EC465 MEMS

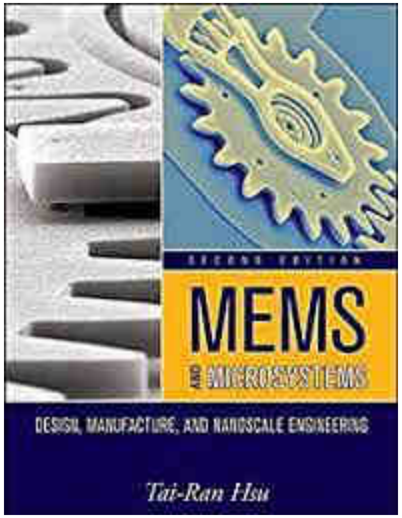
Module 1

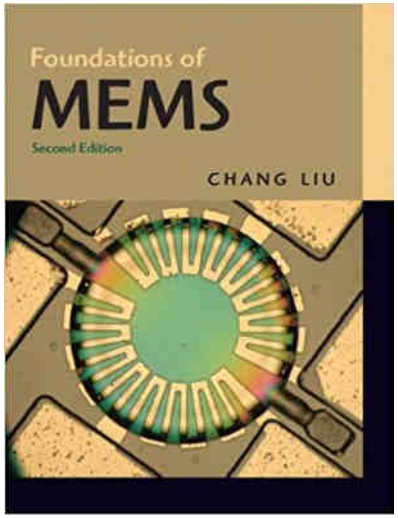
MEMS and Microsystems: Applications – Multidisciplinary nature of MEMS – principles and examples of Micro sensors and micro actuators – micro accelerometer –comb drives - Micro grippers –micro motors, micro valves, micro pumps , Shape Memory Alloys.

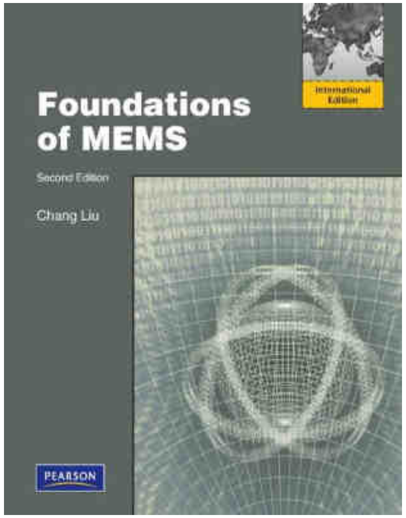
Review of Mechanical concepts: Stress, Strain, Modulus of Elasticity, yield strength, ultimate strength – General stress strain relations – compliance matrix. Overview of commonly used mechanical structures in MEMS - Beams, Cantilevers, Plates, Diaphragms – Typical applications

Text Books

- Tai-Ran Hsu, MEMS and Microsystems Design and Manufacture, TMH, 2002
- Chang Liu, Foundations of MEMS, Pearson 2012

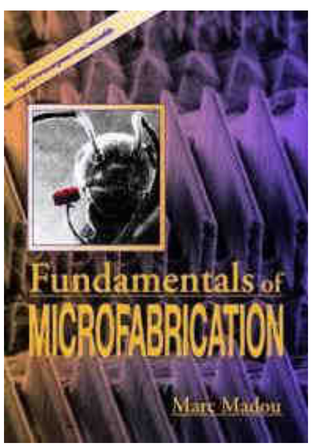


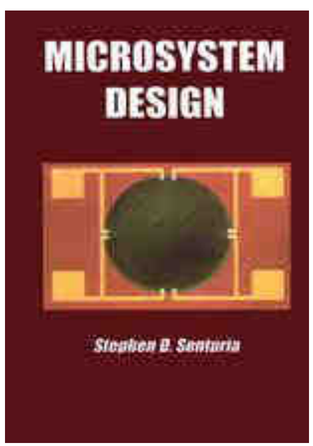


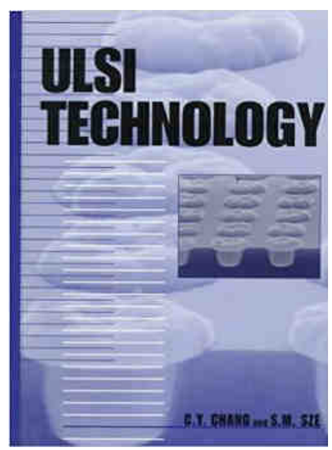


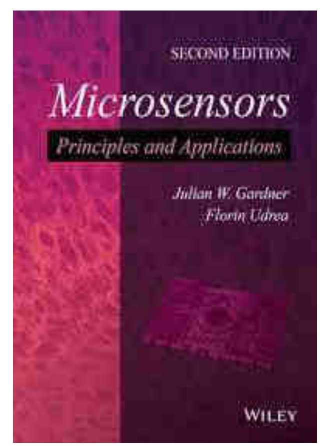
References

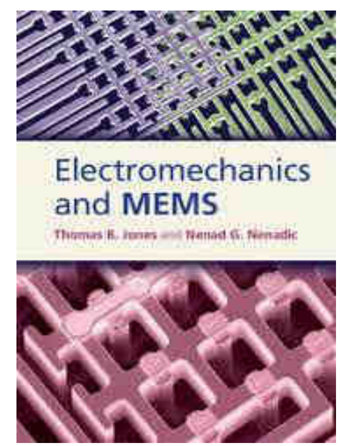
- Mark Madou, “Fundamentals of Micro fabrication”, CRC Press, New York, 1997
- Stephen D. Senturia, Microsystem design, Springer (India), 2006.
- Chang C Y and Sze S. M., “VLSI Technology”, McGraw-Hill, New York, 2000
- Julian W Gardner, “Microsensors: Principles and Applications”, John Wiley & Sons, 1994
- Thomas B. Jones, Electromechanics and MEMS, Cambridge University Press, 2001











What are MEMS?

***M**icro **E**lectro **M**echanical **S**ystem*

Integration of mechanical elements, sensors, actuators and electronics on a common silicon substrate through microfabrication technology

- constructed to achieve a certain engineering function or functions by electromechanical or electrochemical means
- contains *components* of sizes ranging from 1 μm to 1mm.

Available MEMS products include:

- Micro sensors (acoustic wave, biomedical, chemical, inertia, optical, pressure, radiation, thermal, etc.)
- Micro actuators (valves, pumps and microfluidics; electrical and optical relays and switches; grippers, tweezers and tongs; linear and rotary motors, etc.)
- Read/write heads in computer storage systems.
- Inkjet printer heads.
- Micro device components (e.g., palm-top reconnaissance aircrafts, mini robots and toys, micro surgical and mobile telecom equipment, etc.)

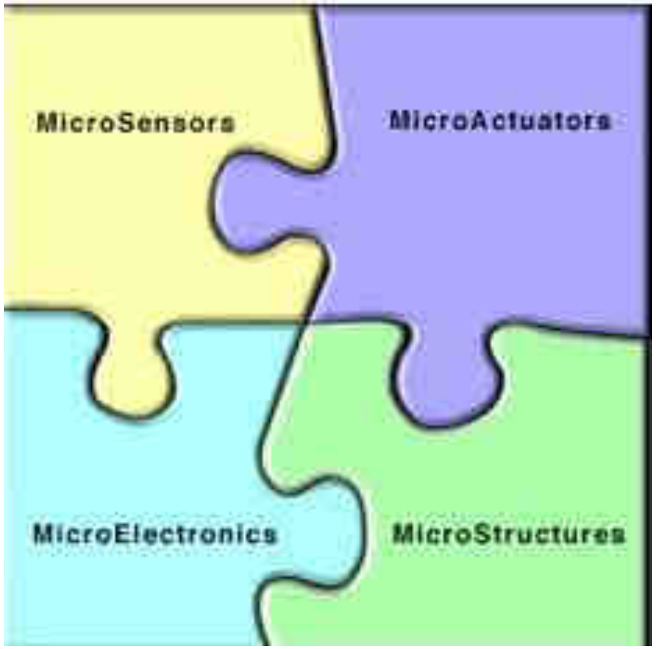
Components

Microelectronics

- It receives, processes, and makes decisions
- data comes from microsensors

Microsensors

- constantly gather data from environment
- pass data to microelectronics for processing
- can monitor mechanical, thermal, biological, chemical, optical, and magnetic readings



Microactuator

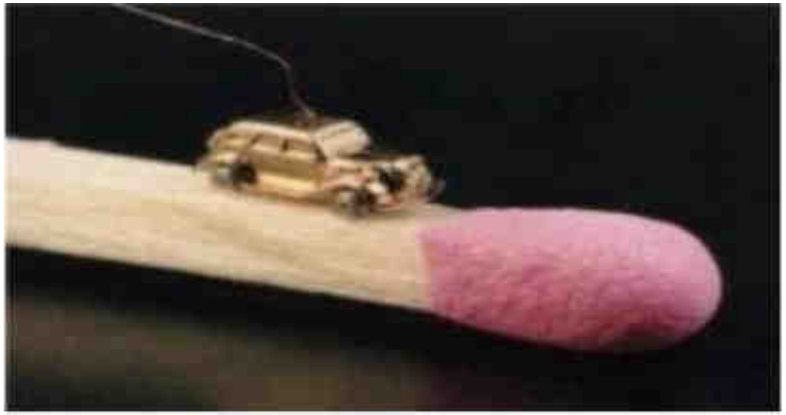
- acts as trigger to activate external device
- microelectronics will tell microactuator to activate device

Microstructures

- extremely small structures built onto surface of chip

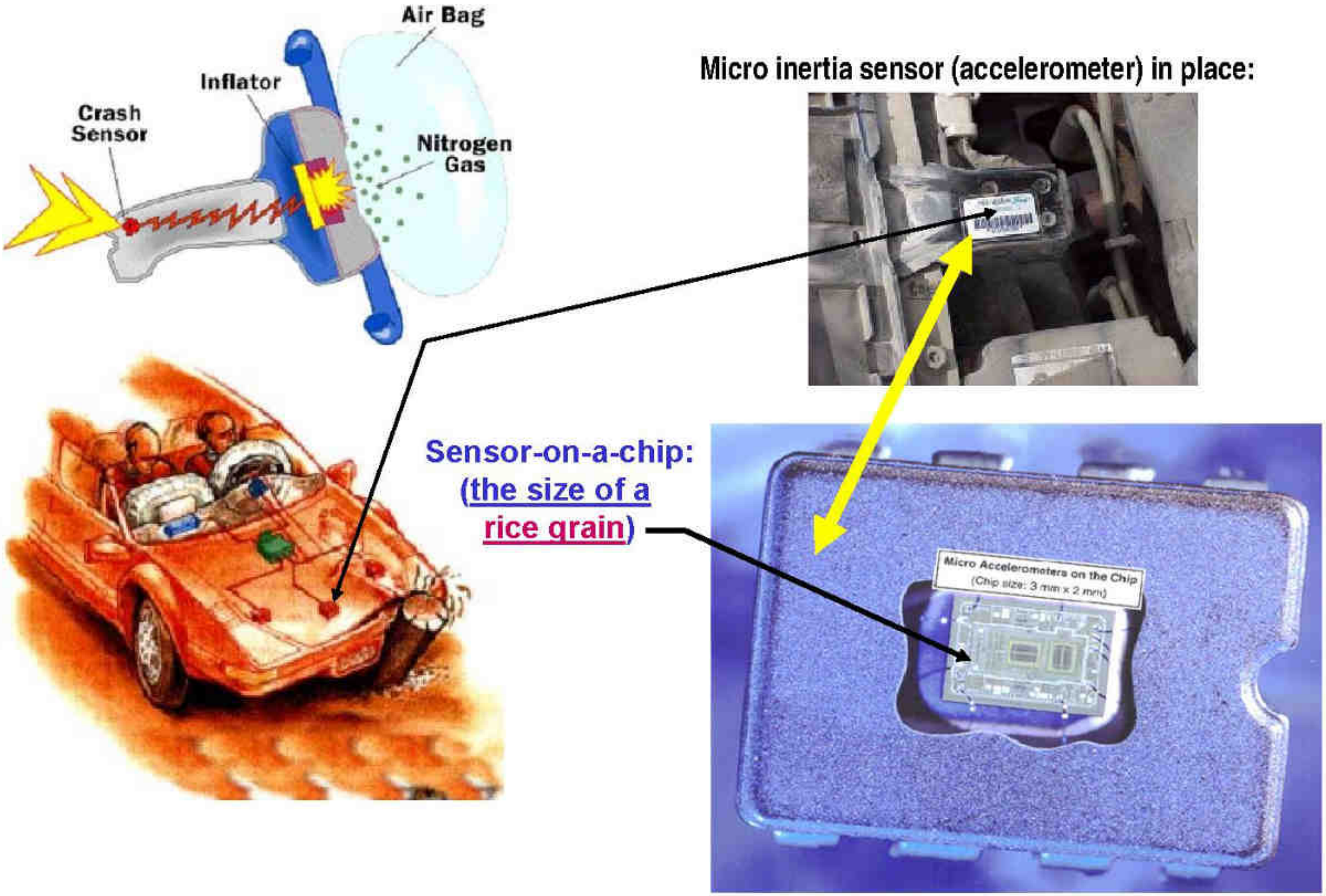
How small are MEMS devices?

They can be of the size of a rice grain, or smaller!



The DENSO Micro-Car is a miniature version of Toyota's first passenger car. Fabricated using MEMS, at 1/1000 th the size of the original.
It consists of a 0.67 mm magnetic-type working motor and when supplied with 3 V 20 mA of alternating current through a 18 μm copper wire,
The engine runs at 600 rpm equivalent to 5-6 mm/s

Fig 1: Micro cars (Courtesy of Denso Research Laboratories, Denso Corporation, Aichi, Japan)



(Courtesy of Analog Devices, Inc)

Fig 2: Inertia Sensor for Automobile "Air Bag" Deployment System

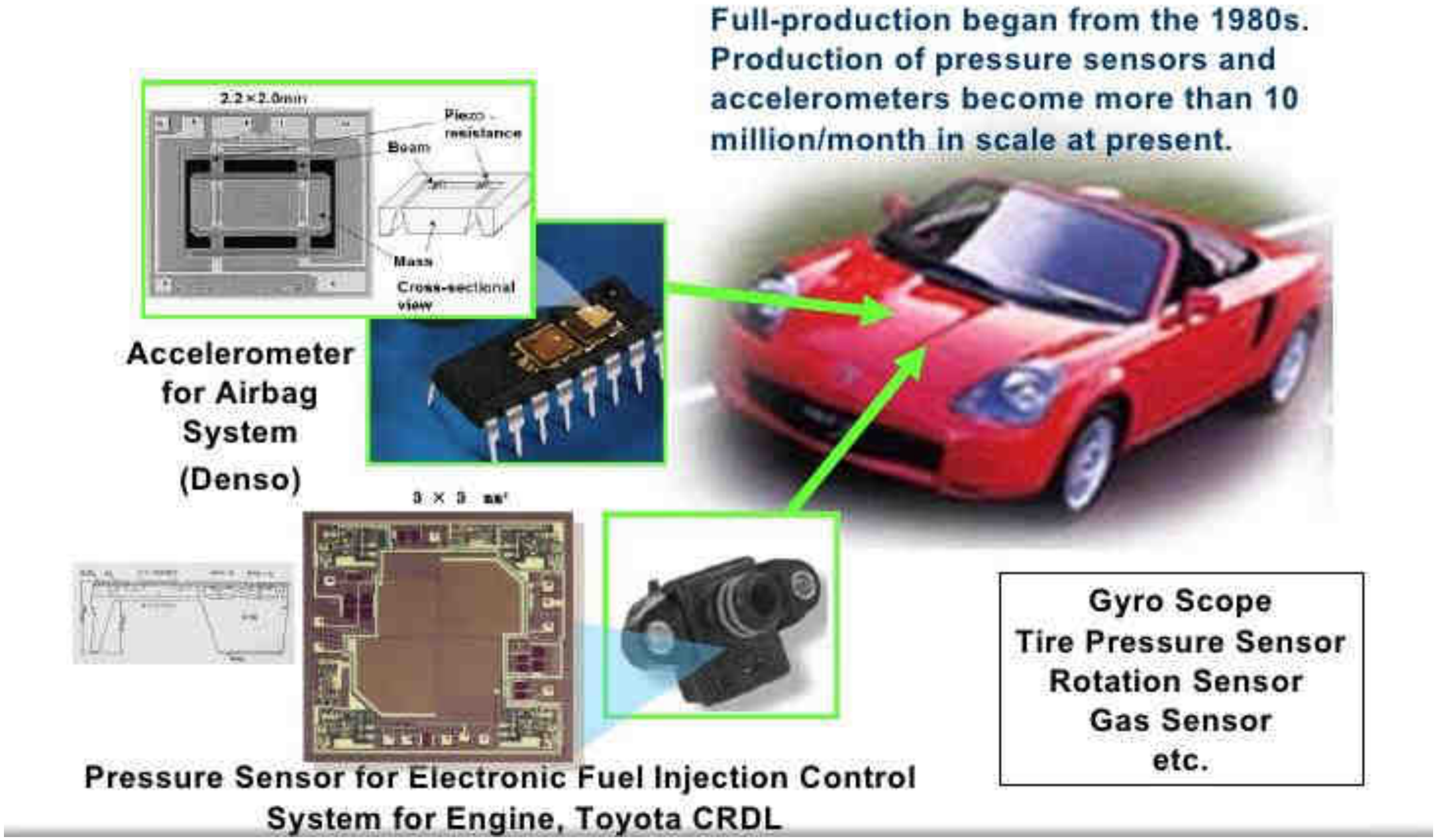


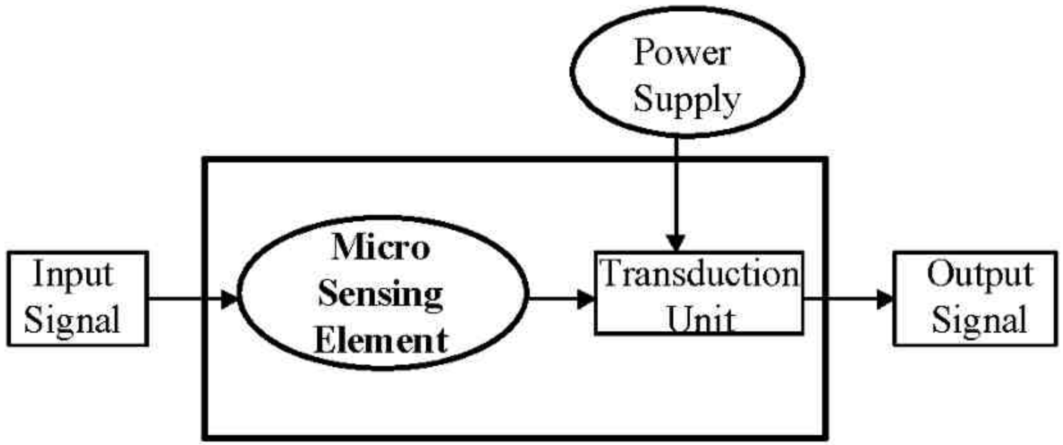
Fig3: MEMS in Automobiles

MEMS

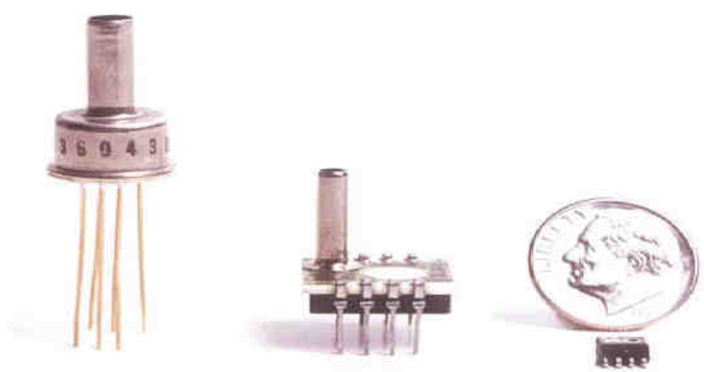
Core elements in MEMS

- A sensing or/and actuating element
- A signal transduction unit

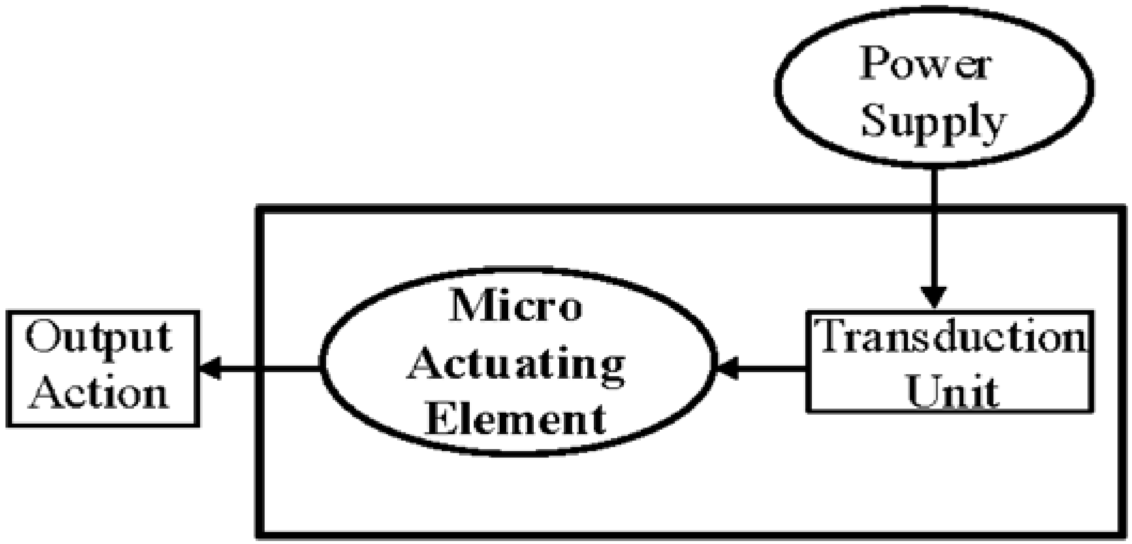
MEMS as microsensors



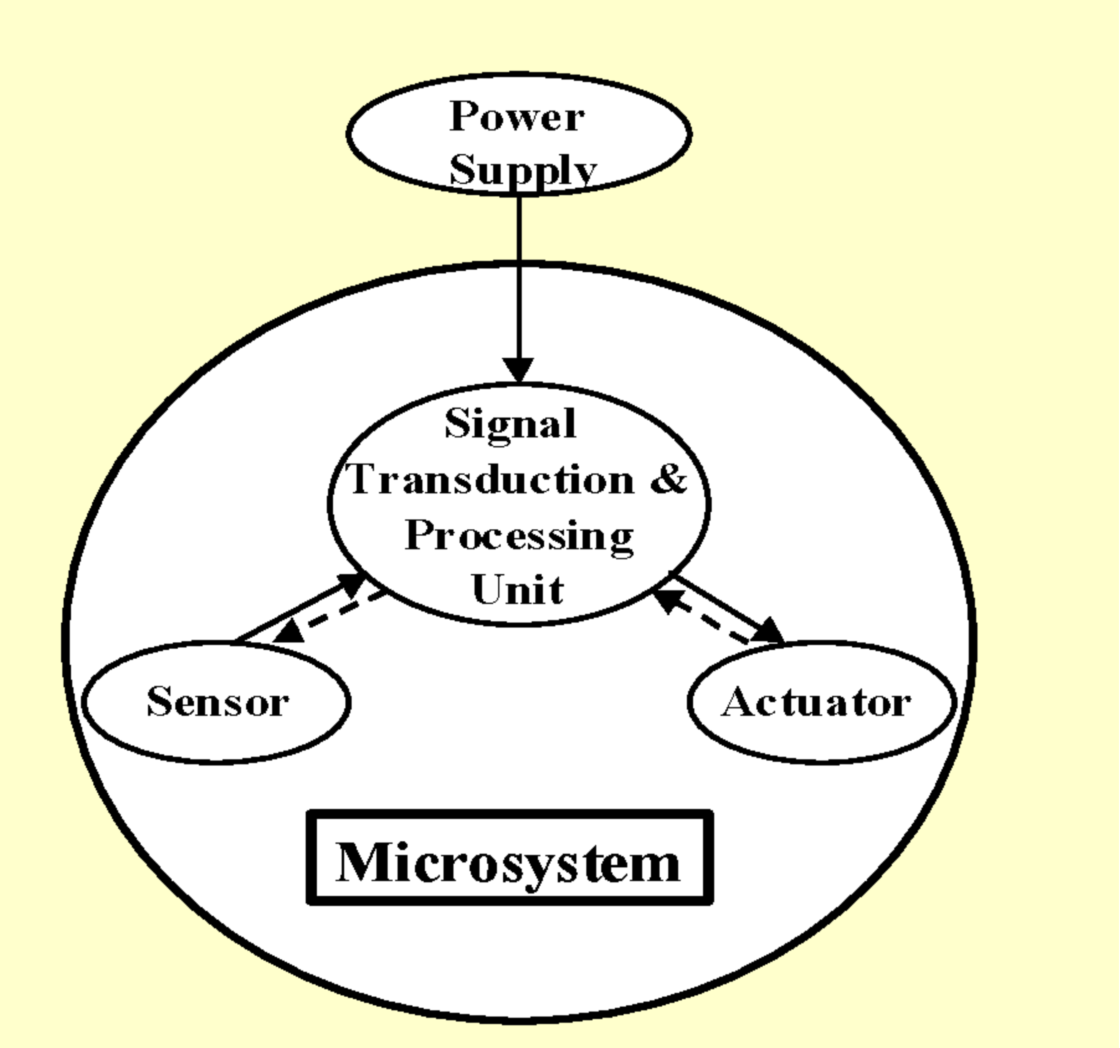
Micro pressure sensors



MEMS as microactuators



Microsystems



Commercial MEMS and Microsystems Products

Micro Sensors:

Acoustic wave sensors
Biomedical and biosensors
Chemical sensors
Optical sensors
Pressure sensors
Stress sensors
Thermal sensors

Micro Actuators:

Grippers, tweezers and tongs
Motors - linear and rotary
Relays and switches
Valves and pumps
Optical equipment (switches, lenses & mirrors, shutters, phase modulators, filters, waveguide splitters, latching & fiber alignment mechanisms)

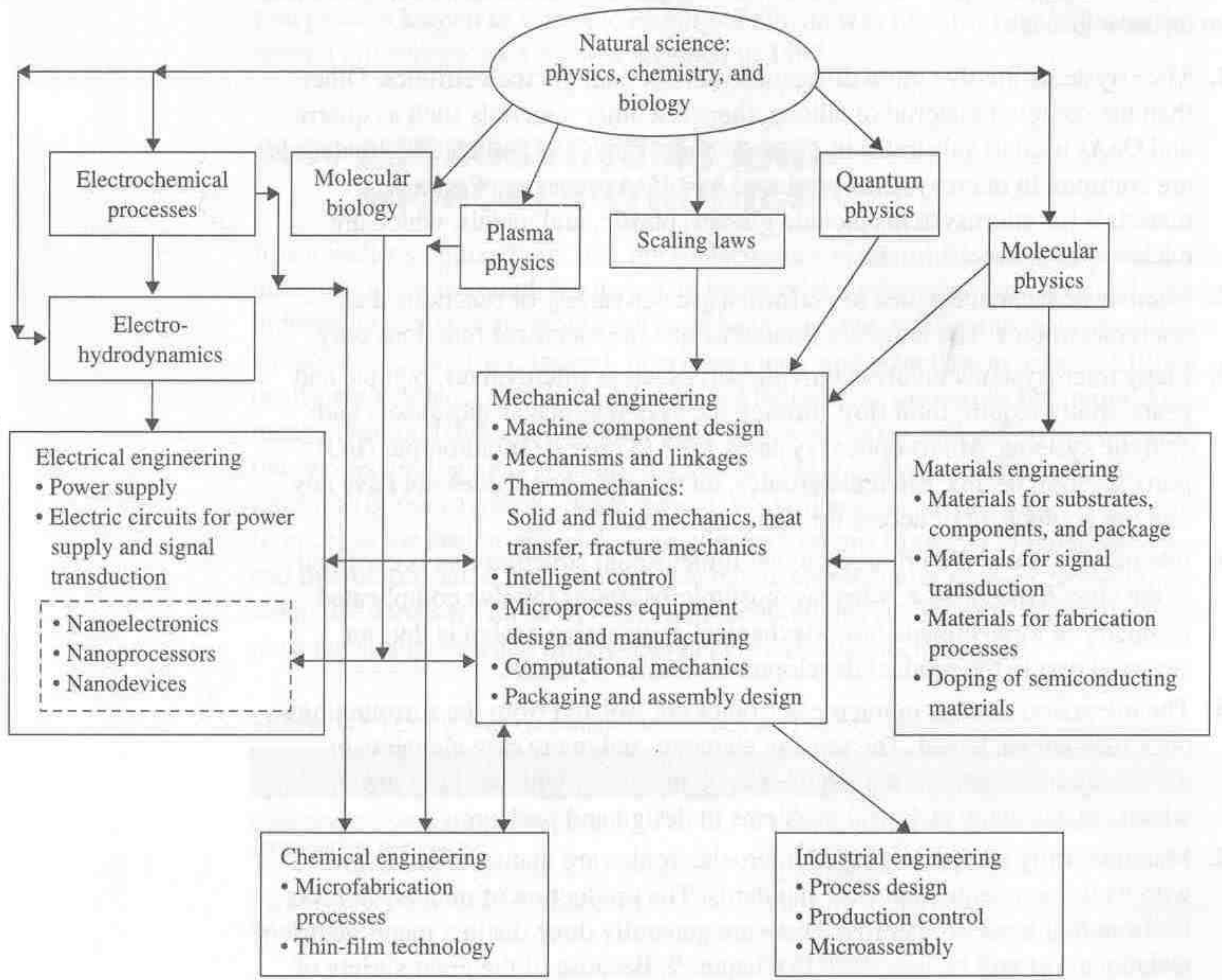
Microsystems = sensors + actuators + signal transduction:

- Microfluidics, e.g. Capillary Electrophoresis (CE)
- Microaccelerometers (inertia sensors)

Comparison of Microelectronics and Microsystems

Microelectronics	Microsystems (silicon based)
Primarily 2-dimensional structures	Complex 3-dimensional structure
Stationary structures	May involve moving components
Transmit electricity for specific electrical functions	Perform a great variety of specific biological, chemical, electromechanical and optical functions
IC die is protected from contacting media	Delicate components are interfaced with working media
Use single crystal silicon dies, silicon compounds, ceramics and plastic materials	Use single crystal silicon dies and few other materials, e.g. GaAs, quartz, polymers, ceramics and metals
Fewer components to be assembled	Many more components to be assembled
Mature IC design methodologies	Lack of engineering design methodology and standards
Complex patterns with high density of electrical circuitry over substrates	Simpler patterns over substrates with simpler electrical circuitry
Large number of electrical feed-through and leads	Fewer electrical feed-through and leads
Industrial standards available	No industrial standard to follow in design, material selections, fabrication processes and packaging
Mass production	Batch production, or on customer-need basis
Fabrication techniques are proven and well documented	Many microfabrication techniques are used for production, but with no standard procedures
Manufacturing techniques are proven and well documented	Distinct manufacturing techniques
Packaging technology is relatively well established	Packaging technology is at the infant stage
Primarily involves electrical and chemical engineering	Involves all disciplines of science and engineering

The Multi-disciplinary Nature of Microsystems Engineering



Working Principles of MEMS and Microsystems

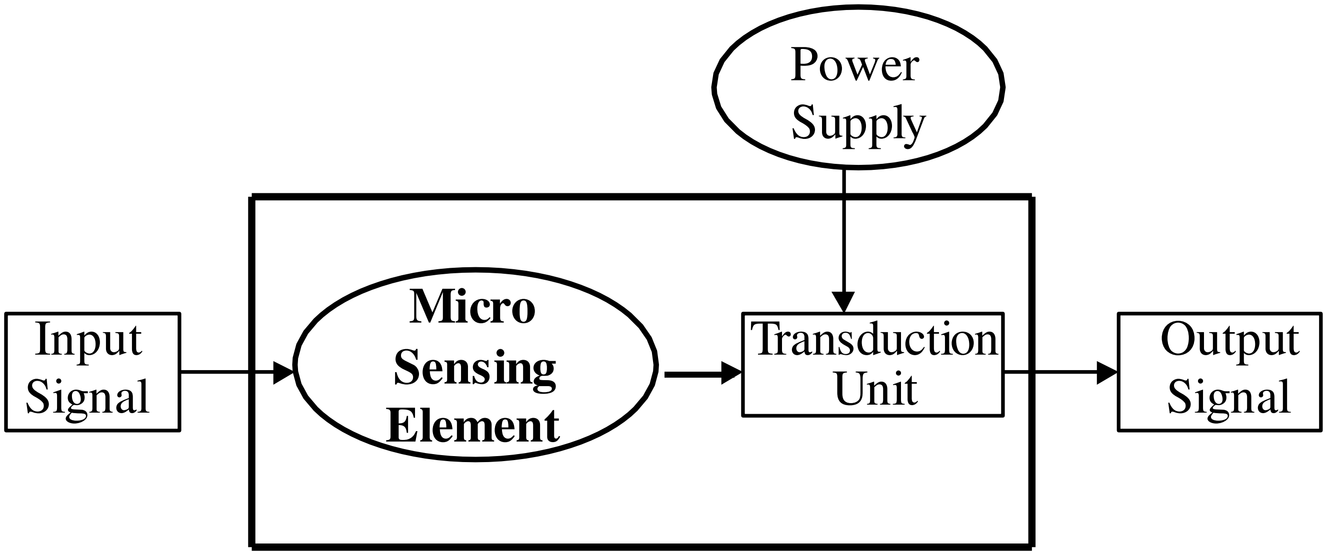
- Minute sensors are expected to detect *a variety of signals* associated with:
  - Accelerations (velocity and forces),
    Biological and biomedical Chemical,
  - Forces (e.g., microaccelerometers and gyroscopes) Optical,
  - Pressure,
  - Thermal (temperatures), etc.

- Input samples may be: motion of a solid, pressurized liquids or gases,
  - biological and chemical substances.

- Due to the minute sizes, microactuators work on *radically different principles* than the conventional *electromagnetic means*, such as solenoids and ac/dc motors.

Instead, electrostatic, thermal, piezoelectric and shape-memory alloys are extensively used in microactuations.

Working Principles for Microsensors



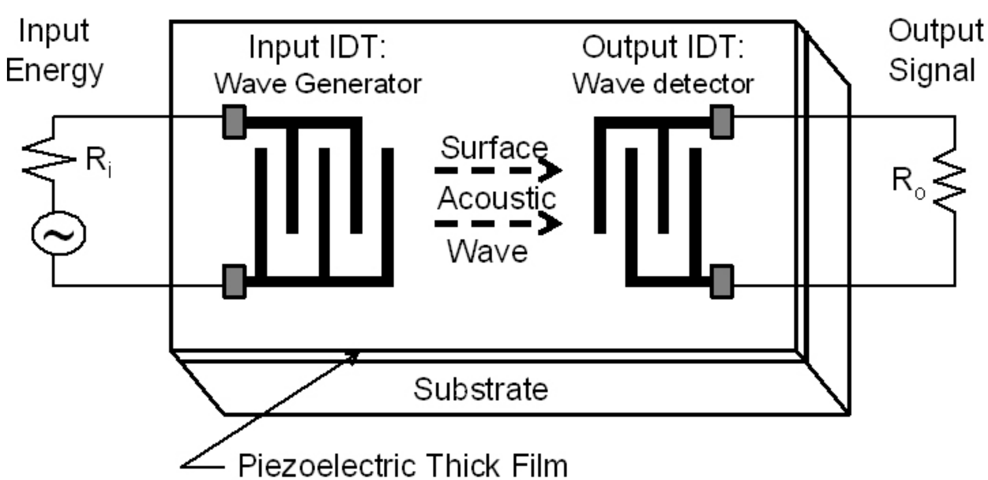
a) Acoustic Wave Sensors

Acoustic wave sensor does not related to the sensing of acoustic waves transmitted in solids or other media, as the name implies.

Primary application of these sensors is to act like "band filters" in mobile telephones and base stations.

Other applications include:

- Sensing of torques and tire pressures
- Sensing biological and chemical substances
- Sensing vapors, humidity and temperature
- Monitor fluid flow in microfluidics



- 2 sets of "Interdigital Transducers" (IDT) are created on a piezoelectric layer attached to a tiny substrate as shown
- Energize by an AC source to the "Input IDT" will close and open the gaps of the finger electrodes, and thus surface deformation/ stresses transmitting through the piezo- electric material
- The surface deformation/stresses will cause the change of finger electrodes in the "Output IDT"
- Any change of material properties (chemical attacks) or geometry due to torques will alter the I/O between the "Input IDT" and "Output IDT."
- The sensing of contact environment or pressure can thus be accomplished

b) BioMEMS

The term "**BioMEMS**" has been a popular terminology in the MEMS industry in recent years due to the many break-through in this technology, which many believe to be a viable lead to mitigate the sky-rocketing costs in healthcare costs in many industrialized countries.

BioMEMS include the following three major areas:

(1) Biosensors for identification and measurement of biological substances,

(2) Bioinstruments and surgical tools, and

(3) Bioanalytical systems for testing and diagnoses.

Major Technical Issues in BioMEMS Products:

(1) **Functionality** for the intended biomedical operations.

(2) **Adaptive** to existing instruments and equipment.

(3) **Compatibility** with biological systems of the patients.

(4) **Controllability, mobility, and easy navigation** for operations such as those required in laparoscope's surgery.

(5) **Fabrication** of MEMS structures with high aspect ratio (defined as the ratio of the dimensions in the depth of the structure to the dimensions of the surface)

Note: Almost all bioMEMS products are subjected to the approval for marketing by the FDA (Food and Drug Administration) of the US government.

Biomedical Sensors and Biosensors

These sensors are extensively used in medical diagnosis, environmental protection, drug discovery and delivery, etc.

1. Biomedcial Sensors

For the measurements of biological substances in the sample and also for medical diagnosis purposes.

Input signal: Biological sample (e.g., blood samples or body fluids typically in minute amount in μL or nL)

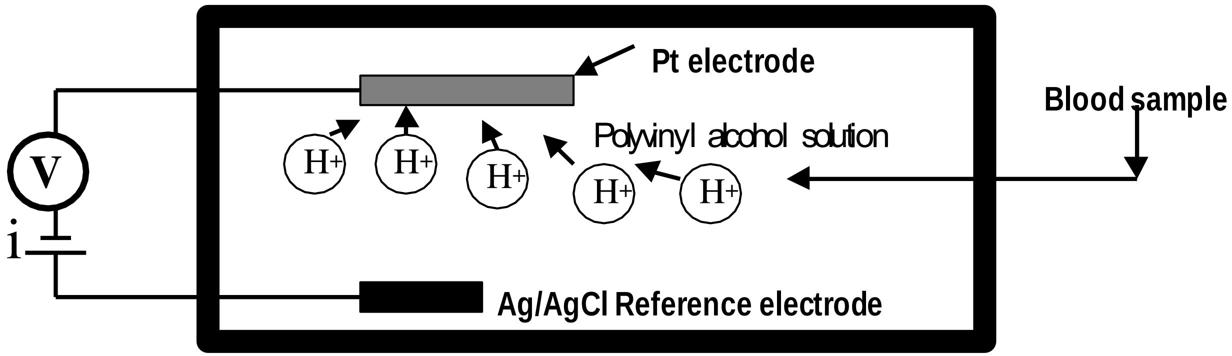
Microsensing element: a chemical that reacts with the sample.

Transduction unit: the product of whatever the chemical reactions between the sample and the chemical in the sensing element will convert itself into electrical signal (e.g. in milli volts, mV).

Output signal: The converted electrical signal usually in mV.

Example of a biomedical sensor:

A sensor for measuring the glucose concentration of a patient.



Working principle:

- The glucose in patient's blood sample reacts with the O_2 in the polyvinyl alcohol solution and produces H_2O_2.

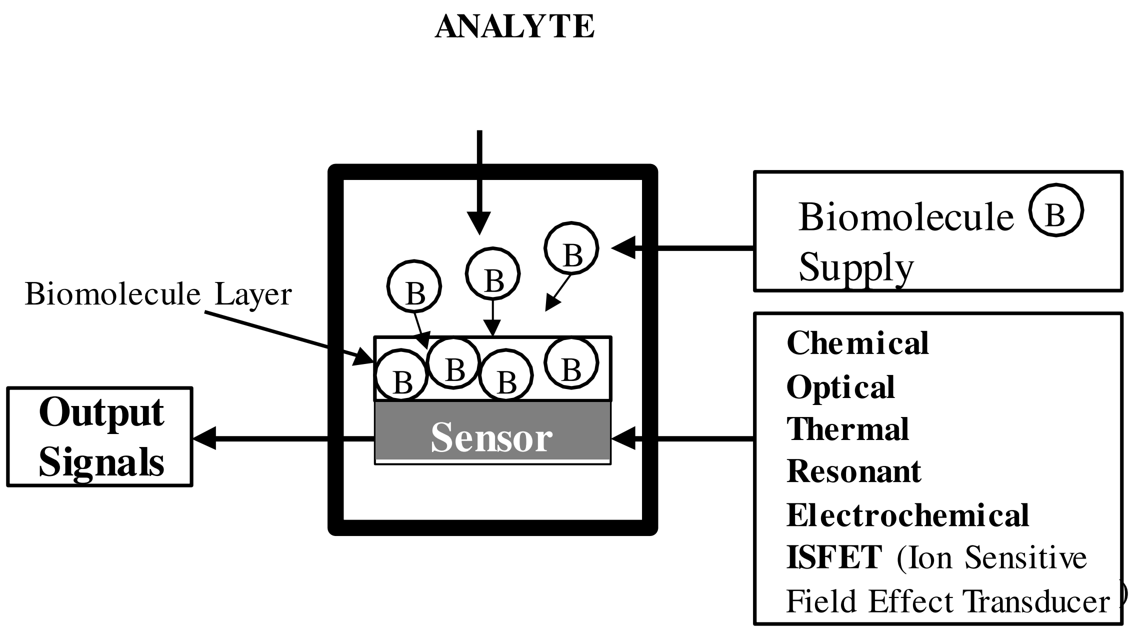
- The H_2 in H_2O_2 migrates toward Pt film in a electrolysis process, and builds up layers at that electrode.

- The difference of potential between the two electrodes due to the build-up of H_2 in the Pt electrode relates to the amount of glucose in the blood sample.

2. Biosensors

These sensors work on the principle of interactions between the biomolecules in the sample and the analyte (usually in solution) in the sensor.

Signal transduction is carried out by the sensing element as shown below:



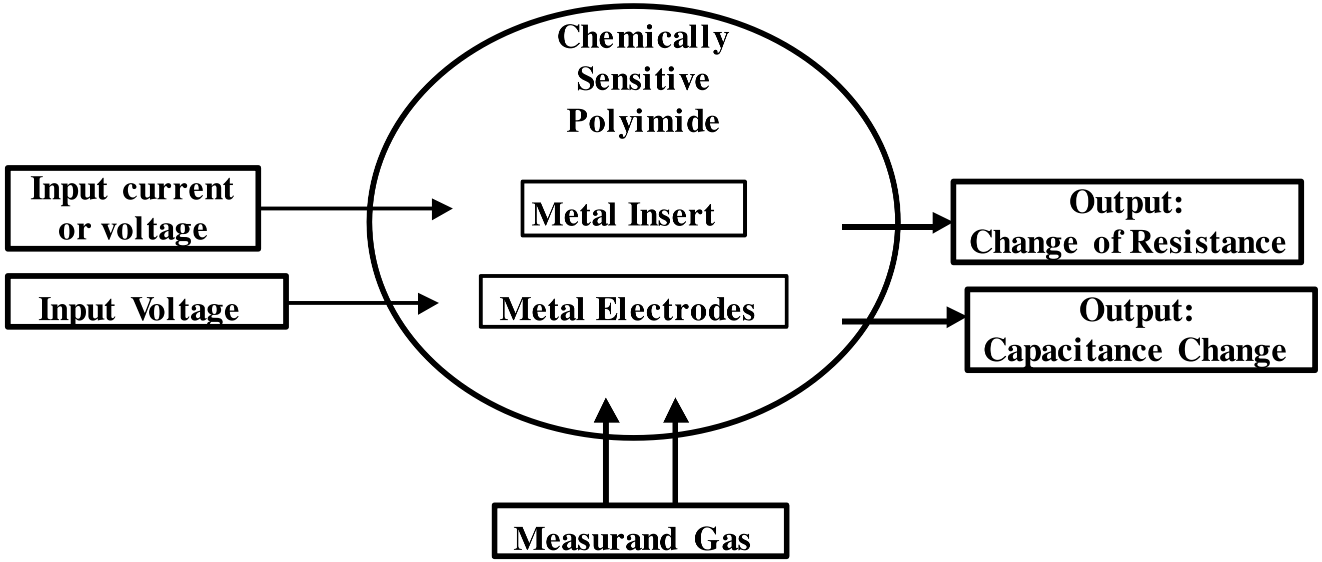
c) Chemical Sensors

- Work on simple principles of chemical reactions between the sample, e.g. O_2 and the sensing materials, e.g., a metal.
- Signal transduction is the changing of the physical properties of the sensing materials after specific type of chemical reactions.

There are four (4) common types of chemical sensors:

(1) Chemiresistor sensors: eg:phthalocyanine used with Cu to sense NH3 and NO2

(2) Chemicapacitor sensors eg: polyphenylacetylene to sense CO,CO2,N2,CH4



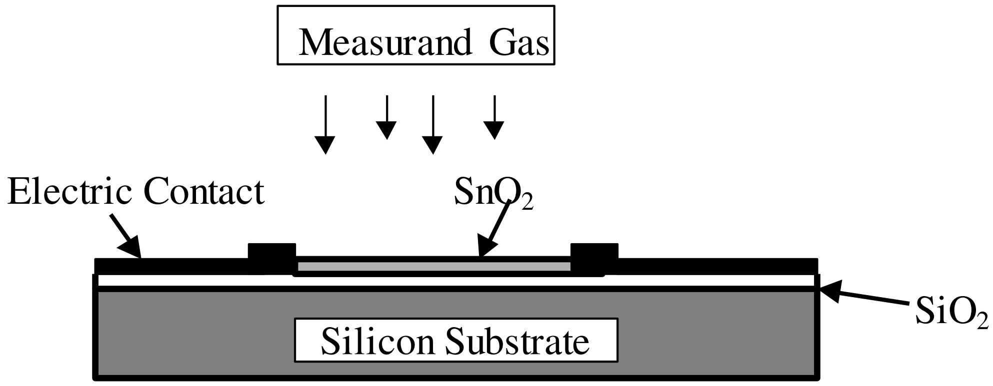
c) Chemical Sensors-Cont'd

(3) Chemimechanical sensors:

Work on certain materials (e.g. polymers) that change shapes when they are exposed to chemicals. Measuring the change of the shape of the sensing materials determines the presence of the chemical. Eg: moisture sensor using pyraline

(4) Metal oxide gas sensors:

Sensing materials: certain semiconducting materials, e.g., SnO_2 change their electrical resistance when exposed to certain chemicals.



c) Chemical Sensors-Cont'd

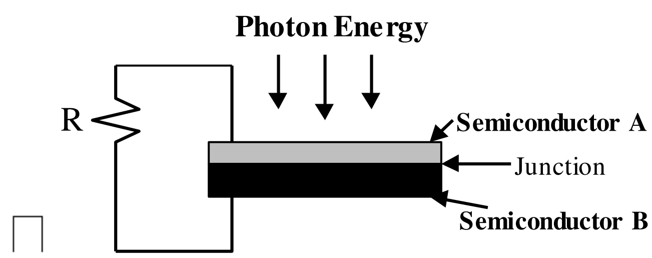
Available metal oxide gas sensors:

Semiconducting Metals	Catalyst Additives	Gas to be Detected
$BaTiO_3/CuO$	La_2O_3, $CaCO_3$	CO_2
SnO_2	Pt + Sb	CO
SnO_2	Pt	Alcohols
SnO_2	Sb_2O_3	H_2, O_2, H_2S
SnO_2	CuO	H_2S
ZnO	V, Mo	Halogenated hydrocarbons
WO_3	Pt	NH_3
Fe_2O_3	Ti-doped + Au	CO
Ga_2O_3	Au	CO
MoO_3	None	NO_2, CO
In_2O_3	None	O_3

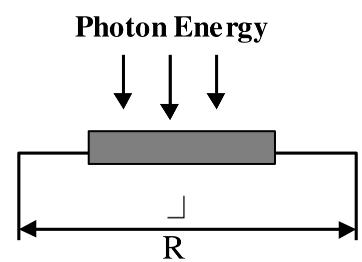
4) Optical Sensors

- These sensors are used to detect the intensity of lights.
- It works on the principle of energy conversion between the photons in the incident light beams and the electrons in the sensing materials.
- The following four (4) types of optical sensors are available:

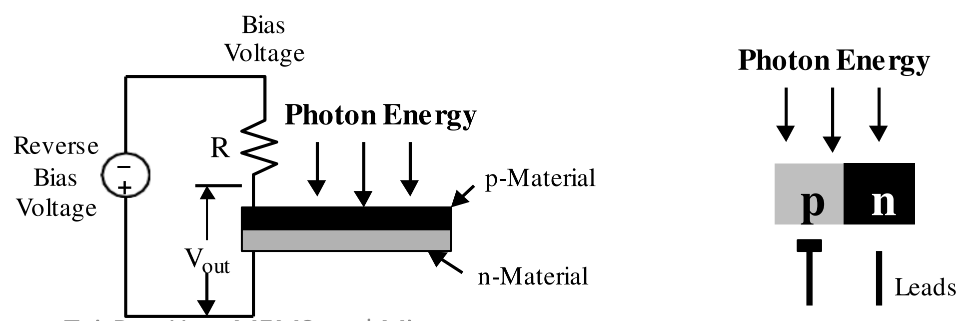
Semiconductor A is more transparent to photon energy in incident light



(a) Photovoltaic junction

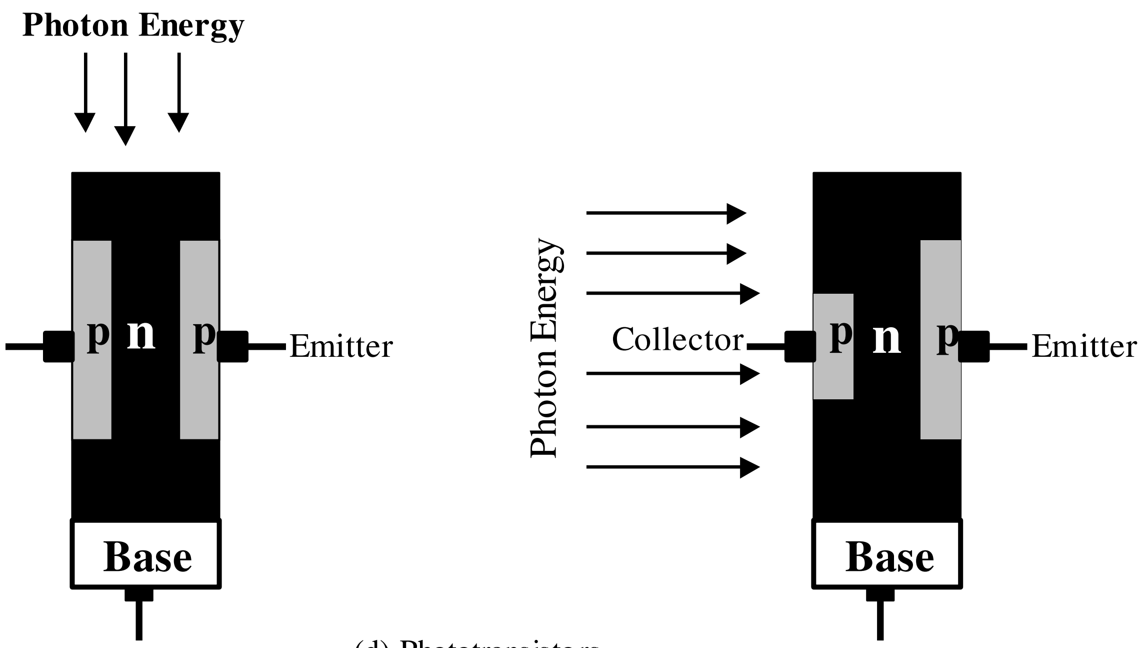


(b) Photoconductive device



(c) Photodiodes

4) Optical Sensors contd..



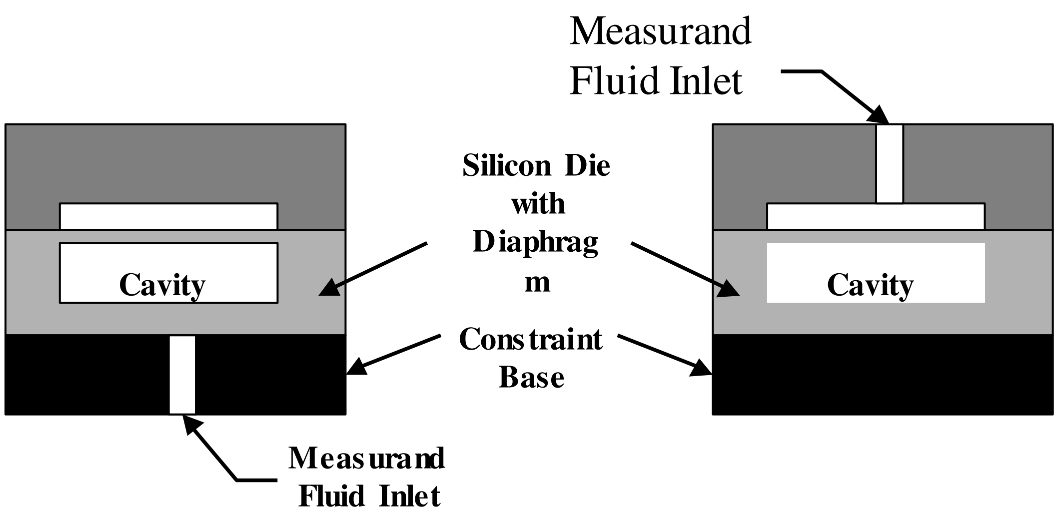
(d) Phototransistors

Silicon (Si) and Gallium arsenide (GaAs) are common sensing materials.
GaAs has higher electron mobility than Si- thus higher quantum efficiency.

Other materials, e.g. Lithium (Li), Sodium (Na), Potassium (K) and Rubidium (Rb) are used for this purpose.

5) Pressure Sensors

- Micro pressure sensors are used to monitor and measure minute gas pressure in environments or engineering systems, e.g. automobile intake pressure to the engine.
- They are among the first MEMS devices ever developed and produced for "real world" applications.
- Micro pressure sensors work on the principle of mechanical bending of thin silicon diaphragm by the contact air or gas pressure.

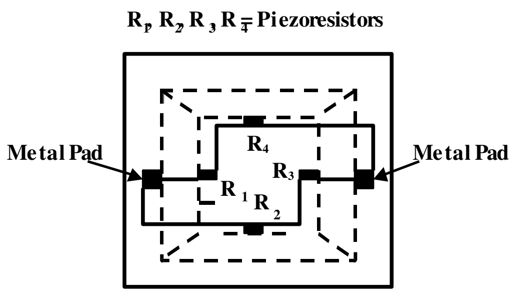


(a) Back side pressurized

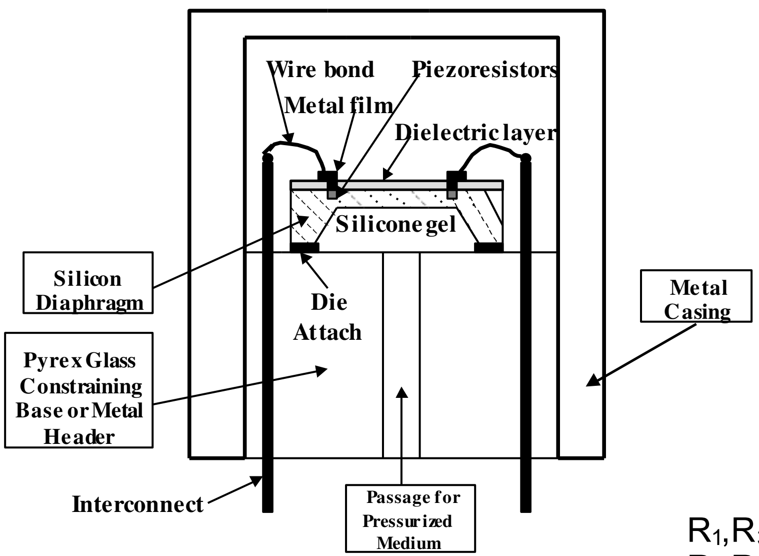
(b) Front side pressurized

5) Pressure Sensors Contd..

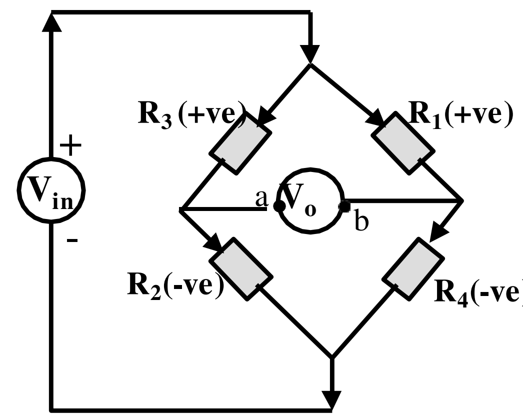
- The strains associated with the deformation of the diaphragm are measured by tiny "piezoresistors" placed in "strategic locations" on the diaphragm.



Top view of silicon die



- These tiny piezoresistors are made from doped silicon. They work on the similar principle as "foil strain gages" with much smaller sizes (in μm), but have much higher sensitivities and resolutions.



Wheatstone bridge for signal transduction

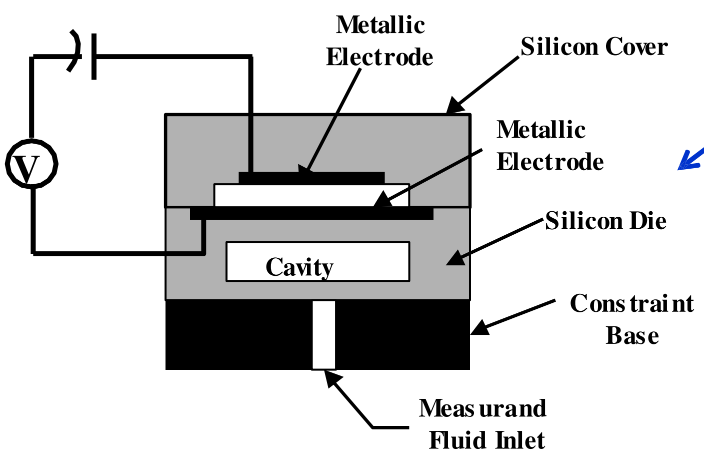
$$V_o = V_{in}\left(\frac{R_1}{R_1 + R_4} - \frac{R_3}{R_2 + R_3}\right)$$

R_1, R_3 = resistance induced by longitudinal and transverse stresses
R_2, R_4 = reference resistors

Tai-Ran Hsu, MEMS and Microsystems Design and Manufacture, TMH, 2002

5) Pressure Sensors Contd..

- Other ways of transducing the deformation of the diaphragm to electronic output signals are available, e.g.,

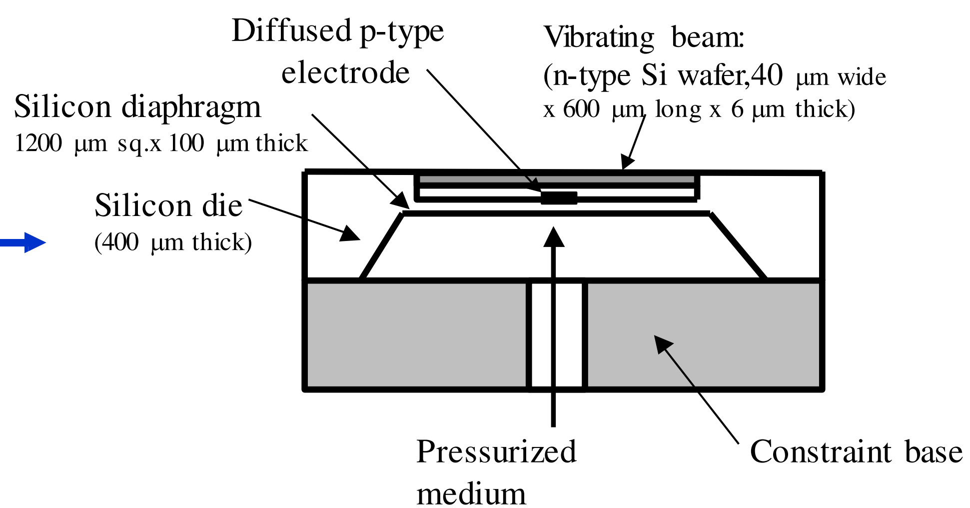


Signal output: capacitance changes (for higher temperature applications)

$$C = \varepsilon_r \varepsilon_o \frac{A}{d}$$

ε_r= Relative permittivity = 1.0 with air
ε_o = Permittivity in vacuum = 8.85 pF/m
A = Overlap area
D = Gap between plate electrodes

By resonant vibration (for higher resolutions) **Signal output:** Shift of resonance frequencies by change of stresses in lower plate electrode by applied pressure loading



Two Common Types of Micro Pressure Sensors

Sensors using piezoresistors:

- Small in size ⌉ Linear I/O relation ⌉ Temperature sensitive

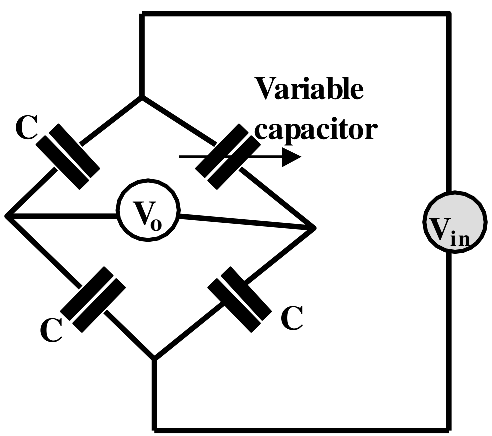
Sensors using capacitances:

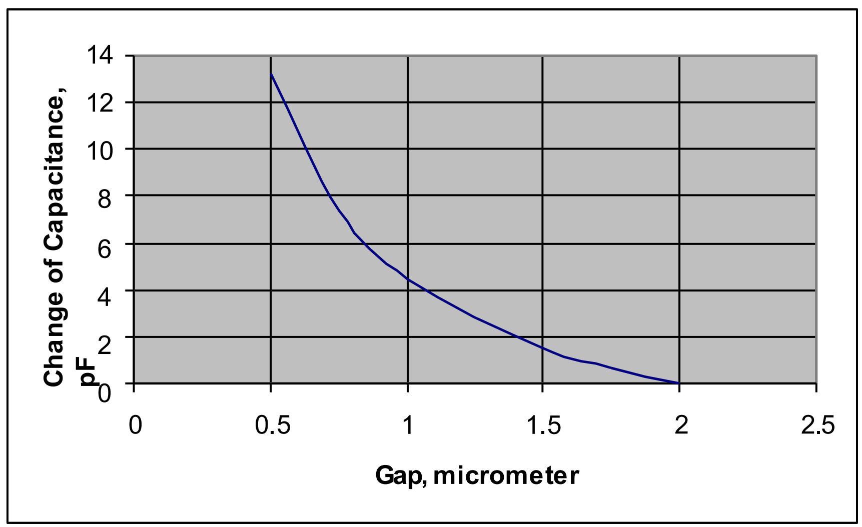
- Tends to be bulky □ Suited for elevated temperature application
- *Nolinear I/O relations* • Lower cost

Nonlinear I/O with plate pressure sensors using electrodes

Electric circuit bridge for converting capacitance changes to voltage output:

$$V_o = \frac{\Delta C}{2(2C + \Delta C)} V_{in}$$





5) Pressure Sensors Contd..

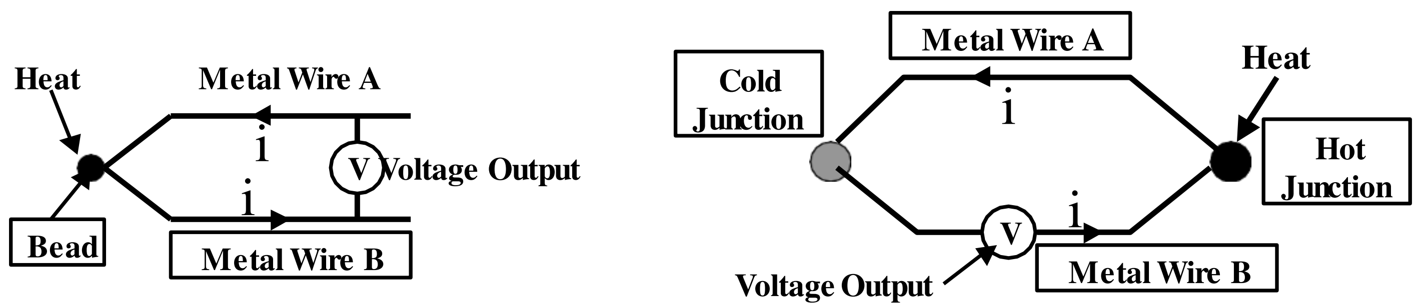
- Major problems in pressure sensors are in the *system packaging* and *protection of the diaphragm* from the contacting pressurized media, which are often corrosive, erosive, and at high temperatures.

6) Thermal Sensors

- Thermal sensors are used to monitor, or measure temperature in an environment or of an engineering systems.

- Common thermal sensors involve thermocouples and thermopiles.

- Thermal sensors work on the principle of the electromotive forces (emf) generated by heating the junction made by dissimilar materials (beads):



(a) A thermocouple (b) A dual junction thermocouple

The generated voltage (V) by a temperature rise at the bead (ΔT) is:

$$V = \beta \Delta T$$

where β = Seebeck coefficient

Thermal Sensors contd..

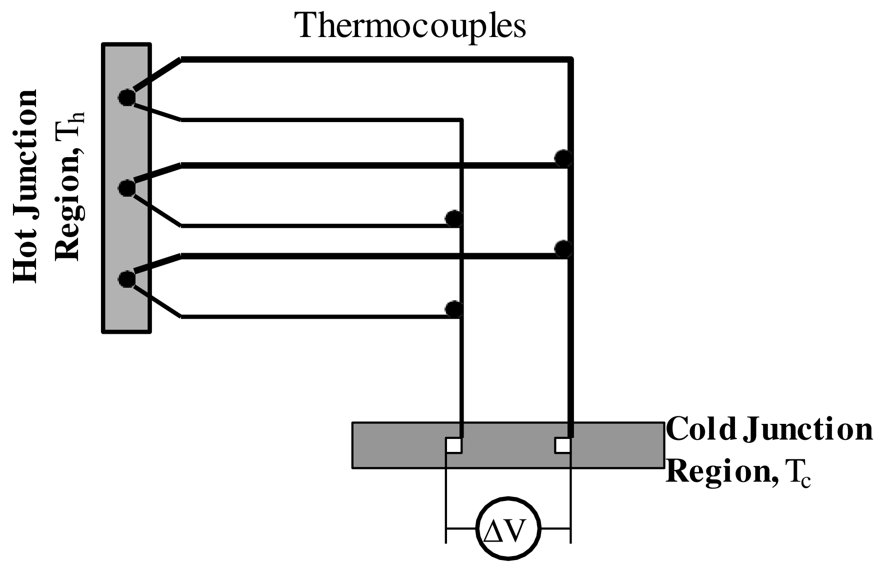
The Seebeck coefficients for various thermocouples are:

Type	Wire Materials	Seebeck Coefficient (μV/°C)	Range (°C)	Range (mV)
E	Chromel/Constantan	58.70 at 0°C	-270 to 1000	-9.84 to 76.36
J	Iron/Constantan	50.37 at 0°C	-210 to 1200	-8.10 to 69.54
K	Chromel/Alumel	39.48 at 0°C	-270 to 1372	-6.55 to 54.87
R	Platinum (10%)-Rh/Pt	10.19 at 600°C	-50 to 1768	-0.24 to 18.70
T	Copper/Constantan	38.74 at 0°C	-270 to 400	-6.26 to 20.87
S	Pt (13%)-Rh/Pt	11.35 at 600°C	-50 to 1768	-0.23 to 21.11

Common thermocouples are of K and T types

Thermal Sensors contd..

Thermopiles are made of connecting a series of thermocouples in parallel:



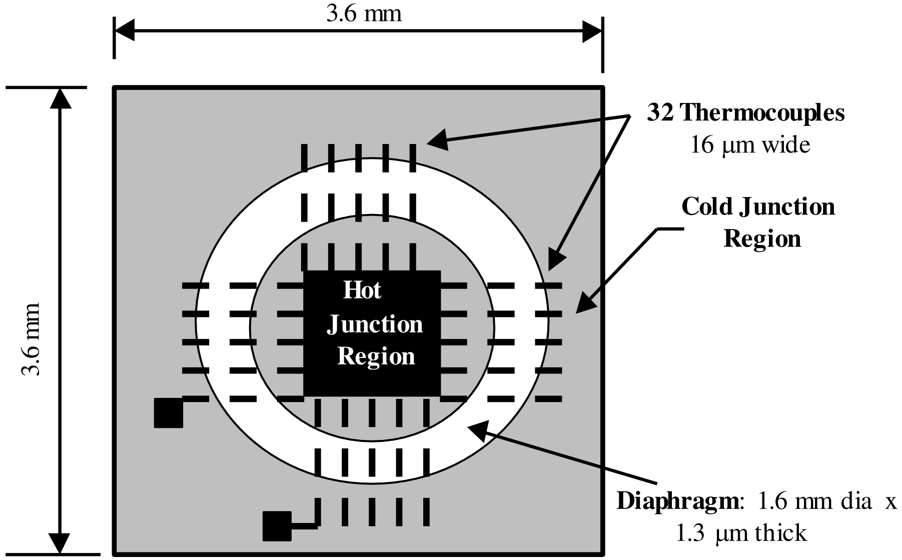
The induced voltage (ΔV) by the temperature change at the hot junction (ΔT) is:

$$\Delta V = N \beta \Delta T$$

with N = number of thermocouple pairs in the thermopile.

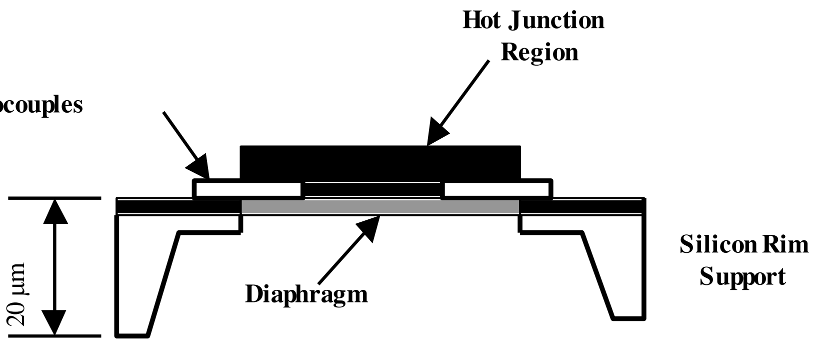
Thermal Sensors contd..

A micro thermal sensor:



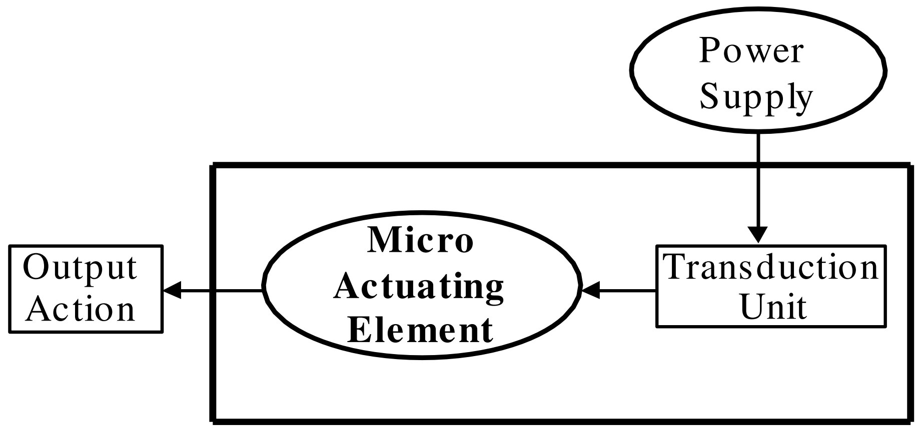
Top view

- 32 polysilicon-gold thermocouples
- dimension of thermopile is:
  3.6 mm x 3.6 mm x 20 μm thick
- Typical output is 100 mV
- Response time is 50 ms



Elevation

Working Principles for Microactuators



Power supply: Electrical current or voltage

Transduction unit: To covert the appropriate form of power supply into the desired form of actions of the actuating element

Actuating element: A material or component that moves with power supply

Output action: Usually in a prescribed motion

Actuators

- A mechanical device for moving or controlling something
- Important part of microsystem
- Four principal means of actuation
  1. Thermal forces
  2. Shape memory alloys
  3. Piezoelectric crystals
  4. Electrostatic forces

An actuator is designed to deliver a desired motion when driven by a power source

Eg: electric relay, inkjet printer heads

- Driving power for actuators depends on its application

Eg: on-off switches→ deflection of bimetallic strip as a result of resistance heating the strip with passing electric current

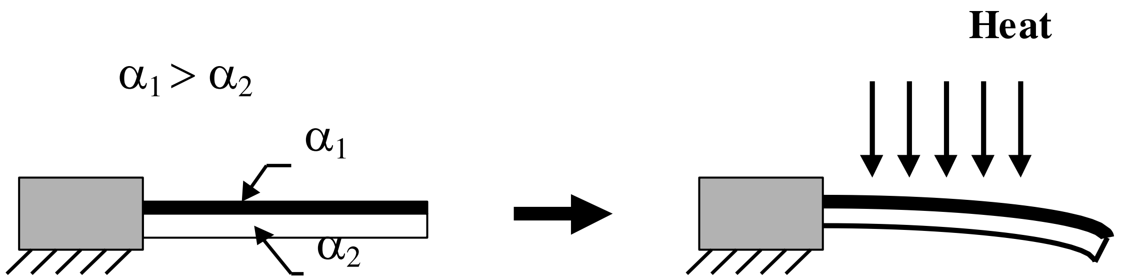
Motors, solenoid devices→ electromagnetic induction

Actuation Using Thermal Forces

- Solids deform when they are subjected to a temperature change (ΔT)

- A solid rod with a length L will extend its length by $\Delta L = \alpha \Delta T$, in which α = coefficient of thermal expansion (CTE) – a material property.

- When two materials with distinct CTE bond together and is subjected to a temperature change, the compound material will change its geometry as illustrated below with a compound beam (bimetallic strip)



- It will return to its original shape after the removal of the heat

- These compound beams are commonly used as microswitches, relays, microclamps and valves in MEMS products.

Actuation Using Shape Memory Alloys (SMA)

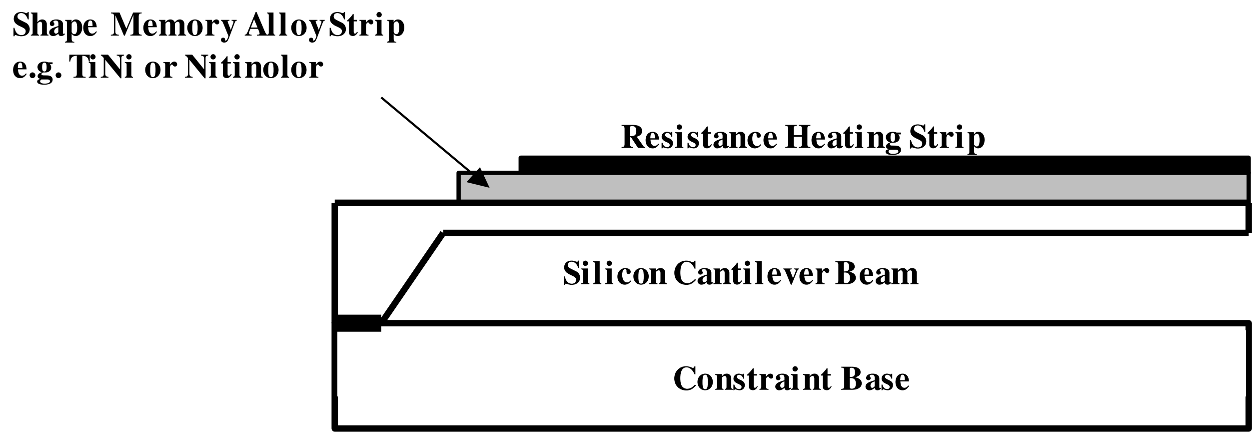
- SMA are the materials that have a “memory” of their original geometry (shape) at a typically elevated temperature of production.

- These alloys are deformed into different geometry at typically room temperature.

- The deformed SMA structures will return to their original shapes when they are heated to the elevated temperature at their productions.

- Nitinolor or Ti-Ni alloys are common SMAs.

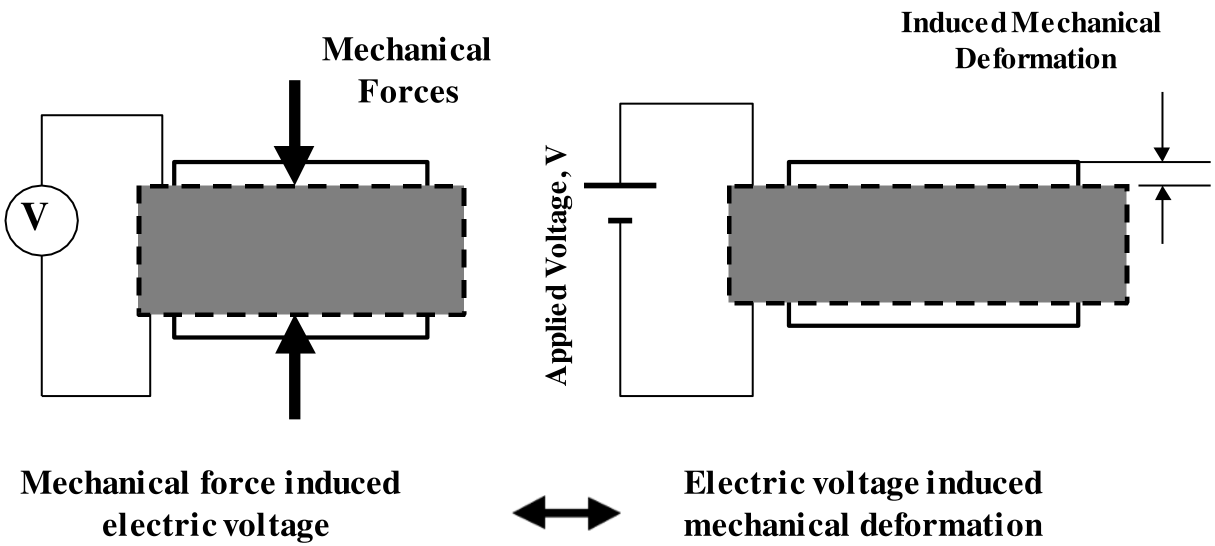
- A microswitch actuated with SMA:



Actuation Using Piezoelectric Crystals

- A certain crystals, e.g., quartz exhibit an interesting behavior when subjected to a mechanical deformation or an electric voltage.→ ***piezoelectric effect***

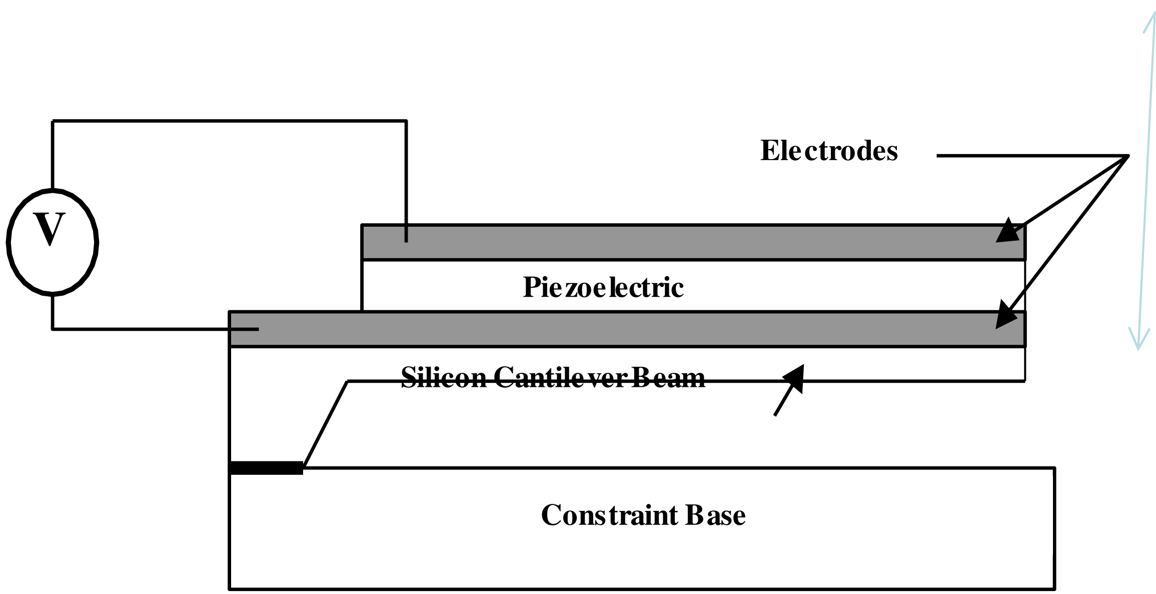
- This behavior may be illustrated as follows:



- This peculiar behavior makes piezoelectric crystals an ideal candidate for microactuation as illustrated in the following case:

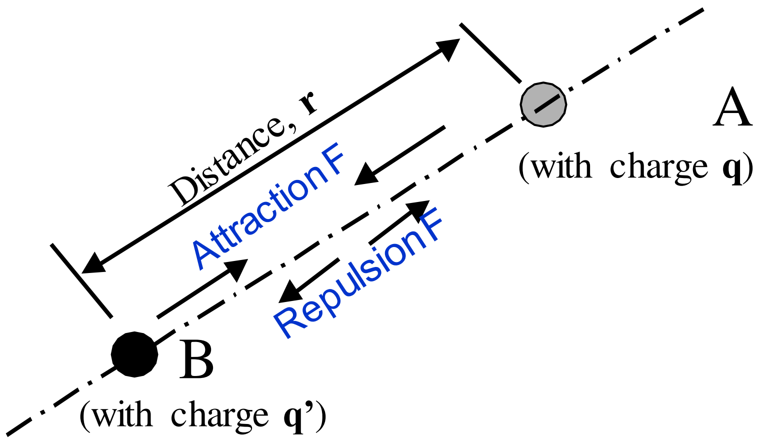
Actuation Using Piezoelectric Crystals

A micro relay or microelectrical switch



Actuation Using Electrostatic Forces

- Electrostatic Force between Two Particles
- **The Coulomb's Law**: The electrostatic force F is defined as the electrical force of attraction or repulsion induced by an electric field E

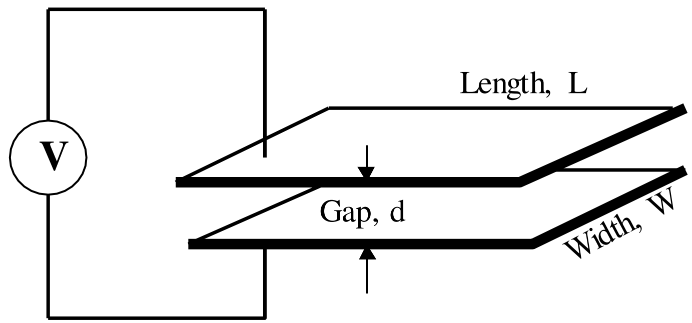


The attraction or repulsive force: $$F = \frac{1}{4\pi\varepsilon}\frac{qq'}{r^2}$$

where ε = permittivity of the medium between the two particles
= 8.85 x 10^{-12} $C^2/N\text{-}m^2$ or 8.85 pF/m in vacuum (= ε_o)
r = Distance between the particles (m)

Actuation Using Electrostatic Forces

- Electrostatic Force Normal to Two Electrically Charged Plates:



- The induced capacitance, C : $C = \varepsilon_r \varepsilon_o \frac{A}{d} = \varepsilon_r \varepsilon_o \frac{WL}{d}$

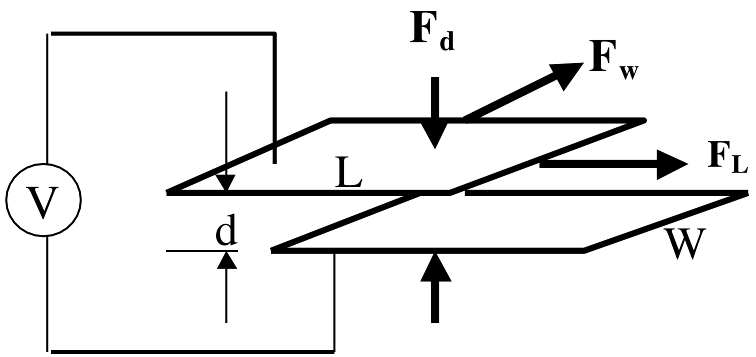
- The induced normal force, F_d is:

$$F_d = -\frac{1}{2} \frac{\varepsilon_r \varepsilon_o WL}{d^2} V^2$$

in which ε_r = relative permittivity of the dielectric material between the two plates

Actuation Using Electrostatic Forces

- Electrostatic Force Parallel to Two Misaligned Electrically Charged Plates:



- Force in the "Width" direction:

$$F_w = -\frac{1}{2}\frac{\varepsilon_r\,\varepsilon_o\,L}{d}V^2$$

- Force in the "Length" direction:

$$F_L = -\frac{1}{2}\frac{\varepsilon_r\,\varepsilon_o\,W}{d}V^2$$

Actuation Using Electrostatic Forces

- These electrostatic forces are prime driving forces of micro motors, comb drivers in micro grippers.

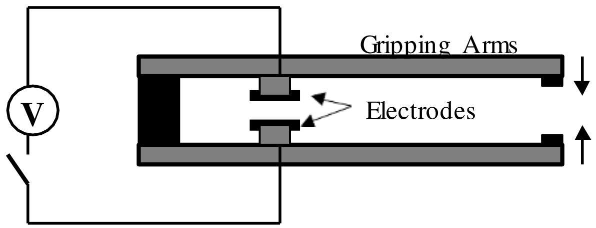
- **Drawback of electrostatic actuation**: force generated by this method is usually low in magnitude.

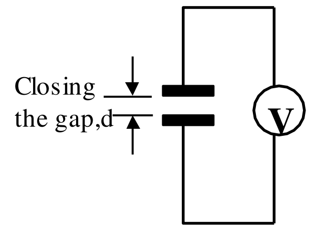
- Application limited to actuators for optical switches, microgrippers.

Applications of Microactuators

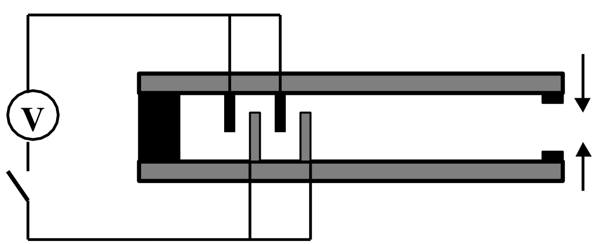
Microgrippers An essential component in microrobots in assembly microassemblies and surgery

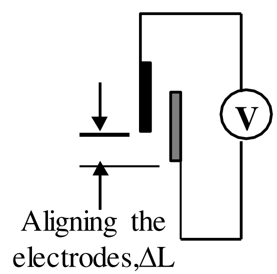
Two gripping methods:





The normal plate electrodes
- Not practical b/c requiring more space.

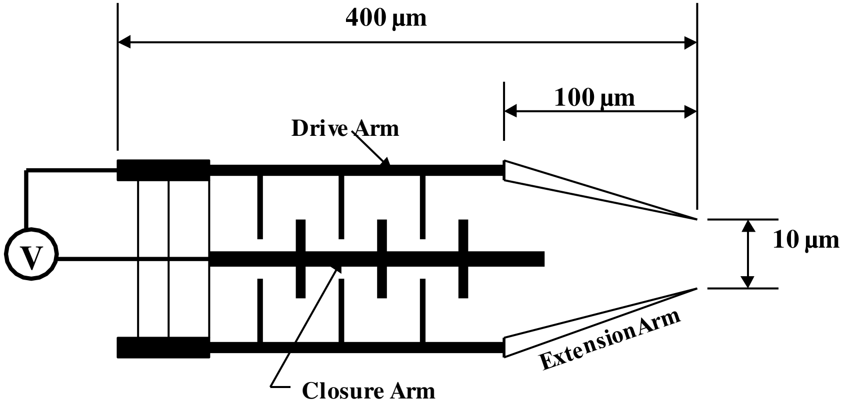




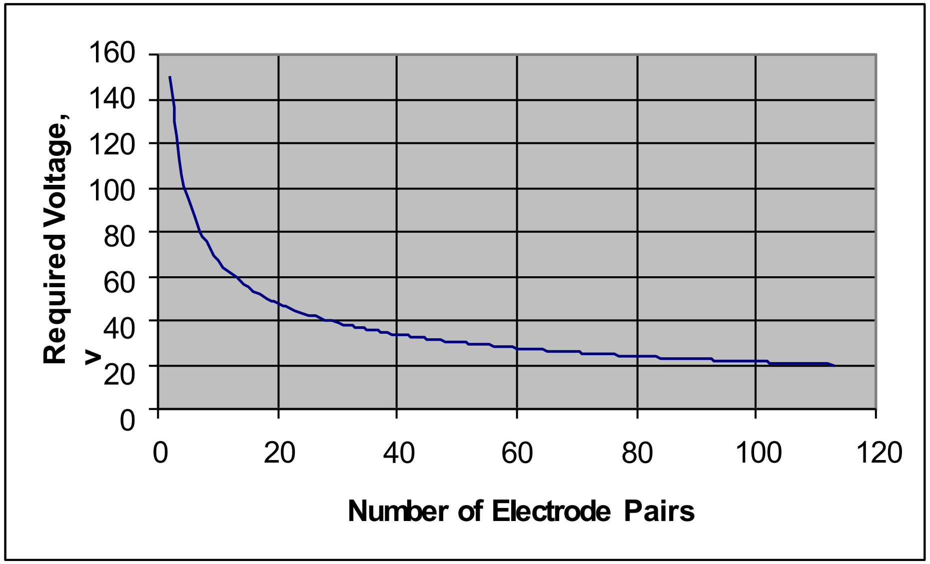
The sliding plate electrodes
- Popular method. Can have many sets to make “Comb drive” actuators

A Typical Microgripper with “Comb drive” Actuators:

Arrangement of electrodes:



Drastic reduction in required actuation voltage with increase of number of pairs of electrodes:



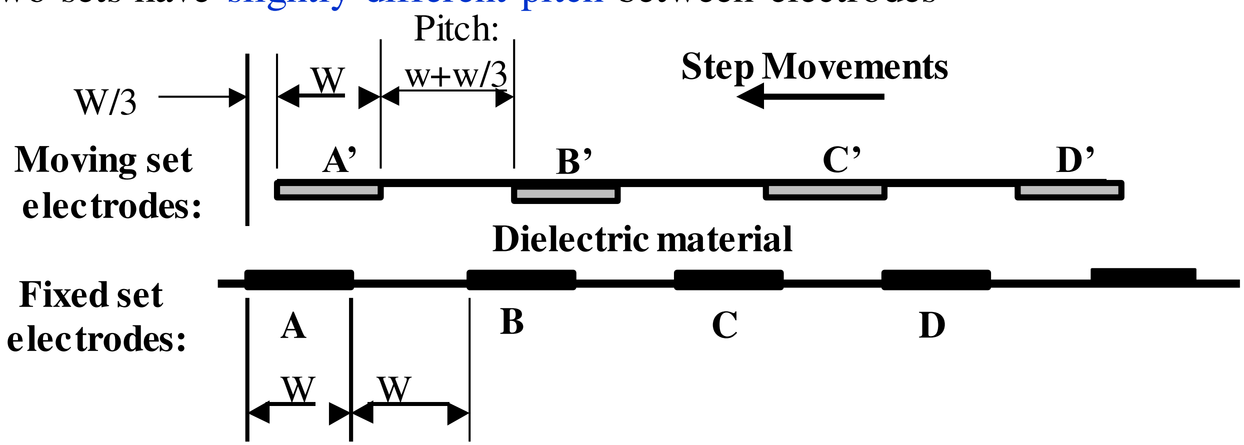
Applications of Microactuators

Micromotors

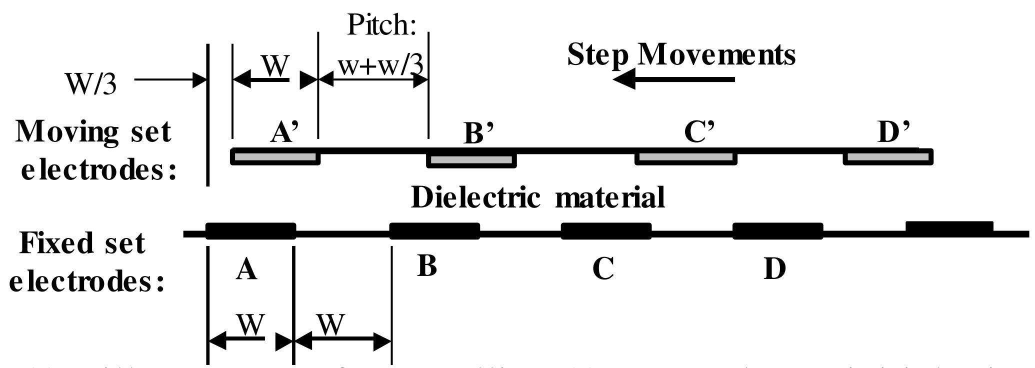
Unlike traditional motors, the *driving forces* for micro motors is primarily the parallel electrostatic forces between pairs of misaligned electrically charged plates (electrodes), as will be demonstrated in the following two cases:

Linear stepping motors:

- Two sets of electrodes in the form of plates separated by dielectric material (e.g. quartz film).
- One electrode set is fixed and the other may slide over with little friction.
- The two sets have slightly different pitch between electrodes



Applications of Microactuators

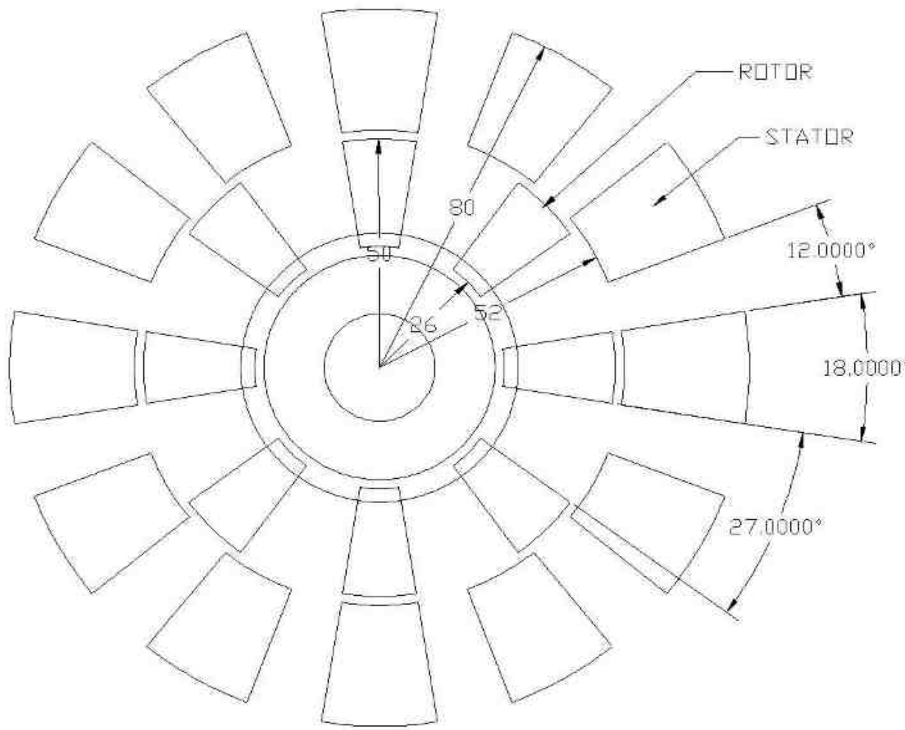


- Energize the set A-A' will generate a force pulling A' over A due to initial misalignment.
- Once A and A' are aligned, the pair B and B' become misaligned.
- Energize the misaligned B-B' will generate electrostatic force pulling B' over B.
- It is now with C' and C being misaligned.
- Energize C' and C will produce another step movement of the moving set over the stationary set.
- Repeat the same procedure will cause continuous movements of the moving sets
- The step size of the motion = w/3, or the size of preset mismatch of the pitch between the two electrode sets.

Applications of Microactuators

Rotary stepping motors:

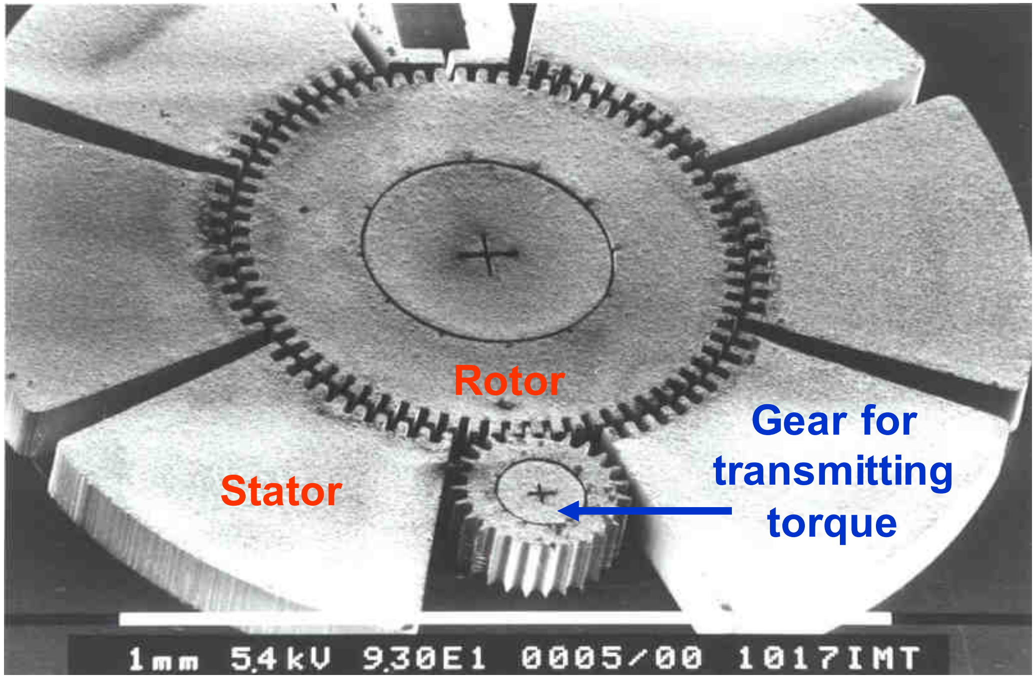
- Involve two sets of electrodes- one set for the rotor and the other for the stator.
- Dielectric material between rotor and stator is air.
- There is preset mismatch of pitches of the electrodes in the two sets.



Applications of Microactuators

- Working principle of this rotary motor is similar to that in linear motors.

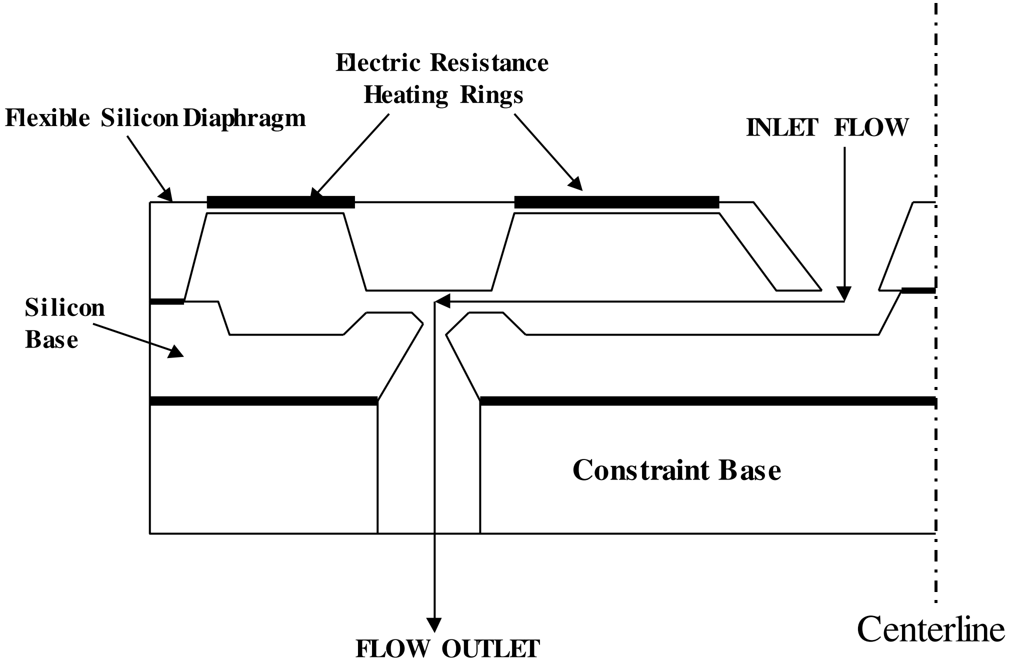
A micro motor produced by Karlsruhe Nuclear Research Center, Germany:



Applications of Microactuators

Microvalves

- A special microvalve designed by Jerman in 1990.
- Circular in geometry, with diaphragm of 2.5 mm in diameter x 10 μm thick.
- The valve is actuated by thermal force generated by heating rings.
- Heating ring is made of aluminum films 5 μm thick.
- The valve has a capacity of 300 cm^3/min at a fluid pressure of 100 psig.
- Power consumption is 1.5 W.

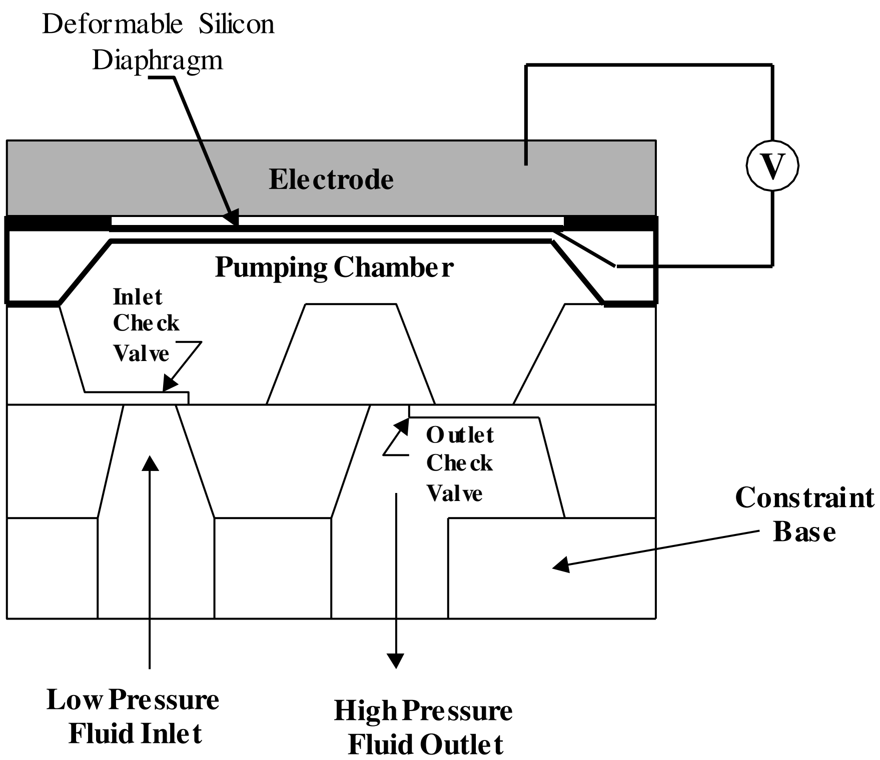


Applications of Microactuators

Micropumps

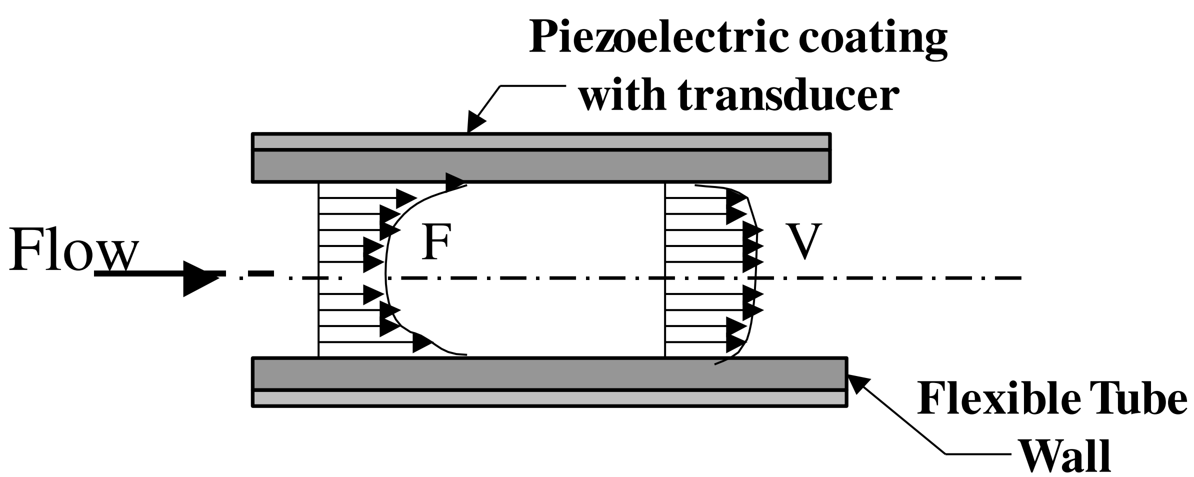
1. Electrostatically actuated micropump:

- An electrostatic actuated pump in 1992.
- The pump is of square geometry with 4 mm x 4mm x 25 μm thick.
- The gap between the diaphragm and the electrode is 4 μm.
- Pumping rate is 70 μL/min at 25 Hz.



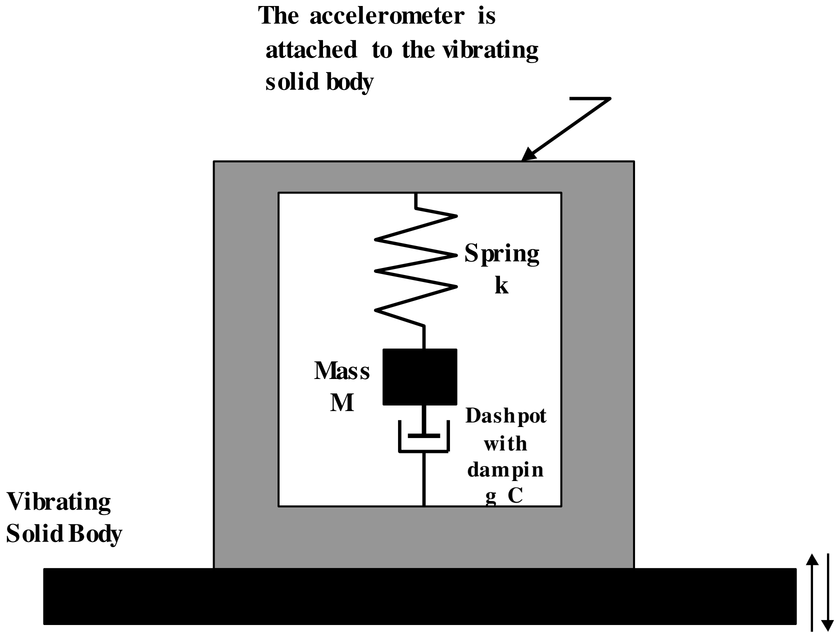
2. Piezoelectrically actuated pump:

- An effective way to pump fluid through capillary tubes.
- Tube wall is flexible.
- Outside tube wall is coated with piezoelectric crystal film, e.g. ZnO with aluminum interdigital transducers (IDTs).
- Radio-frequency voltage is applied to the IDTs, resulting in mechanical squeezing in section of the tube (similar to the squeezing of toothpaste)
- Smooth flow with "uniform" velocity profile across the tube cross section.

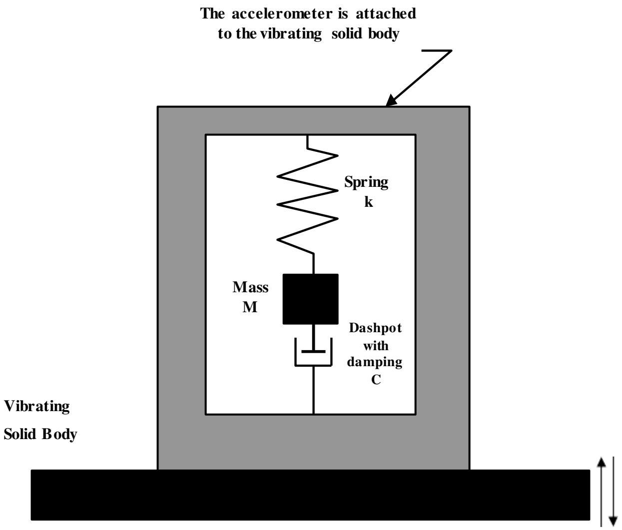


Microaccelerometers

- Accelerometers are used to measure dynamic forces associated with moving objects.
- These forces are related to the velocity and acceleration of the moving objects.
- Traditionally an accelerometer is used to measure such forces.
- A typical accelerometer consists of a "proof mass" supported by a spring and a "dashpot" for damping of the vibrating proof mass:



Microaccelerometers contd..



- The instantaneous displacement of the mass y(t) induced by the attached moving solid body is measured and recorded with respect to time, t.

- The associated velocity, V(t) and the acceleration α(t) may be obtained by the following derivatives:

$$V(t) = \frac{dy(t)}{dt} \quad and \quad \alpha(t) = \frac{dy(t)}{dt} = \frac{d^2y(t)}{dt^2}$$

- The associated dynamic force of induced by the moving solid is thus obtained by using the Newton's law, i.e. F(t) = M α(t), in which M = the mass of the moving solid.

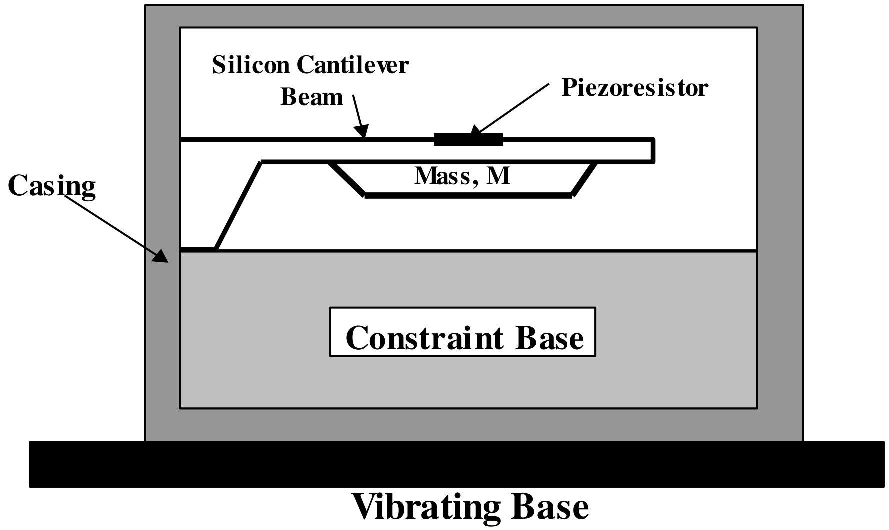
- In miniaturizing the accelerometers to the micro-scale, there is no room for the coil spring and the dashpot for damping on the vibrating mass.

- Alternative substitutes for the coil spring, dashpot, and even the proof mass need to be found.

Microaccelerometers contd..

- There are two types micro accelerometers available.

(1) The cantilever beam accelerometer:



In this design: Cantilever beam = coil spring;
Surrounding viscous fluid = dashpot for damping of the proof mass

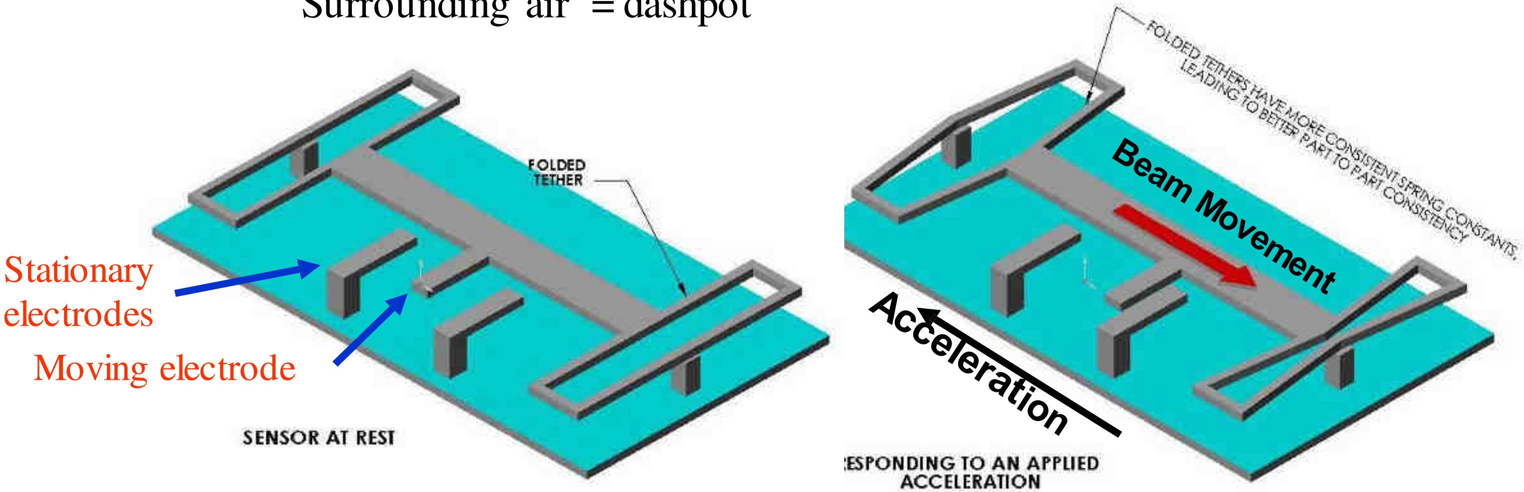
The movement of the proof mass is carried out by the attached piezoresistor.

Microaccelerometers contd..

(2) Balanced force micro accelerometer:

- This is the concept used in the "air-bag" deployment sensor in automobiles

- In this design: Plate beam = proof mass;
  Two end tethers = springs
  Surrounding air = dashpot



- The movement of the proof mass is carried out by measuring the change of capacitances between the pairs of electrodes.

Review of Mechanical concepts

Stress

- Stress is developed in response to mechanical loading.
- Newton's three laws of motion is the foundation for analyzing the static and dynamic behaviors of MEMS devices under loading.

Newton's laws	Statement
Newton's First Law of Motion (The Law of Inertia)	Every object in a state of uniform motion tends to remain in the state of motion unless an external force is applied to it.
Newton's Second Law of Motion	The relationship between an object's mass m, its acceleration a, and the applied force F is $F = ma$. Acceleration and forces are vectors. The direction of the force vector is the same as the direction of the acceleration vector.
Newton's Third Law of Motion	For every action there is an equal and opposite reaction.

- These laws are used to analyze force distribution inside a material, which gives rise to stress and strain.

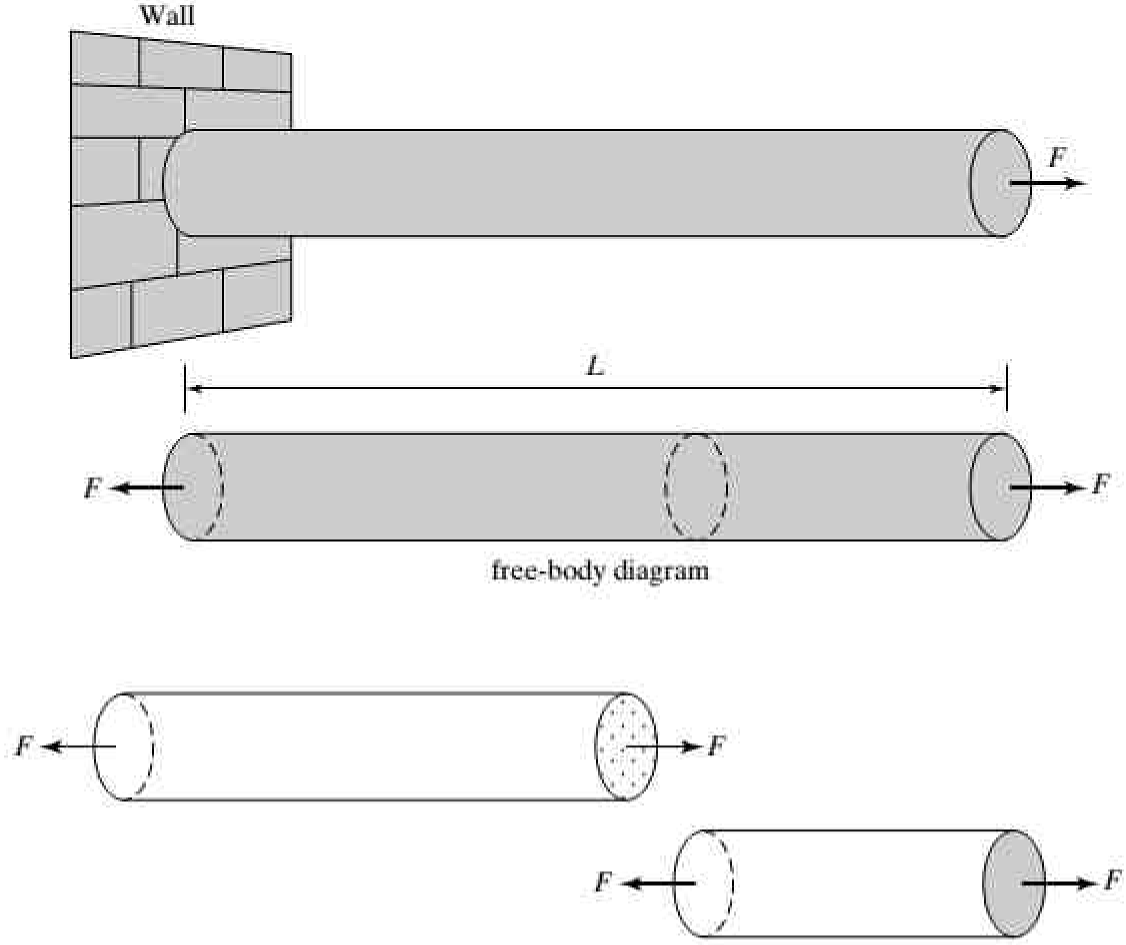


Fig: Force balance analysis

Chang Liu, Foundations of MEMS, Pearson
2012

- Consider a bar firmly embedded in a brick wall with an axial force F applied at the end.
- Since the force is transmitted through the bar to the wall, the wall must produce a reaction according to the Newton's Third Law.
- The wall would act on the left end of the bar with an unknown force.
- To expose and quantify this force, we imaginarily remove the wall, and replace it with the actions it imparts on the bar.
- This free-body diagram of the bar clearly reveals that the wall must provide an axial force with equal magnitude but opposite direction to the applied force, so that the total force on the bar is zero to maintain its stationary status (Newton's First Law).
- We can use this technique to expose and quantify hidden forces and stresses at any section.
- Since the bar is in equilibrium, any part of it must be in equilibrium as well.
- We can pick an arbitrary section of interest, and imaginarily cut the bar into two halves.
- Since a force is applied at the free end of the bar, an equal but opposite force must develop at the cross section.

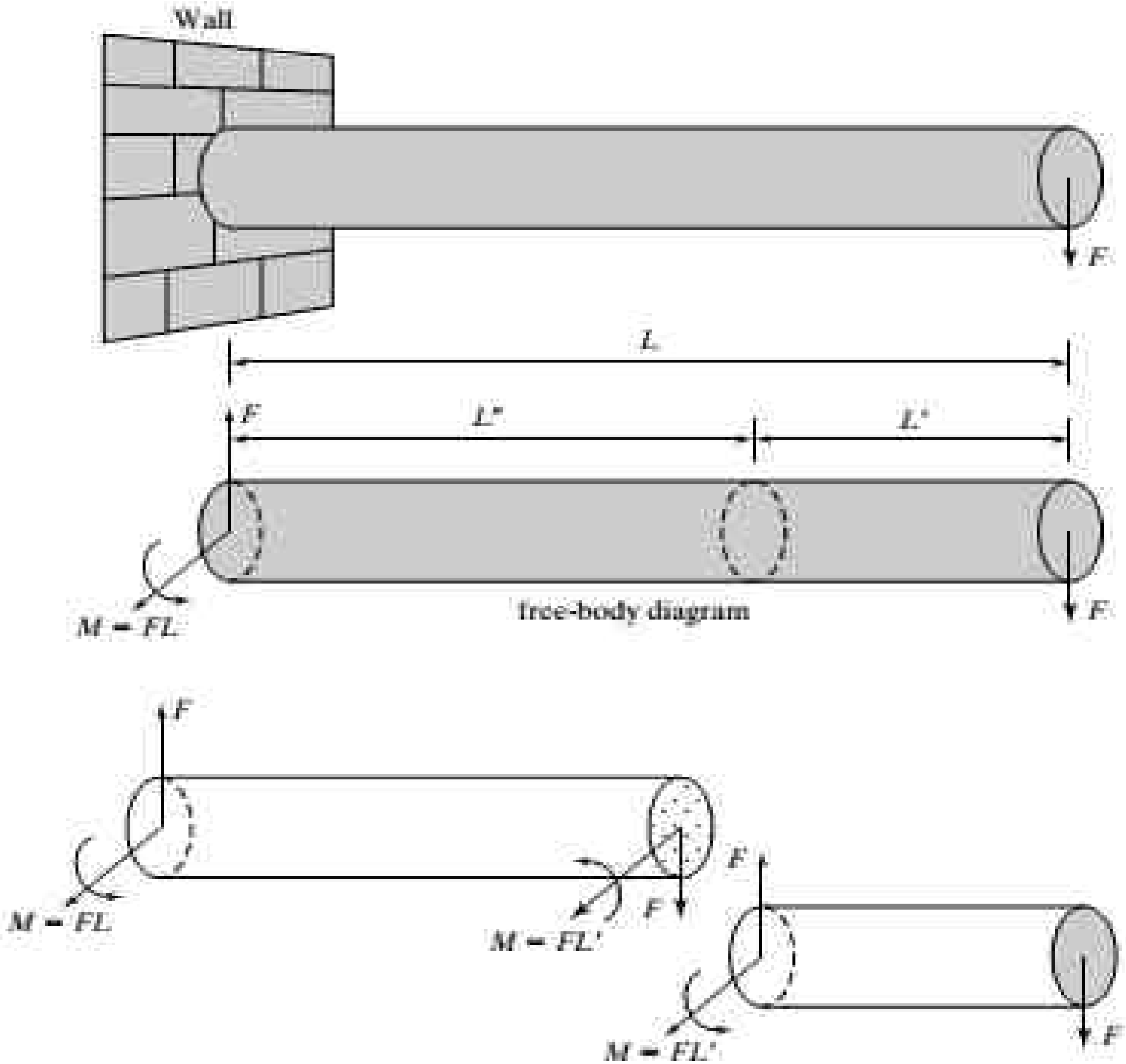


Fig: Force and moment balance analysis.

- Now let us consider the same bar under a force acting in the transverse direction.
- Isolate the bar: The sum of forces and moments acting on the isolated bar must be zero.
- For the net force to be zero, a force of same magnitude but opposite sign must act on the end of the bar attached to the wall.
- The pair of force, however, creates a torque (also referred to as a couple or a moment in mechanics) with the magnitude being F times L, the length of the bar.
- A reactive torque, with the magnitude of F times L but opposite sign, must act on the end of the bar attached to the wall. The imaginarily cut section on the piece to the left would have exactly opposite force and torque as the opposing surface (according Newton's Third Law).
- The magnitude of the sum of torques on the left-hand piece is equal to , which equals the force multiplied by the length of the left-hand piece.
- The net force and torque acting on the left hand piece are both zero.

Definitions of Stress and Strain

Mechanical stresses fall into two categories

1. normal stress
2. shear stress.

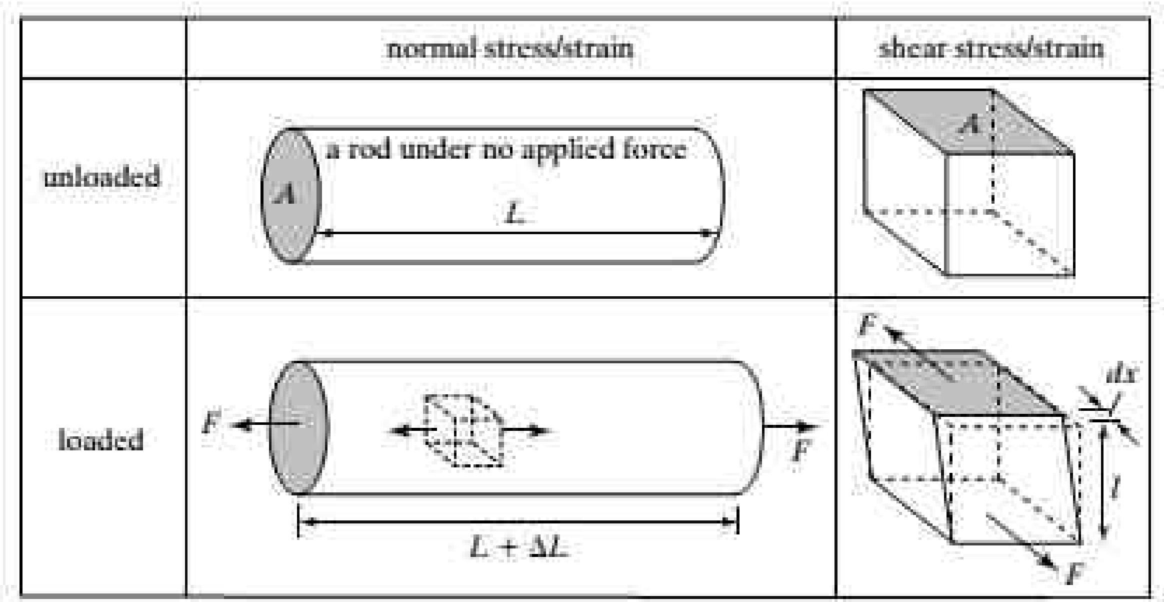


Fig: Normal stress and shear stress.

- If we pull on the rod in its longitudinal direction, it will experience tension and the length of the rod will increase.
- The internal stress in the rod is exposed if we make an imaginary cut through the rod at section.
- The intensity of this force is called the stress.
- If the stress acts in a direction perpendicular to the cross section, it is called a **normal stress**.
- The normal stress, commonly denoted as is defined as the force applied on a given area (A).

$$\sigma = \frac{F}{A}.$$

The SI unit of stress is N/m2 or Pa.

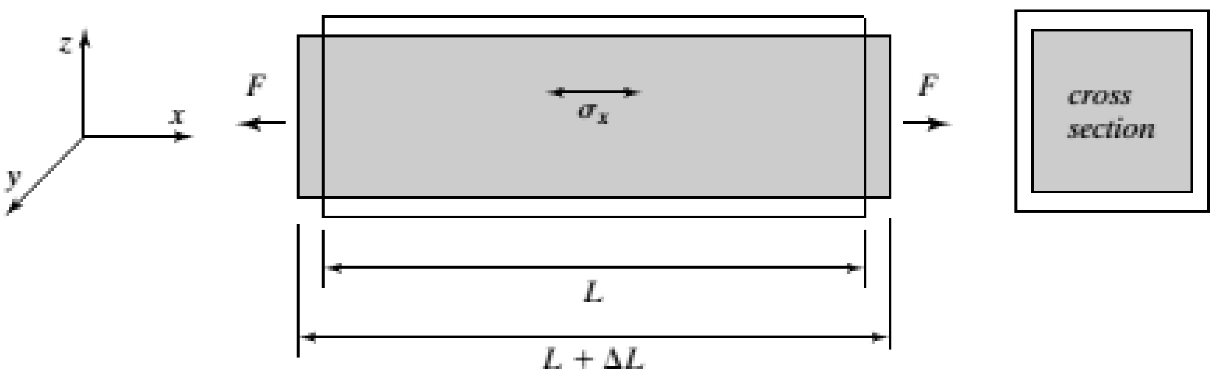
- A normal stress can be **tensile** (as in the case of pulling along the rod) or **compressive** (as in the case of pushing along the rod).
- The polarity of normal stress can also be determined by isolating an infinitesimally small volume inside the bar.
- If the volume is pulled in one particular axis, the stress is tensile; if the volume is pushed, the stress is compressive.

- The unit elongation of the rod represents the strain.
- In this case, it is called **normal strain** since the direction of the strain is perpendicular to the cross section of the beam.
- Suppose the steel bar has an original length L0. Under a given normal stress the rod is extended to a length of L.
- The resultant strain in the bar is defined as

$$s = \frac{L - L_0}{L_0} = \frac{\Delta L}{L_0}.$$

- The applied longitudinal stress along the x-axis not only produces a longitudinal elongation in the direction of the stress, but a reduction of cross-sectional area as well.
- The relative dimensional change in the y and z directions can be expressed as εy and εz.
- This general material characteristic is captured by a term called the **Poisson's ratio**, which is defined as the ratio between transverse and longitudinal elongations

$$\nu = \left|\frac{S_y}{S_x}\right| = \left|\frac{S_z}{S_x}\right|.$$



- Stress and strain are closely related.
- Under small deformation, the stress and the strain terms are proportional to each other according to the **Hooke's law**.

$$\sigma = E\varepsilon.$$

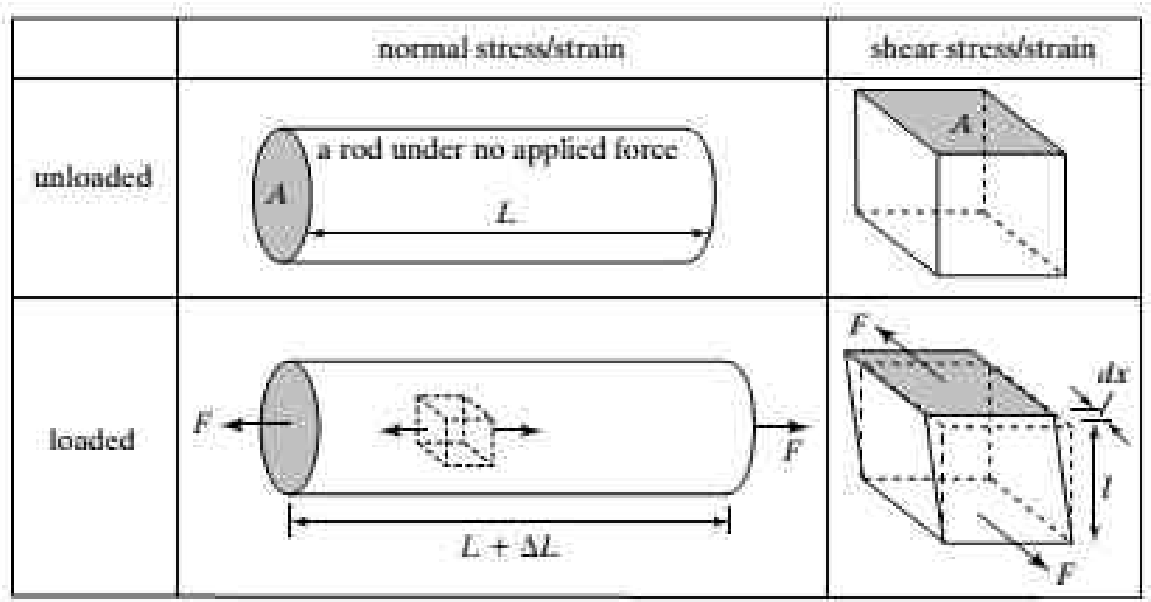
- The proportion constant, E, is called the **modulus of elasticity**.

- The modulus of elasticity, often called the **Young's modulus**, is an intrinsic property of a material.

- It is a constant for a given material, irrespective of <u>the shape and dimensions of the mechanical element</u>.

- Atoms are held together with atomic forces.
- If one imagines inter-atomic force acting as springs to provide restoring force when atoms are pulled apart or pushed together, the modulus of elasticity is the measure of the stiffness of the inter-atomic spring near the equilibrium point.

- Shear stresses can be developed under different force loading conditions.
- One of the simplest ways to generate a pure shear loading is illustrated in the Figure, with a pair of forces acting on opposite faces of a cube



- The magnitude of the shear stress is defined as

$$\tau = \frac{F}{A}.$$

The unit of τ is N/m2

- Shear stress has <u>no tendency to elongate or shorten the element in the x, y, and z directions.</u>

- Shear stresses produce a change in the shape of the element.

- The original element shown here, which is a rectangular parallelepiped, is deformed into an oblique parallelepiped.

- Shear strain ϒ, defined as the extent of rotational displacement, is

$$\gamma = \frac{\Delta X}{L}.$$

- The shear stress is unit less
- It represents the angular displacement expressed in the unit of radians.

- The shear stress and strain are also related to each other by a proportional constant, called **the shear modulus of elasticity, G**.

- The expression of G is simply the ratio of τ and ϒ

$$G = \frac{\tau}{\gamma}.$$

The unit of G is N/m2.

- The value of G depends on the material, not the shape and dimensions of an object.

- For a given materials, E, G, and the Poisson's ratio are linked through the relationship

$$G = \frac{E}{2(1 + \nu)}.$$

Stress and Strain relation

- Stress–strain relation is decided by a tensile test.

- A rod with precise dimensions, calibrated crystalline orientation and smooth surface finish is subjected to a tension force applied in the longitudinal direction.

- The amount of relative displacement and the applied stress are plotted on a stress-strain curve until the beam breaks.

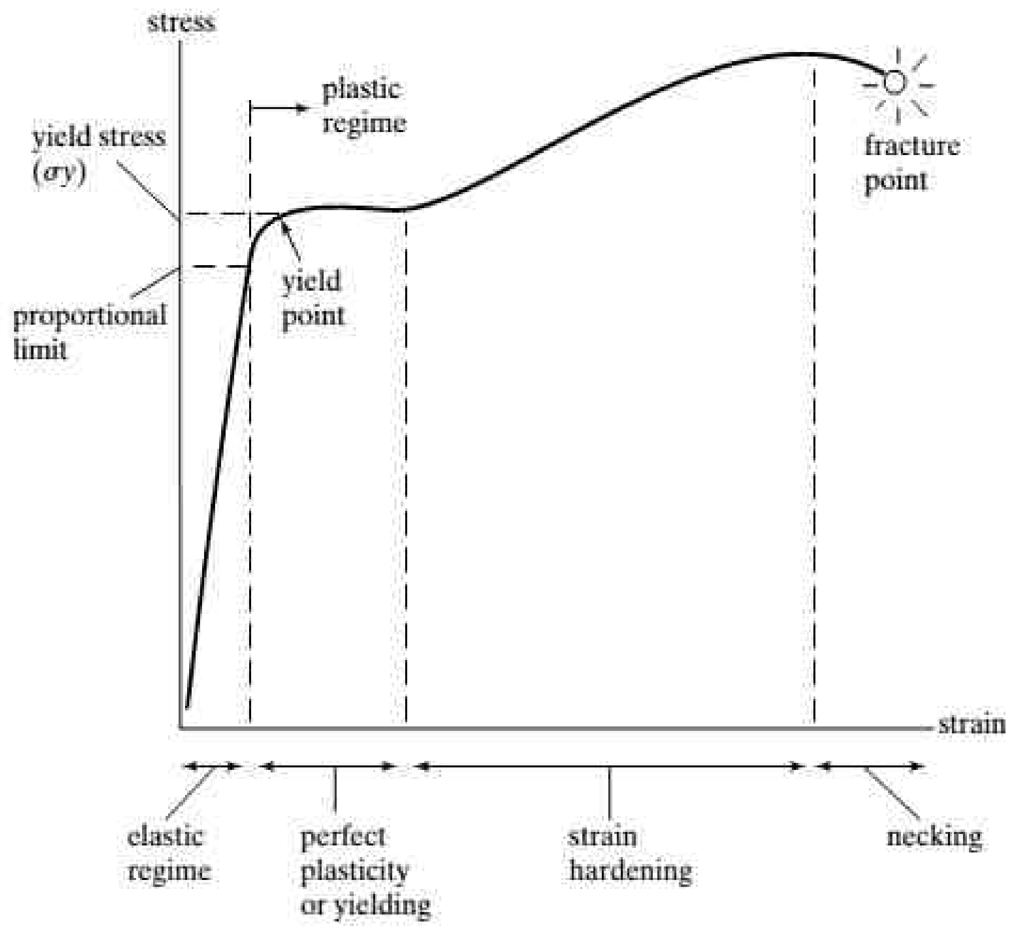


Fig: generic stress-strain curve

Inferences from stress-strain curve

1. At low levels of applied stress and strain, the stress value increases proportionally with respect to the developed strain, with the proportional constant being the **Young's modulus**.
2. This segment of the stress-strain curve is called the **elastic deformation regime.**
3. If the stress is removed, the material will return to its original shape.
4. This force loading can be repeated for many times.
5. As the stress exceeds a certain level, the material enters the **plastic deformation regime.**
6. In this regime, the amount of stress and strain does not follow a **linear relationship** anymore.
7. Furthermore, deformation cannot be fully recovered after the external loading is removed.

   Bend a metal paper clip wire slightly, it will always return to its original shape.

   If the wire is bent beyond a certain angle, the clip will never return to original shapes again.

Inferences from stress-strain curve

8. Plastic deformation is said to have occurred.
9. Stress-strain curves for materials in compression differ from those in tension
10. The stress-strain curve has two noticeable **points—yield point and fracture point.**
11. Before the yield point is reached, the material remains elastic.
12. Between the yield point and the fracture point, the specimen undergoes plastic deformation.
13. At the **fracture point**, the specimen suffers from irreversible failure.
14. The y-coordinate of the yield point is the **yield strength** of the material.
15. The y-coordinate of the fracture point is designated the **ultimate strength** (or the fracture strength) of the material.

A material is strong if it has high yield strength or ultimate strength

silicon is even stronger than stainless steel.

- For many metals the generic relationship depicted in previous figure is true.
- All materials do not exhibit this generic stress–strain relationship.
- Some representative curves for different classes of materials are shown in Figure.
- Includes brittle materials (such as silicon) and soft rubber, both are used extensively in MEMS.

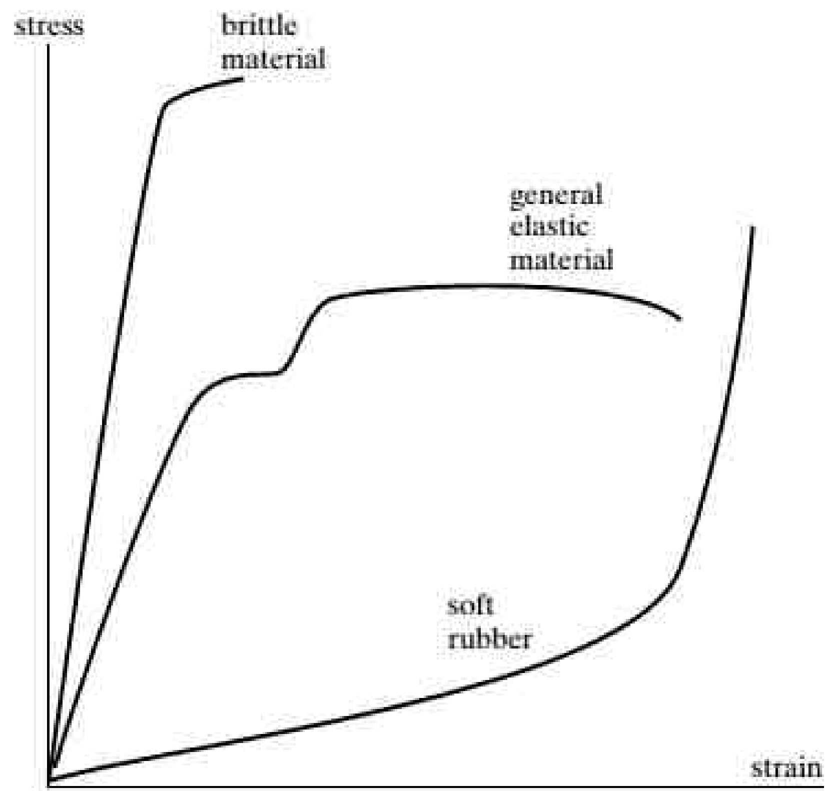


Fig: Stress-strain relation

Important properties

- **Ductility** is an important mechanical property.
- It is a measure of the degree of plastic deformation that has been sustained at the point of fracture.
- A material that experiences very little or no plastic deformation upon fracture is termed **brittle**.
- **Silicon** is a brittle material, which fails in tension with only little elongation after the proportional limit is exceeded.
- Ductility may be expressed quantitatively as either percent elongation or percent reduction in area.

Important properties

- **Toughness** is a mechanical measure of the ability of a material to absorb energy up to fracture.

- For a static situation, toughness may be ascertained from the <u>result of the tensile stress-strain test</u>.

- It is the area under the stress-strain curve up to the point of fracture.

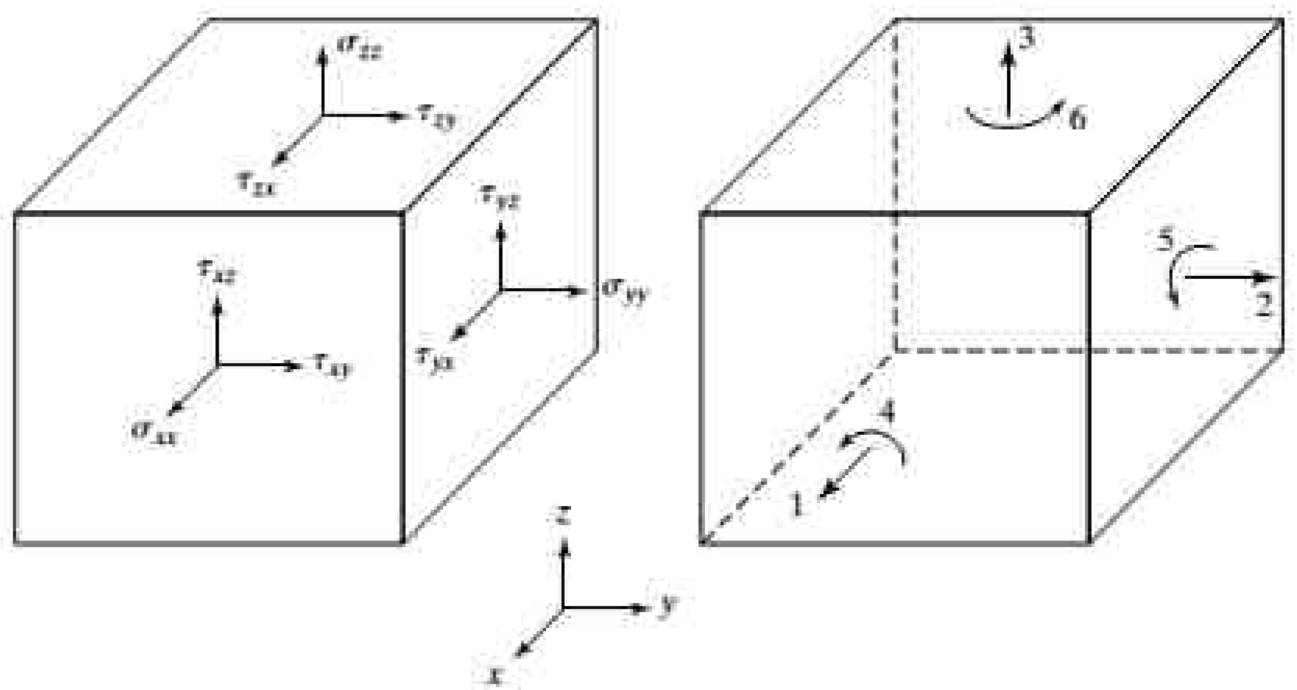
- For a material to be tough, it must display both strength and ductility.

- **Resilience** is the capacity of a material to absorb energy when it is deformed elastically and then, upon unloading, to have this energy recovered.

Stress and Strain relation

- Stress and strain are tensors.
- Their relationship can be conveniently expressed in matrix form in which stress and strain are expressed as vectors.

To visualize vector components of stress and strain, let us isolate a unit cube from inside a material and consider stress components acting on it.



- A cube has six faces.
- Consequently, there are 12 possible shear force components—two for each face. These are not all independent
- Each pair of shear stress components acting on parallel faces but along the same axis have equal magnitude and opposite directions for force balance (Newton's first law).
- This reduces the number of independent shear stress components to six.
- Each component is identified by two subscript letters.
- The first letter in the subscript indicates the normal direction of the facet on which the stress is applied to, while the second letter indicates the direction of the stress component.
- Based on torque balance, two shear stress components acting on two facets but pointing towards a common edge have the same magnitude.

$\tau_{xy}=\tau_{yx}$ $\tau_{xz}=\tau_{zx}$ $\tau_{zy}=\tau_{yz}$

- In other words, equal shear stresses always exist on mutually perpendicular planes.
- The independent number of shear stress components is reduced to three.

- There are six possible normal stress components—one for each face of a cube.
- Under equilibrium conditions, the normal stress components acting on opposite facets must have the same magnitude and point to opposite directions.
- Therefore, there are three independent normal stress components.
- Normal stress components are labeled σ with two subscript letters.

Overall, in a rectangular coordinate system under motion equilibrium, there are three independent normal stresses and three shear ones.

1. Normal stress components σ_{xx}, σ_{yy}, and σ_{zz} are simply noted as T_1, T_2 and T_3 respectively.
2. Shear stress components τ_{yz}, τ_{xz}, and τ_{xy} are simply noted as T_4, T_5 and T_6 respectively.

- Correspondingly, there are three independent strains (through) and three shear strains (through).

- The general matrix equation between stress and strain, is

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \end{bmatrix}.$$

- In short-hand form, the expression is

$$\overline{T} = C\overline{s}.$$

- The coefficient matrix C is called the stiffness matrix.

- The strain matrix is a product of the compliance matrix, S, and the stress tensor, according to the following matrix expression

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \\ s_5 \\ s_6 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix}.$$

- The expression in short hand form is

$$\bar{s} = S\,\overline{T}.$$

- The compliance matrix S is the inverse of the stiffness matrix.

In short hand notation,

$$S = C^{-1}$$

Note the stiffness matrix is denoted by the letter C, whereas the compliance matrix is denoted by the letter S