G_2

G_1 is hamiltonian each $\delta(v) = 2$, It is a complete graph but G_2 is not a hamiltonian, because, It is not a complete graph.

Therefore, A complete graph by satisfying Dirac's theorm, will be a hamiltonian.

Let G' be a maximal non-hamiltonian graph with $n \geq 3$ & $\delta(v) \geq n/2$. and G' is not a complete graph.



Let u & v be non adjacent vertices in G'.

Since G' is maximal non-hamiltonian, G'+uv is hamiltonian also, since, G' is non-hamiltonian. Each hamiltonian circuit of G'+uv must contain the edge u-v

There is a hamiltonian path $V_1, V_2 \ldots V_n$ in G' with orgin $u = V_1$ & ends $v = V_n$.

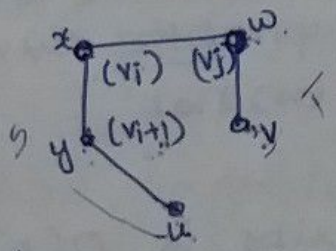
hamiltonian path $(p) = u - y - x - w - v$

Let, $S = \{ v_i \mid u v_{i+1} \in P \}$ such that there is an edge from u to v_{i+1} in G then add V_i vertex in set S.

Let, $T = \{ v_j \mid v_j \in P \}$ such that there is an edge from v to v_j in G then add V_j vertex in T

$S = \{ x \}$

$T = \{ w \}$



u & v → ending vertex is not an element of S & T

(ie) $V_n \notin |S \cup T|$ and $|S \cup T| < n$ also $|S \cap T| = \phi$

Let $|S|$, $|T|$ & $|S \cup T|$ be the number of elements in S, T and $S \cup T$ respectively,

$$|S| + |T| + |S \cup T| < n. \qquad [\because n = 5]$$

$$1 + 1 + 2 < 5.$$

every edge incident with u, there correspond one vertex V_i in S. Therefore $|S| = d(u)$

Similarly, $|T| = d(v)$

$$|S \cup T| = |S| + |T| - |S \cap T|$$

$$= d(u) + d(v) - \phi$$

$$\leq n/2 + n/2 - 0.$$

$$|S \cup T| < n \qquad \therefore d(u) + d(v) < 1 + 1$$

$$d(u) + d(v) < n \; [n = 5]$$

As per Ore's theorem, $\boxed{d(u) + d(v) \geq n}$ then the graph is hamiltonian. but, here $d(u) + d(v) < n$. therefore, [illegible] contradiction. Hence, the theorem proved.

Comparision of Euler and Hamiltonian Circuits:

Euler circuit	Hamiltonian circuit.
Repeated visits to a given Vertices are allowed	visit each Vertex exactly once.
Traverse each edge exactly once	Repeated traversal of a given edges are not allowed.
No need to consider the graph type (ie) weighted graph (or) not	Consider the graph type while visiting the Vertices (shortest route is considered)
Complexity is $O(n)$	Complexity is $O(n!)$

Application of Hamiltonian circuit:

Travelling Salesman problem:

A salesman is required to visit a number of cities during a trip. Given the distances between the cities, in what order should he travel so as to visit every city precisely once and return home, with minimum mileage travelled.

Representing the cities by Vertices and the roads between cities by edges

Weighted graph → every edge is associated a real number

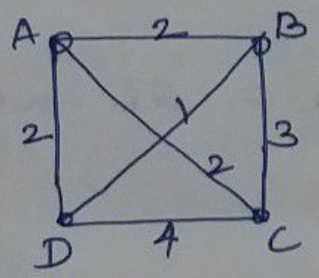
Completed weighted graph → each of city has a roads between other cities

TSP Algorithm:

1. List all possible Hamiltonian Circuits (not edge disjoint hamiltonian circuits)
2. Find the weight of each hamiltonian circuit
3. Choose the one with the smallest weight

$$\boxed{\left.\begin{array}{c}\text{Total number of different}\\ \text{hamiltonian circuits of}\\ n \text{ vertices}\end{array}\right\} = \frac{(n-1)!}{2}}$$

Example:



weighted complete graph.

$n = 4,$

possible hamiltonian circuits $= \frac{(n-1)!}{2} = \frac{(4-1)!}{2} = \frac{3!}{2} = \frac{3\times 2}{2}$

$\therefore$ $\boxed{\text{3 possible hamiltonian circuits}}$

$HC_1 = ABCDA = 2+3+4+2 = 11$ weights

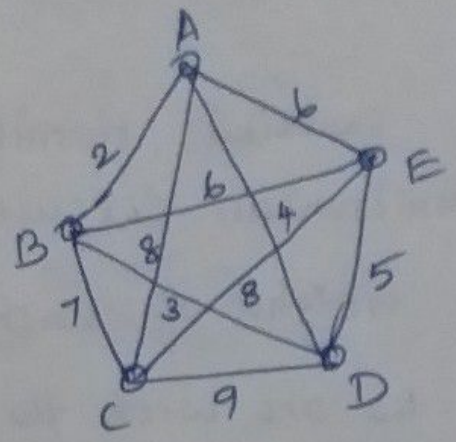
$HC_2 = ACBDA = 2+3+1+2 = 8$ weights

$HC_3 = ADBCA = 2+1+3+2 = 8$ weights.

$\therefore$ Minimum mileage travelling is 8 weights path

$\therefore$ Consider circuit HC_2 (or) HC_3

Example:



$n = 5$, $\frac{(n-1)!}{2} = \frac{4!}{2} = \frac{4 \times 3 \times 2 \times 1}{2} = 12$

12 possible hamiltonian circuits for this graph.

$\therefore$ $HC_1 =$

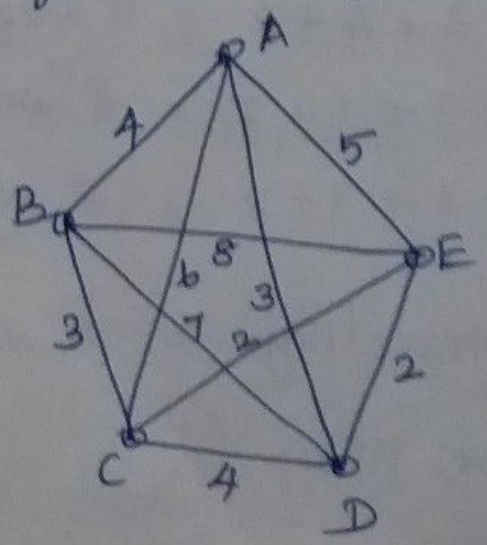
$HC_2 = A - B - D - E - C - A = 26$ weights.

$\vdots$

$HC_{12} =$

Among 12 hamitonian circuits HC_2 is having minimum weight (26).

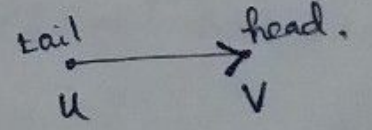
Exercise: Solve the travelling salesman problem for the weighted graph.



DIRECTED GRAPH. (or) DIGRAPH:

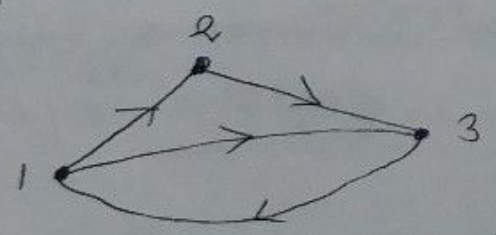
A directed graph D is a pair (V, A) where V is a finite nonempty set and A is a subset of $V \times V - \{(x,x) / x \in V\}$. The elements of V and A are called vertices and arcs respectively.

If $(u,v) \in A$ then the arc (u,v) is said to have u as its initial vertex (tail) and v as its terminal vertex (head). Also the arc (u,v) is said to join u to v.



$(u,v) \rightarrow$ An arc.

Example:



Directed graph.

Def:

A graph in which every edge is directed is called a directed graph or simply a digraph.

Note:

1. In a digraph an edge is not only incident on a vertex but is also incident out of a vertex and incident into a vertex.

2. The vertex v_i, which edge e_k is incident out of is called the initial vertex of e_k

The vertex v_j, which e_k is incident into, is called the terminal vertex of e_k

3. An edge for which the initial and terminal vertices are the same forms a self-loop

4. The number of edges incident out of a vertex v_i is called the out degree of v_i. It is denoted by $d^+(v_i)$

5. The number of edges incident into v_i is called the in degree of v_i. It is denoted by $d^+(v_i)$.

6. An isolated vertex is a vertex in which the in-degree and the out-degree are both equal to zero.

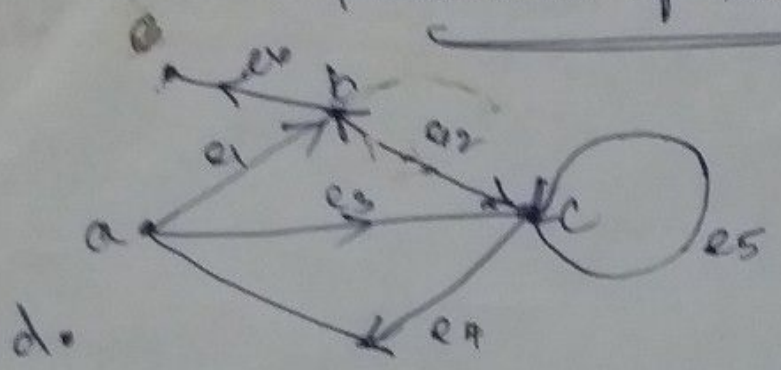
7. A vertex v in a digraph is called endant if it is of degree one.

ie, $d^+(v) + d^-(v) = 1$.

8. Two directed edges are said to be parallel if they are mapped onto the same ordered pair of vertices.

Explanation of the points

(10)



1) e_1 is incident out of a vertex a
e_1 is incident in of a vertex b

2) Vertex c, which edge e_4 is incident out is called the initial vertex of e_4.
Vertex a, which edge e_4 is incident into is called the terminal vertex of e_4

3) e_5's initial & terminal vertex are same then e_5 is a selfloop

4) out degree (a) = no; of edges incident out of a
$\boxed{deg^+(a) = 2}$ out valence (or) outward demidegree

5) In degree (a) = no; of edges incident into a
$\boxed{deg^-(a) = 1}$ in valence (or) inward - demidegree

6) · Vertex d is an isolated. In & out degree of d is zero.

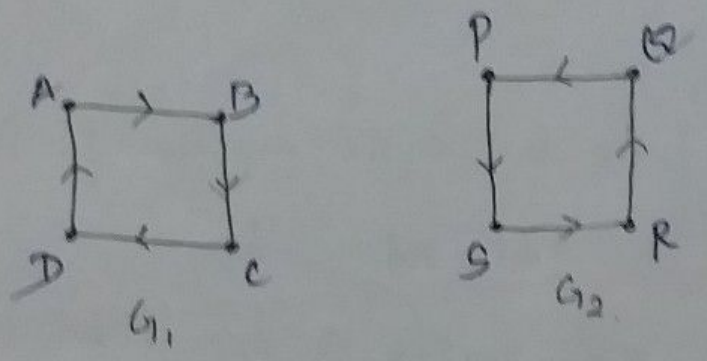
7). ~~Isolated~~ pendant vertex e's $deg^+(e) + deg^-(e) = 1$
e_6 is incident into e. $0 + 1 = 1$

8) edge e_2 is parallel edge, Two directed edge

ISOMORPHIC DIGRAPH.

A directed graph is said to be an Isomorphic digraph if they have identical behaviour together in terms of graph properties.

Example:



Let us consider the following 1-1 correspondence between the vertices of the di-graphs

$A \leftrightarrow Q$, $B \leftrightarrow P$, $C \leftrightarrow S$, $D \leftrightarrow R$

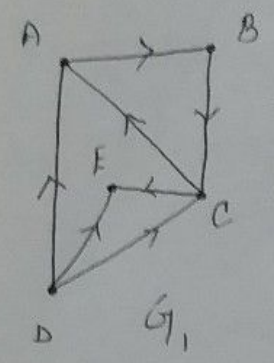
The directed edges of the two graphs correspond with each other as shown below.

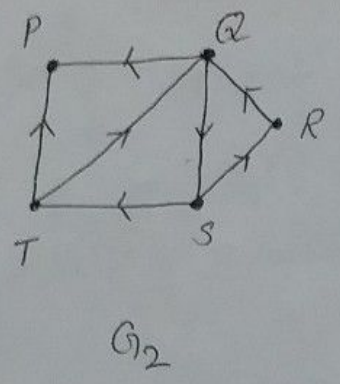
$(A, B) \leftrightarrow (Q, P)$, $(B, C) \leftrightarrow (P, S)$, $(C, D) \leftrightarrow (S, R)$,

$(D, A) \leftrightarrow (R, Q)$

The adjacency of vertices including directions of the edges is preserved. Hence the given digraphs are isomorphic.

Check the following digraphs are not isomorphic





Solution:

The two digraphs have the same number of vertices and the same number directed edges 7.

From the graph G_1, the vertex A has 1 as its out-degree and 2 as its in-degree. There is no such vertex in the graph G_2.

∴ There cannot be any 1-1 correspondence between the vertices of the two digraphs which reserves the direction of edges.

Hence the two digraphs are not isomorphic.

Theorem 5:

In every digraph D, the sum of the out-degree of all vertices is equal to the sum of the in-degrees of all vertices, each sum being equal to the number of edges in the digraph D.

Proof:-

let q denote the number of edges in $D = (V, A)$

Let $B = \sum_{v \in V} d^+(v)$ and $C = \sum_{v \in V} d^-(v)$.

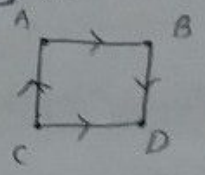
An arc (u,w) $u \to w$ contributes one to the out degree of u and one to the in-degree of w. Hence each arc contributes 1 to the sum B and 1 to the sum C.

Hence $B = C = q$.

TYPES OF DIRECTED GRAPHS (Digraphs)

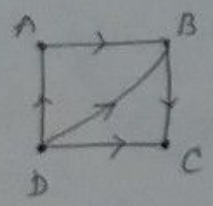
1. Oriented Graph

A digraph without parallel edges is called an oriented graph.



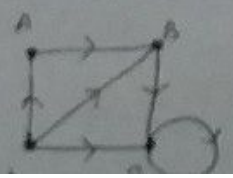
2. Strict digraph (or) Simple Digraph:

A digraph without self loops and parallel edges) arcs is called a simple digraph or a strict digraph.



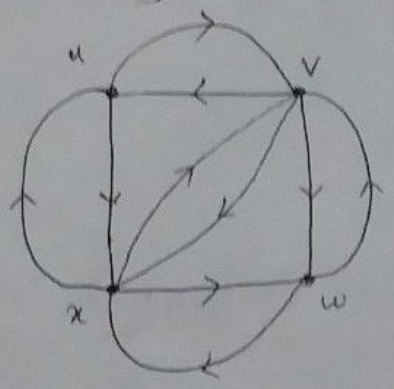
3. Asymmetric Graph:

A digraph having atmost one arc between a pair of vertices and with or without self loops is called an asymmetric or antisymmetric digraph.



4. Symmetric digraph:

A digraph in which for each arc (u,v) there is also an arc (v,u) is called a symmetric digraph.



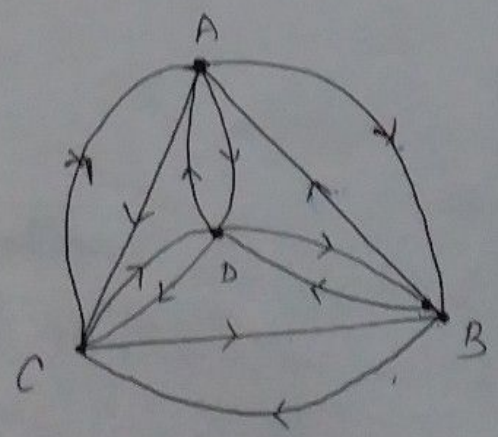
5. Complete Digraphs.

- a) Complete Symmetric digraph
- b) Complete assymmetric digraph

a) Complete Symmetric digraph

A complete symmetric digraph is a simple digraph in which there is exactly one arc directed from every vertex to every other vertex.

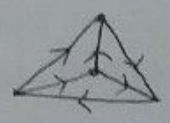
If D is a complete symmetric digraph on n vertices, the number of arcs is $n(n-1)$.



7. Complete asymmetric digraph (or) Tournament

A complete asymmetric digraph is an asymmetric digraph in which there is exactly one arc between every pair of vertices.

If n is the number of vertices of a complete asymmetric digraph the number of arcs is $\frac{n(n-1)}{2}$.

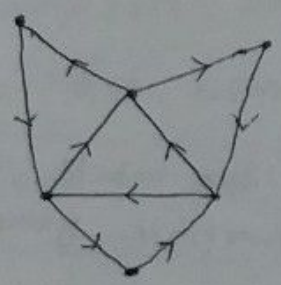


8) Isograph (or) pseudosymmetric (or) Balanced digraph:

A digraph is said to balanced if for every vertex v the indegree and out degree are equal.

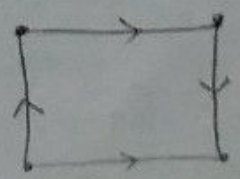
ie $d^+(v) = d^-(v) \ \forall \ v \in D$.

It is also called a pseudo symmetric digraph or an isograph.



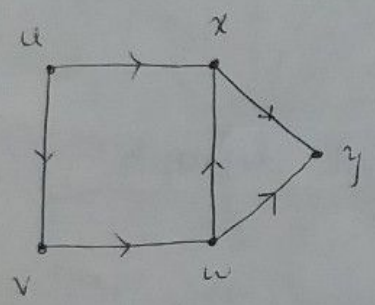
9) Regular graph:

A balanced digraph in which all vertices have the same equal in-degrees and out-degrees is called a regular graph.

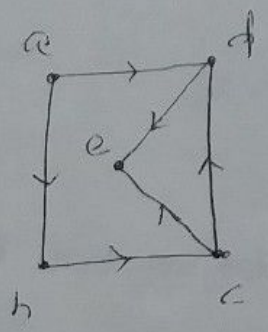


Isomorphism of digraphs

Two digraphs $D_1 = (V_1, A_1)$ and $D_2 = (V_2, A_2)$ are said to be isomorphic if there exists a bijective map $f: V_1 \to V_2$ such that $(u,v) \in A_1$ iff $(f(u), f(v)) \in A_2$. and it is denoted by $D_1 \cong D_2$.



D_1



D_2

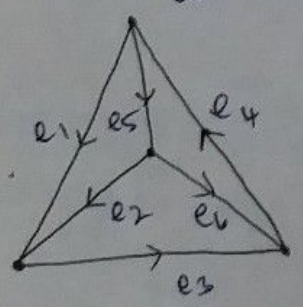
Isomorphic graphs were defined such that they have identical behaviour interms of graph properties.

Consider the digraphs D_1 and D_2 and the map $: V_1 \to V_2$ given by

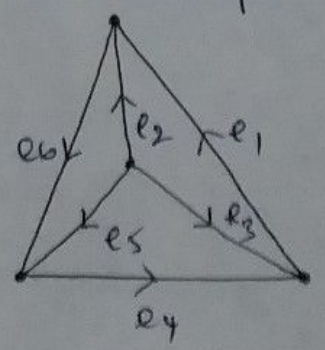
$f(u) = a,\ f(v) = b,\ f(w) = c,\ f(x) = d,\ f(y) = e$

Clearly f is bijective.

Non-isomorphic digraphs: If the labels of graphs are removed then two isomorphic graphs are indistinguishable



I_1



I_2

I_1 & I_2 are nonisomorphic graphs

Types of Digraphs: (Short notes)

1. oriented graph → without parallel edges.
2. Strict digraph (or) Simple digraph → without selfloops & Parallel edges.
3. Asymmetric digraph (or) Antisymmetric digraph → Atmos one arc (edge) b/w a pair of vertices with (or) without selfloops.
4. Symmetric digraph → A digraph with parallel edges
5. Complete digraph < complete symmetric digraph / Complete assymetric digraph
6. complete symmetric digraph → It is a simple graph with exactly one arc directed from every vertex to every other vertex

$\boxed{\text{If } n \text{ vertices} \Rightarrow n(n-1) \text{ arcs}}$

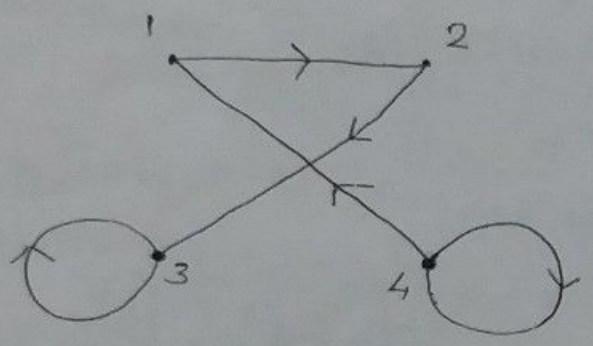
7. complete Assymmetric digraph (or) Tournament → exactly one arc between every pair of vertices.

$\boxed{\text{If } n \text{ vertices} \Rightarrow \frac{n(n-1)}{2} \text{ arcs}}$

8. Iso-graph (or) pseudosymmetric (or) Balanced digrap for every vertex the indegree & outdegree are eq
9. Regular graph. → It is a balanced graph ~~with~~ in which all vertices have the same equal indegree & out degrees.

1. Draw the digraph $G = (V, E)$ where $V = \{1, 2, 3, 4\}$ and $E = \{(1,2), (2,3), (3,3), (4,4), (4,1)\}$.

Solution:-



Digraphs and Binary Relations:-

In a set of objects X, where $X = \{x_1, x_2, \ldots\}$ a binary relation R between pairs (x_i, x_j) may exist. It is denoted by $x_i R x_j$ and it say that x_i has a relation R to x_j.

Note:-

1. Every binary relation on a finite set can be represented by a directed graph without parallel edges.

2. Conversely, every directed graph without parallel edges defines a binary relation on the set of its vertices.

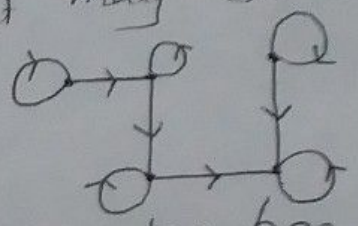
3. Each $x_i \in X$ is represented by a vertex x_i. If x_i has the relation R to x_j then a directed edge is drawn from vertex x_i to x_j for every pair (x_i, x_j).

Reflexive relation:- < Reflexive digraph / Irreflexive digraph

A relation R on a set A is said to be reflexive if $aRa \ \forall \ a \in A$.

Thus a directed graph having a self-loop - each of its vertices defines a reflexive relation and vice versa

Such a digraph represents a reflexive binary relation on its vertex set may be called reflexive digraph:



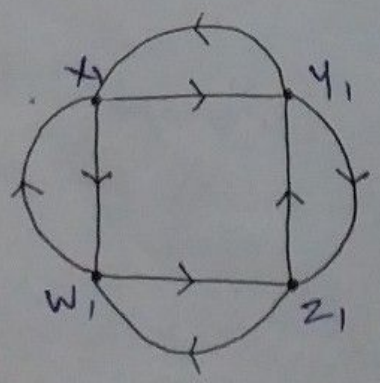
Irreflexive digraph

A digraph in which no vertex has a self-loop is called an irreflexive digraph.

Symmetric relation:

A relation R on set A is said to be symmetric if $aRb \Rightarrow bRa$.

The digraph of a symmetric relation is a symmetric digraph because for every directed edge from vertex x_i to x_j there is a directed edge from x_j to x_i.



aRb ; bRa

aRa

(1)

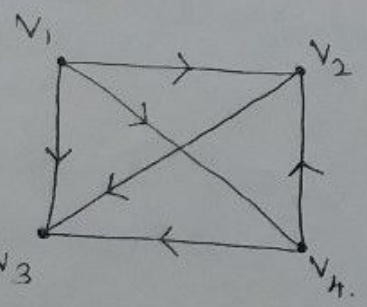
Transitive Relation (or) Transitive directed graph.

A relation R on A is said to be transit

if aRb and bRc implies aRc.

A digraph representing a transitive relatio

is called a transitive directed graph.



aRb & bRc = aRc

Note:

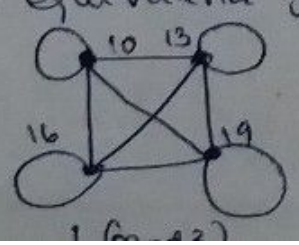
A digraph for transitive relation should contain a directed path between from a to c whenever there is a directed path from a to b and b to c.

Equivalence Relation:

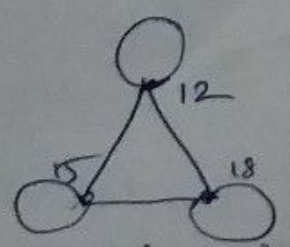
A relation R on a non-empty set A is called an equivalence relation if its is reflexive, symmetric and transitive.

The graph representing an equivalence relatio may be called an equivalence graph.

Ex: equivalence graph.



$\equiv 1 \pmod 3$



$\equiv 0 \pmod 3$

Relation Matrices:

A binary relation R on a set can be represented by a matrix, called a relation matrix. It is a (0,1), n by n matrix. where n is the number of vertices in the set.

Graph Theory & Combinatorics

III - Year - S_5 RB - ASSIGNMENT - I & CLASS TEST

Module - 1 (University Questions)

Max Marks: 55

① Define pendent vertex, isolated vertex & null graph with an example. (3)

② Consider a graph (Completed) G with 11 vertices. find the maximum number of edges in G. (3)

③ Define subgraph. Give two subgraph of Fig 1 a) & b) (3)

④ Define Isomorphism. Check fig 1 a) & b) are Isomorphic. (3)

fig 1 a)

fig 1 b)

⑤ Draw a simple disconnected graph with 10 vertices, 4 components & calculate maximum number of edges (3)

⑥ Are the below two graphs are Isomorphic (5)

fig 2 a)

fig 2 b)

⑦ Explain two applications of graph (3)

⑧ prove that a simple graph with n vertices must be connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges (6)

⑨ prove that the number of odd degree vertices in a graph is always even. (4)

⑩ Draw a graph G with 4 vertices v_1, v_2, v_3, v_4 and the degree of vertices are 3, 5, 2, 1 respectively. Is it possible to construct such a graph G. If not why? (3)

① Define Isomorphism between two graphs. Are the following graphs are Isomorphic, Justify (5)

fig 3 a)

fig 3 b)

② What are the basic conditions to be satisfied for two graphs to be isomorphic. Are the two graphs below Isomorphic (6)

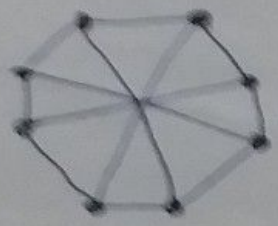
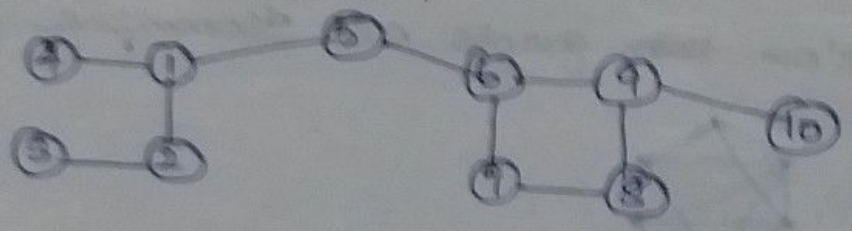


fig 4 a)

fig 4 b)

③ Define walk & print a walk, trail, path & cycle (3)



④ 19 Students in a nursery school play a game each day, where they hold hand to form a circle. For how many days can they do this, with no students holding hands with the same playmates more than once? Substantiate your answer with graph theoritic concepts. (3)

⑤ There are 37 telephones in the City of Istanbul, Turkey. Is it possible to connect them with wires so that each telephone is connected with exactly 7 others? Substantiate your answer with graph concept. (3)

III - Year - S_5 (B) Assignment - II & Class test

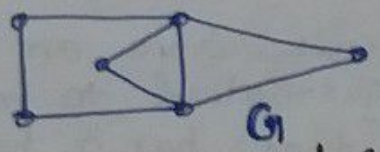
Module - 2 (University Questions)

Max. Marks: 75

① State Dirac's theorem for hamiltonicity and why it is not a necessary condition for a simple graph to have a hamitonian circuit (5)

② Differentiate between symmetric and asymmetric digraphs with examples and draw a complete symmetric diagraph of four vertices. (3)

③ Consider a graph G. Define Euler graph. Is G an Euler. If yes, write an Euler line from G. (4)



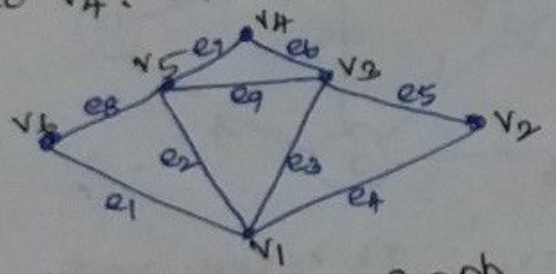
G

④ What is necessary and sufficient condition for a graph to be Euler? And also prove it. (4)

⑤ Define hamiltonian circuits and paths with examples. Find out the number of edge joint hamiltonian circuits possible in a comple graph with five vertices. (5)

⑥ State Travelling-salesman problem and how TSP solution is related with hamiltonian circuits? (3)

⑦ For the following graph, find the shortest path between from v_1 to v_4. Also find a Euler circuit. (3)

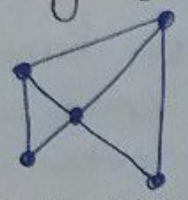


⑧ Define hamiltonian graph. Find an example of a non-hamiltonian graph with a hamiltonian path. (3)

⑨ For a Eulerian graph G, prove the following properties
i) the degree of each vertex of G is even. (6)
ii) G is an edge disjoint union of cycles

(10) Discuss the Konigsberg bridge problem. Is there any solution to the problem? Justify your answer. (3)

(11) State Dirac's theorem and check its applicability in the following graph, G (3)



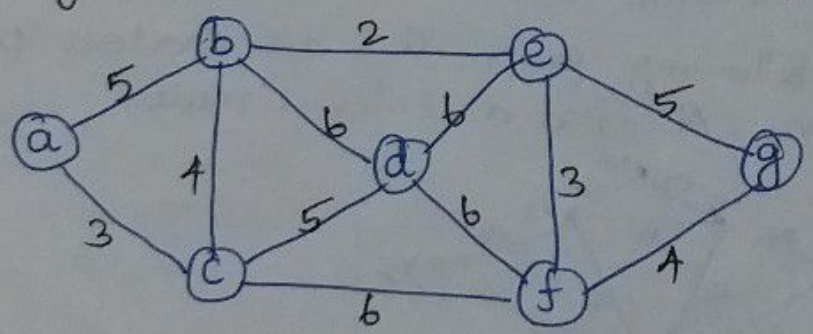
(12) A connected graph G is an Euler graph if and only if all vertices of G are of even degree. Prove the statement. (6)

(13)* Consider a complete graph G with 11 vertices
i) Find the maximum number of edges possible in G. (4)
ii) Find the number of edge disjoint hamiltonian circuits in G.

(14) Define hamiltonian circuit. Give an example. What general class of graphs is guaranteed to have a hamiltonian circuit. Also draw a graph that has a hamiltonian path but not a hamiltonian circuit. (4)

(15) Prove that if a connected graph G is decomposed into two subgraphs g_1 & g_2 there must be at least one vertex common between g_1 and g_2. (3)

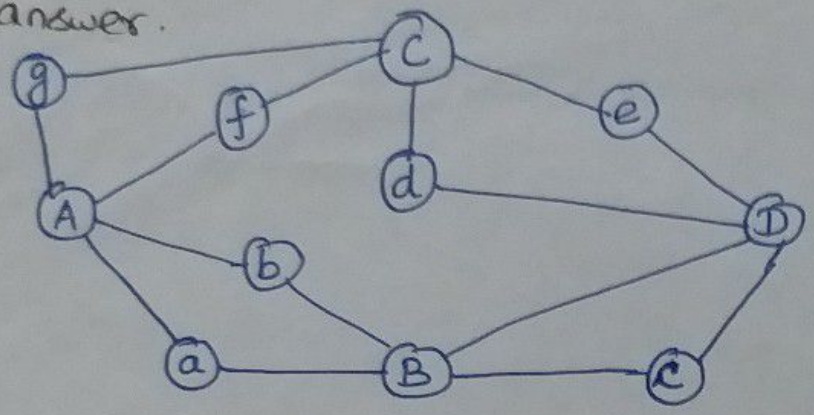
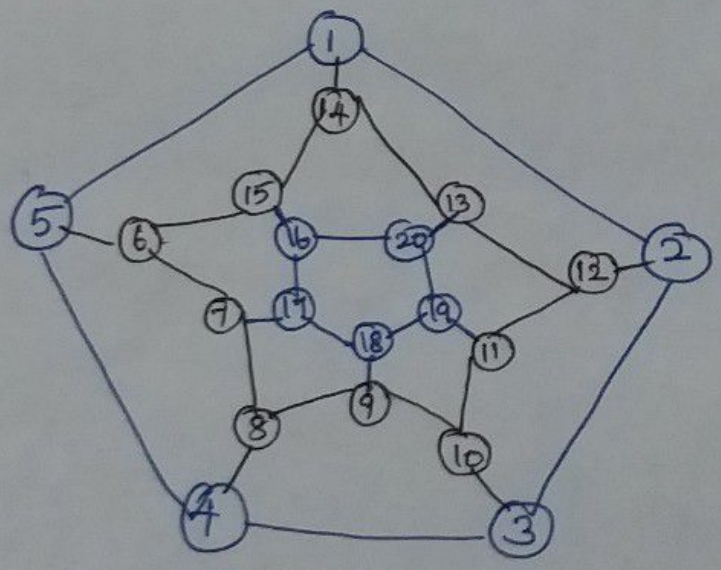
(16) State travelling salesman problem. Print a travelling salesman's tour on the graph below (3)



(17) Prove (or) disprove: If every vertex of a simple graph G has degree 2, then G is a cycle (3)

(18) In a Digraph, the sum of the out-degree vertices is equal to the sum of the in-degree vertices. each sum being equal to the number of edges in the Digraph, prove the statement (8)

(19) Check whether the given graph is an Euler graph and if yes, give the Euler line. Justify your answer. (4)



(20) Give Hamiltonian circuits of the following graph. (4)