

①

MODULE-1
NUMBER SYSTEMS AND CODES

1). Convert binary to decimal.

a). 1101101_2

$$\begin{aligned} &= 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 + 1 \times 2^3 + 0 \times 2^4 + 1 \times 2^5 + 1 \times 2^6 \\ &= 1 + 4 + 8 + 32 + 64 \\ &= \underline{\underline{109_{10}}} \end{aligned}$$

b). 0.0011_2

$$\begin{aligned} &= 1 \times 2^{-4} + 1 \times 2^{-3} + 0 \times 2^{-2} + 0 \times 2^{-1} + 0 \times 2^0 \\ &= 2^{-4} + 2^{-3} \\ &= 0.0625 + 0.125 \\ &= \underline{\underline{0.1875}} \end{aligned}$$

c). 10111101.011_2

$$\begin{aligned} &= 1 \times 2^{-3} + 1 \times 2^{-2} + 0 \times 2^{-1} + 1 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 \\ &\quad + 1 \times 2^3 + 1 \times 2^4 + 1 \times 2^5 + 0 \times 2^6 + 1 \times 2^7 \\ &= 2^{-3} + 2^{-2} + 1 + 4 + 8 + 16 + 32 + 128 \\ &= \underline{\underline{189.375_{10}}} \end{aligned}$$

2). Convert decimal to binary

(2)

i). 29_{10}

2		29
2		14 - 1
2		7 - 0
2		3 - 1
		1 - 1

MSB \rightarrow Most significant byte.

LSB \rightarrow Lowest significant byte

$$29_{10} = \underline{\underline{011001_2}}$$

MSB LSB

ii). 57_{10}

2		57
2		28 - 1
2		14 - 0
2		7 - 0
2		3 - 1
		1 - 1

$$57_{10} = \underline{\underline{111001_2}}$$

iii). 256_{10}

2		256
2		128 - 0
2		64 - 0
2		32 - 0
2		16 - 0
2		8 - 0
2		4 - 0
2		2 - 0
		1 - 0

$$256_{10} = \underline{\underline{100000000_2}}$$

iv). 0.065_{10}

(3)

$$\begin{array}{l} 2 \times 0.065 = 0.13 \\ 2 \times 0.13 = 0.26 \\ 2 \times 0.26 = 0.52 \\ 2 \times 0.52 = 1.04 \\ 2 \times 0.04 = 0.08 \end{array}$$

$$0.065_{10} = \underline{\underline{0.00010_2}}$$

v). 0.625_{10}

$$\begin{array}{l} 2 \times 0.625 = 1.25 \\ 2 \times 0.25 = 0.5 \\ 2 \times 0.5 = 1 \end{array}$$

$$0.625_{10} = \underline{\underline{0.101_2}}$$

vi). 0.726_{10}

$$\begin{array}{l} 2 \times 0.726 = 1.452 \\ 2 \times 0.452 = 0.904 \\ 2 \times 0.904 = 1.808 \\ 2 \times 0.808 = 1.616 \\ 2 \times 0.616 = 1.232 \\ 2 \times 0.232 = 0.464 \end{array}$$

$$0.726_{10} = \underline{\underline{0.10111_2}}$$

BINARY ADDITION

$$\begin{array}{r} (1) \cdot 1101_2 + \\ 1111_2 \\ \hline 11100_2 \\ \hline \end{array}$$

Verification:

$$\begin{array}{r} 13 + \\ 15 \\ \hline 28 \\ \hline \end{array}$$

④

$$\begin{array}{l} 0_{10} \rightarrow 0_2 \\ 1_{10} \rightarrow 1_2 \\ 2_{10} \rightarrow 10_2 \\ 3_{10} \rightarrow 11_2 \\ 4_{10} \rightarrow 100_2 \end{array}$$

$$\begin{array}{r} (2) \cdot 1101101_2 + \\ 1111011_2 \\ \hline 11101000_2 \\ \hline \end{array}$$

BINARY SUBTRACTION

$$\begin{array}{r} (1) \cdot \overset{0}{1} \overset{1}{\cancel{1}} \overset{10}{\cancel{0}} \overset{10}{\cancel{1}} 0_2 - \\ 10111_2 \\ \hline 00011_2 \\ \hline \end{array}$$

NEGATIVE Nos.

(5)

1) → we should take one's complement. (1's comp)

$$\begin{aligned} \text{Eg: } 1110010_2 \\ \Rightarrow 0001101_2 \end{aligned}$$

2). 2's complement

Take 1's complement + 1

$$\begin{array}{r} \text{Eg: } 0001101_2 \\ 1110010_2 + \\ \hline 1110011_2 \end{array}$$

$$\text{Eg: (1). } -25_{10}$$

$$\begin{array}{r|l} 2 & 25 \\ \hline 2 & 12 - 1 \\ 2 & 6 - 0 \\ 2 & 3 - 0 \\ & 1 - 1 \end{array} \quad 00011001_2$$

* we take 8 bits for -ve No.

MSB

-ve $\Rightarrow 1$

+ve $\Rightarrow 0$

1's complement

$$11100110_2$$

2's complement

$$\underline{\underline{11100111_2}}$$

$$\Rightarrow 11100110_2$$

$$\begin{array}{ccccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ -2^7 & +2^6 & -2^5 & +2^4 & -2^3 & +2^2 & -2^1 \end{array}$$

$$= -128 + 64 + 32 + 4 + 2$$

$$= -26 + 1 = \underline{\underline{-25}}$$

$$\begin{array}{ccccccc} \textcircled{1} & 1 & 1 & 0 & 0 & 1 & 1 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ -2^7 & +2^6 & -2^5 & +2^4 & -2^3 & +2^2 & -2^1 \end{array}$$

$$= -128 + 64 + 32 + 4 + 2 + 1$$

$$= -25_{10}$$

For 1's complement, when converted to decimal,
(weighted sum)
we need to add one.

whereas for 2's complement, we need not add 1.

2). -39

2	39	
2	19	-1
2	9	-1
2	4	-1
2	2	-0
	1	-0

$\Rightarrow 00100111_2$

1's complement

$$11011000_2$$

$$= \overset{\downarrow}{2^7} \overset{\downarrow}{2^6} \overset{\downarrow}{2^4} \overset{\downarrow}{2^3}$$

$$= -40 + 1 = \underline{\underline{-39}}$$

2's complement

~~$$00100111_2$$~~

$$11011001_2$$

$$= -2^7 + 2^6 + 2^4 + 2^3 + 2^0$$

$$= \underline{\underline{-39}}$$

Q. Find decimal value of the signed binary no. in 1's complement (7)

Need not add 1 for +ve Nos.

a) 0001011_2

$$= 2^0 + 2^1 + 2^2 + 2^4$$

$$= \underline{\underline{23_{10}}}$$

b) 11101000_2

$$= 2^3 + 2^5 + 2^6 - 2^7 = -24 + 1$$

$$= \underline{\underline{-23_{10}}}$$

Q. Find decimal value of signed binary no. in 2's complement.

a) 01010110_2

$$= 2^1 + 2^2 + 2^4 + 2^6$$

$$= \underline{\underline{86}}$$

b) 10101010_2

$$= 2^1 + 2^3 + 2^5 - 2^7$$

$$= \underline{\underline{-86}}$$

Q. Add the signed nos:

$$\begin{array}{r}
 \text{(1) (ve)} \quad 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 1_2 \\
 \text{(ve)} \quad 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0_2 \\
 \hline
 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 1_2 \\
 \hline
 \end{array}$$

$$\Rightarrow \begin{array}{r} 7 \\ + \\ 4 \\ \hline 11 \end{array}$$

$$\begin{array}{r}
 \text{(2) (ve)} \quad 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 1 \ 1_2 \\
 \text{(ve)} \quad 1 \ 1 \ 1 \ 1 \ 0 \ 1 \ 0_2 \\
 \hline
 1 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1_2 \\
 \hline
 \end{array}$$

$$\Rightarrow \begin{array}{r} 15 \\ - 6 \\ \hline 9 \end{array}$$

Discard this 1

$$\begin{array}{r}
 \text{(3) (ve)} \quad 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0_2 \\
 \text{(ve)} \quad 1 \ 1 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0_2 \\
 \hline
 1 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0_2 \\
 \hline
 \end{array}$$

$$\begin{array}{r} 16 \\ - 24 \\ \hline -8 \end{array}$$

discard

2. Add the signed numbers:

(9)

$$\begin{array}{r}
 (1). \quad 11111011_2 + \\
 \quad 1110111_2 \\
 \hline
 \quad 111110010 \\
 \hline
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 -5 + \\
 -9 \\
 \hline
 -14 \\
 \hline
 \hline
 \end{array}$$

2. Subtract the signed nos.

When you have a
-ve sign with a binary no.

take 2's complement & add.

take 2's complement & add.

$$(1). \quad 00001000_2 - 0000011_2$$

$$\begin{array}{r}
 00001000_2 + \\
 1111101_2 \\
 \hline
 100000101_2 \\
 \hline
 \hline
 \end{array}
 \qquad
 \Rightarrow
 \begin{array}{r}
 8 + \\
 -3 \\
 \hline
 5 \\
 \hline
 \hline
 \end{array}$$

$$(2). \quad 00001100_2 - 1110111_2$$

$$12 - (-9) = 21$$

$$\begin{array}{r}
 00001100_2 + \\
 00001001_2 \\
 \hline
 00010101 \\
 \hline
 \hline
 \end{array}$$

(3) $11100111_2 - 00010011_2 \Rightarrow -25 - 19$
 $= -25 + (-19)$
 $= \underline{\underline{-44}}$

$$\begin{array}{r} 11100111_2 \\ 11101101_2 \\ \hline 111010100_2 \end{array}$$

HEXADECIMAL

DECIMAL

BINARY

HEXADECIMAL

0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Q. Convert binary to hexadecimal

(11)

(1) 1100101001010111_2

Group into 4 each.

$$1100, 1010, 0101, 0111_2$$

$$C \quad A \quad 5 \quad 7_{16}$$

$$\Rightarrow \underline{\underline{CA57_{16}}}$$

(2) 00011111000101101001_2

$$1 \quad F \quad 1 \quad 6 \quad 9_{16}$$

$$\Rightarrow \underline{\underline{1F169_{16}}}$$

Q. Hexa to binary

(1) $10A4_{16}$

$$\begin{array}{cccc} 1 & 0 & A & 4_{16} \\ \swarrow & \downarrow & \downarrow & \searrow \\ 0001 & 0000 & 1010 & 0100_2 \end{array}$$

(2) $4CFA_{16}$

$$\underline{\underline{010011001111010_2}}$$

Q. Hexa to decimal

①. Weighted Sum technique

(a). $1C_{16}$
 $\downarrow \quad \quad \quad \searrow$
 $1 \times 16^1 + 12 \times 16^0 = \underline{\underline{28_{10}}}$

②. Hexa \rightarrow Binary \rightarrow decimal

(b). $2F_{16}$
 $\downarrow \quad \downarrow$
 $00101111_2 \rightarrow 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$
 $+ 1 \times 2^3 + 1 \times 2^5 = \underline{\underline{47_{10}}}$

Q. Convert Decimal to hexa:

(a). $16 \overline{) 65_{10}}$
 $\quad 4 - 1 \Rightarrow \underline{\underline{41_{16}}}$

(b). 860_{10}

$16 \overline{) 860_{10}}$
 $16 \overline{) 53 - 12} \Rightarrow \underline{\underline{35C_{16}}}$
 $\quad 3 \quad - 5$

$16 \overline{) 860} \quad 16 \overline{) 53}$
 $\quad 80 \quad \quad 48$
 $\quad \quad 60 \quad \quad 5$
 $\quad \quad 48$
 $\quad \quad 12$

OCTADECIMAL

DECIMAL	BINARY
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

13

Q. Convert binary to Octal

11, 101, 101₂
3 5 5₈

Q. Convert octal to binary

203₈
010000011₂

Q. Convert octal to decimal

127₈

$$\Rightarrow 7 \times 8^0 = 7$$

$$2 \times 8^1 = 16$$

$$1 \times 8^2 = 64$$

$$\underline{\underline{87_{10}}}$$

Q. Convert decimal to Octal

(4)

1). 256_{10}

$$\begin{array}{r} 8 \overline{) 256} \\ 8 \overline{) 32} - 0 \\ 4 - 0 \end{array}$$

$$= \underline{\underline{400_8}}$$

ASCII \rightarrow American Standard Codes for Information Index

- ①. Weighted coding
 - ②. Non-weighted coding
- Ex-3 code
Gray code

(a). Ex-3 code :

Eg: $1 + 4 = 5$

$$\begin{array}{r} 1 \Rightarrow 0001_2 + \\ (\text{add } 3) \quad 0011_2 \\ \hline 0100_2 \end{array}$$

$$\begin{array}{r} 4 \Rightarrow 0100_2 + \\ (\text{add } 3) \quad 0011_2 \\ \hline 0111_2 \end{array}$$

Now add,

$$\begin{array}{r} 0100_2 + \\ 0111_2 \\ \hline 1000 \end{array}$$

$\Rightarrow 81$
(excess 3)

BINARY TO GRAY

① . $1 \xrightarrow{\text{add}} 0_2 = \underline{\underline{11_2}}$
1st no. write as it is

② . $1011 = \underline{\underline{1110_2}} \rightarrow \text{ignore the carry 1.}$

③ . $1101_2 = \underline{\underline{1011_2}}$

GRAY TO BINARY

Eg: 1011

$\downarrow \uparrow \text{add}$
 $\underline{\underline{1101_2}}$
write 1st no. as it is

LOGIC GATES

The term gate is used to describe a circuit that performs logic operation

NOT

The inverter or NOT performs the operation called inversion or complementation. The inverter changes one logic level to the opposite level. In terms of bits it changes bit '1' to '0' & '0' to '1'

Symbol:



Truth Table

A	Y
0	1
1	0

The negative indicator is a bubble (o) that indicates inversion or complementation when it appears on the i/p or o/p of any logic element. Generally, inputs are on the left of the logic symbol & the o/p is on the right

0 \Rightarrow Active low

1 \Rightarrow Active high

In boolean algebra, a variable is represented by a letter. The complement of a variable is designated by a bar over the letter. A variable can take on a value of either 1 or 0.

Boolean algebra uses variables & operators to describe a logic circuit.

The logic operation of NOT gate can be expressed as by taking A as input variable & Y as o/p variable

$$Y = \bar{A}$$



TRUTH INPUT		TABLE o/p
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

If A & B are two o/p variables and Y is the o/p variable, the logical OR fn. can be expressed by a + b/w two variables.

$$Y = A + B$$

NAND GATE

It is a universal gate i.e. NAND gates can be used in combination to perform AND, OR & NOT operations. The term NAND is a contraction of NOT-AND & implies ^{an} AND function with complemented output.

SYMBOL



The NAND gate produces a low o/p only when all the inputs are high when any of the i/p is low o/p is high.

Truth Table

Input		Output
A	B	
0	0	1
0	1	1
1	0	1
1	1	0

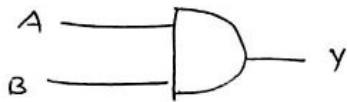
The boolean expression for NAND gate is

$$Y = \overline{AB}$$

AND GATE

An AND gate produces a high o/p only when all the inputs are high. When any one of the i/p is low o/p is low. If A & B are the two inputs then Y is the o/p variable. Its truth table is shown below. It performs logical multiplication

SYMBOL



Truth Table

i/p		o/p
A	B	
0	0	0
0	1	0
1	0	0
1	1	1

The logic expression of AND gate using two variables is represented mathematically either by placing a dot (.) between the two variables as $A \cdot B$ or by simply writing the adjacent letters without the dot as AB .

Thus the Boolean expression for AND gate is

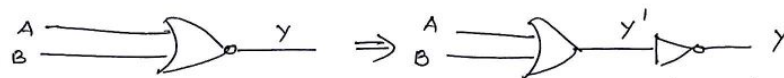
$$\boxed{Y = AB.}$$

OR GATE

An OR gate can have two or more inputs and performs logical addition. An OR gate produces a high on the o/p when any of the i/p is high. The o/p is low only when all the i/ps are low. \therefore an OR gates determine when one or more inputs are high produces a high on the o/p.

NOR GATE

It is also used as universal gate because it can be used in combination to perform AND OR & NOT operation. The term NOR is a contraction NOT-OR & implies an OR fn. with an inverted o/p.



A NOR gate produces a low o/p when any of the inputs is high & it produces a high on the o/p when all the i/p's are low. Thus the truth table is

Truth table		
Input		Output
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

The boolean expression for NOR gate is

$$Y = \overline{A+B}$$

EXCLUSIVE OR (EXOR)

The Ex-OR gate has only two inputs. The o/p Ex OR gate is high only when the two inputs are at opposite logic levels.

Truth Table		
A	B	O/p
0	0	0
0	1	1
1	0	1
1	1	0

Boolean expression for

Ex-OR gate is

$$Y = \bar{A}B + A\bar{B} = A \oplus B$$

SYMBOL



REALISATION OF AND, OR, NOT, AND EX-OR GATES USING NAND GATE

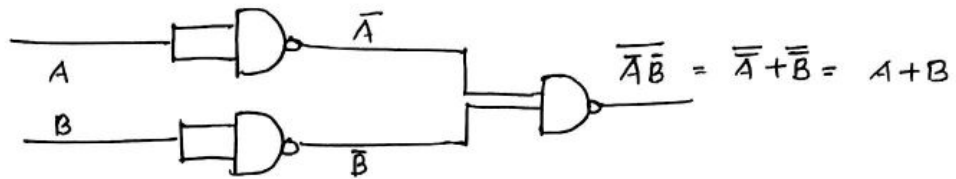
AND

$$Y = AB$$

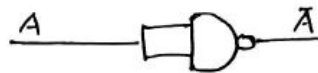


OR

$$Y = A + B$$

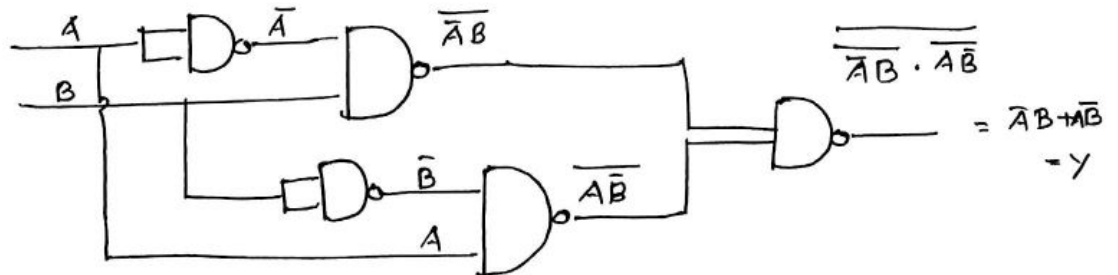


NOT



EX-OR

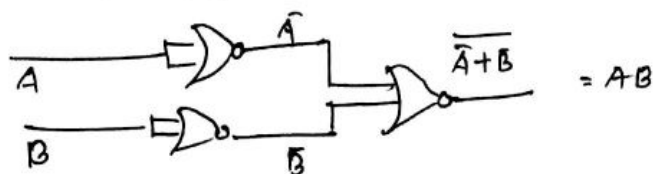
$$Y = \bar{A}B + A\bar{B}$$

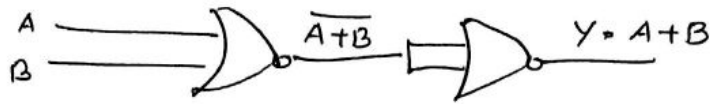


REALIZATION OF AND, OR, NOT & EX-OR GATES USING NOR GATE

AND

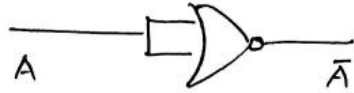
$$Y = AB$$





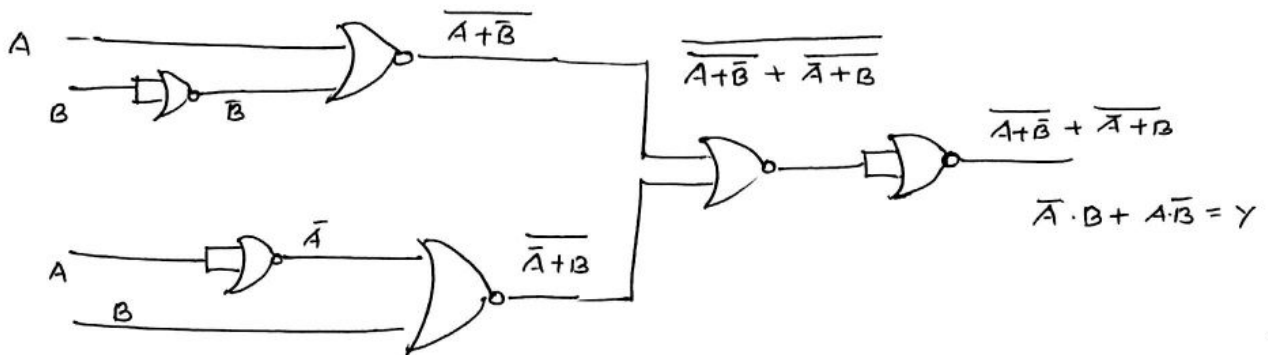
NOT

$$Y = \bar{A}$$



EX-OR GATE

$$Y = \bar{A}B + A\bar{B}$$



SIMPLIFICATION OF BOOLEAN EXPRESSIONS

RULE 1: Let 'a' be a variable then

$$a + \bar{a} = 1 \text{ (sum of a variable \& its complement = 1)}$$

$$a \cdot \bar{a} = 0 \quad (\text{product " " " " } = 0)$$

110. a (double complement)

a	\bar{a}	$a + \bar{a}$	$a \cdot \bar{a}$	$a \oplus \bar{a}$
0	1	1	0	0
1	0	1	0	1

RULE 2 Let 'a' be a variable then

$$a + 1 = 1 \quad (\text{any variable} + 1 = 1)$$

$$a + 0 = a \quad (\text{any variable} + 0 = 0)$$

$$a \cdot 1 = a$$

$$a \cdot 0 = 0$$

Proof

a	a+1	a+0	a.1	a.0
0	1	0	0	0
1	1	1	1	0

E3 Absorption law: If a & b are two variables

then $a + ab = a$ or $a(1+b) = a \cdot 1 = a$

Here the variable 'b' is absorbed by variable 'a'

a	b	ab	a+b
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

$$\text{Ily } a + \bar{a}b = a + b$$

$$a(\bar{a} + b) = ab$$

$$\bar{a} + ab = \bar{a} + b$$

$$\bar{a}(a+b) = \bar{a}b$$

$$\text{Proof: } a + \bar{a}b = a + ab + \bar{a}b = a + (a + \bar{a})b = a + b$$

$$a(\bar{a} + b) = a\bar{a} + ab = ab + 1 = ab$$

$$\bar{a}(a+b) = \bar{a}a + \bar{a}b = 1 + \bar{a}b = \bar{a}b$$

RULE 4: Idempotency: Let 'a' be a variable then

$$a + a = a \quad \& \quad a \cdot a = a$$

a	a+a	a.a
0	0	0
1	1	1

RULE 5: COMMUTATIVE LAW: If a & b are two variables then

$$a+b = b+a \quad \& \quad a \cdot b = b \cdot a$$

RULE 6: Associative law: If a, b & c are three variables then

$$a+(b+c) = (a+b)+c \quad \& \quad a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

RULE 7: DISTRIBUTIVE LAW If a, b & c are three variables then

$$a(b+c) = ab+ac \quad \&$$

$$a+bc = (a+b)(a+c)$$

↳ special property of boolean algebra

RULE 8: DEMORGAN'S LAW

If a & b are two variables then

$$\overline{a+b} = \bar{a} \cdot \bar{b} \quad \&$$

$$\overline{a \cdot b} = \bar{a} + \bar{b}$$

RULE 9 Consensus theorem: If a, b & c are three variable then

$$ab+bc+ca = ab+c\bar{a}$$

$$\begin{aligned} ab+bc+c\bar{a} &= ab+(a+\bar{a})bc+c\bar{a} \\ &= ab+abc+\bar{a}bc+c\bar{a} \\ &= abc(1+c) + \bar{a}c(1+b) \\ &= \underline{\underline{ab+c\bar{a}}} \end{aligned}$$

REPRESENTATION OF BOOLEAN EXPRESSION

Boolean expressions can be represented in two forms

1. Sum of Products (SOP) form
2. Product of Sums (POS) form

SOP FORM

This form is also called Disjunctive Normal form (DNF)

eg: $f(A, B, C) = \bar{A}B + \bar{B}C$

Standard SOP form

This form is also called Disjunctive canonical form (DCF). It is also called the expanded sum of product form or canonical sum of product form. In this form, the fn. is the sum of a number of product terms where each product term contains all the variables of the fn. either in complement or uncomplemented form. This can be derived from the truth table by finding the sum of all the terms that correspond to those combination for which f assumes the value 1, or It can be obtained from the SOP form as follows.

$$\begin{aligned} f(A, B, C) &= \bar{A}B + \bar{B}C = \bar{A}B(C + \bar{C}) + (A + \bar{A})\bar{B}C \\ &= \bar{A}BC + \bar{A}\bar{B}C + A\bar{B}C + \bar{A}\bar{B}C \end{aligned}$$

A product term which contains all the variables of the fn either in complemented or uncomplemented form is called a minterm. A minterm assumes the value 1 only for one combination of the variables.

An n -variable fn can have in all 2^n minterms.

The sum of the minterms whose value = 1 is the std. SOP form of the fn.

The minterms are often denoted as m_0, m_1, m_2 where the suffixes are the decimal codes of the combinations. for a 3-variable fn $m_0 = \bar{A}\bar{B}\bar{C}$ $m_1 = \bar{A}\bar{B}C$ $m_2 = \bar{A}B\bar{C}$. . . $m_7 = ABC$. Another form of representing fn in std SOP form is by showing the sum of minterms for which the fn equal 1.

$$f(A, B, C) = m_1 + m_2 + m_3 + m_4$$

or by listing the decimal codes of the minterms for which $f = 1$

$$f(A, B, C) = \sum m(1, 2, 3, 5)$$

where $\sum m$ represents the sum of all the minterms whose decimal codes are given in the parenthesis

CONVERT THE FOLLOWING SOP FORM INTO STD SOP FORM

$$\begin{aligned} 1. \quad \bar{A} + \bar{B} &= \bar{A}(B + \bar{B}) + (A + \bar{A})\bar{B} \\ &= \bar{A}B + \bar{A}\bar{B} + A\bar{B} + \bar{A}\bar{B} \\ &= \bar{A}B + A\bar{B} + \bar{A}\bar{B} \\ &= 01 + 10 + 00 \\ &= m_1 + m_2 + m_0 \\ &= \sum m(0, 1, 2) \end{aligned}$$

$$\begin{aligned} 2. \quad AB + \bar{A}\bar{B} + AC + \bar{A}\bar{C} \\ &= AB(C + \bar{C}) + \bar{A}\bar{B}(C + \bar{C}) + A(C + \bar{C}) + \bar{A}(\bar{C} + C) \\ &= ABC + AB\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + AC + A\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} \end{aligned}$$

$$\begin{aligned}
 &= ABC + AB\bar{C} + \bar{A}\bar{B}C + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} \\
 &= 111 + 110 + 001 + 101 + 010 + 000 \\
 &= \Sigma(7, 6, 1, 5, 2, 0) \\
 &= \underline{\underline{\Sigma(0, 1, 2, 5, 6, 7)}}
 \end{aligned}$$

SIMPLIFICATION OF BOOLEAN EXPRESSION BY USING BOOLEAN ALGEBRA

A simplified Boolean expression uses the fewest gates possible to implement a given expression.

1. using Boolean algebra technique, simplify the following expression

$$Y = AB + A(B+C) + B(B+C)$$

$$\begin{aligned}
 Y &= AB + AB + AC + BB + BC \\
 &= AB + AC + B + BC \\
 &= AB + AC + B \\
 &= AB + B + AC = \underline{\underline{B + AC}}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad Y &= A\bar{B} + A(\bar{B}+C) + B(\bar{B}+C) \\
 &= A\bar{B} + A\bar{B}\bar{C} + A\bar{B}C + B\bar{B} + BC \\
 &= A\bar{B} + A\bar{B}\bar{C} + A\bar{B}C + 0 + BC \\
 &= A\bar{B}(1 + \bar{C} + C) + BC = A\bar{B} + BC
 \end{aligned}$$

$$\begin{aligned}
 3. \quad Y &= [A\bar{B}(C+BD) + A\bar{B}]C \\
 &= [A\bar{B}C + A\bar{B}BD + A\bar{B}]C \\
 &= A\bar{B}CC + A\bar{B}C \\
 &= A\bar{B}C + A\bar{B}C \\
 &= (A+A)\bar{B}C = \underline{\underline{\bar{B}C}}
 \end{aligned}$$

$$[AB(C + \bar{B}D) + \bar{A}\bar{B}]CD$$

$$\Rightarrow [ABC + AB\bar{B}D + \bar{A}\bar{B}]CD \Rightarrow [ABC + AB(\bar{B} + D) + \bar{A} + \bar{B}]CD$$

$$\Rightarrow ABCD + AB\bar{B}D + \bar{A}BCD + \bar{A}CD + \bar{B}CD$$

$$\Rightarrow ABCD + 0 + 0 + \bar{A}CD + \bar{B}CD$$

$$\Rightarrow ABCD + \bar{A}CD + \bar{B}CD$$

$$\Rightarrow CD[AB + \bar{A} + \bar{B}]$$

$$\Rightarrow CD[A + B + \bar{B}] = CD[\bar{A} + 1] = \underline{\underline{CD}}$$

$$\bar{A}B\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}C + AB\bar{C}$$

$$\Rightarrow (\bar{A} + A)B\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + A\bar{B}C$$

$$\Rightarrow 1(B\bar{C}) + A\bar{B}(\bar{C} + C) + \bar{A}\bar{B}\bar{C}$$

$$\Rightarrow B\bar{C} + A\bar{B} + \bar{A}\bar{B}\bar{C}$$

$$\Rightarrow B\bar{C} + \bar{B}(A + \bar{A}\bar{C})$$

$$\Rightarrow B\bar{C} + \bar{B}(A + \bar{C})$$

$$\Rightarrow \underline{\underline{B\bar{C} + A\bar{B} + \bar{B}\bar{C}}}$$

$$AB\bar{C} + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C}$$

$$\Rightarrow AB\bar{C} + \bar{A}\bar{B}(C + \bar{C}) + \bar{A}B\bar{C}$$

$$\Rightarrow AB\bar{C} + \bar{A}\bar{B} + \bar{A}B\bar{C}$$

$$\Rightarrow AB\bar{C} + \bar{A}(\bar{B} + B\bar{C})$$

$$\Rightarrow AB\bar{C} + \bar{A}(\bar{B} + \bar{C})$$

$$= \underline{\underline{AB\bar{C} + \bar{A}\bar{B} + \bar{A}\bar{C}}}$$

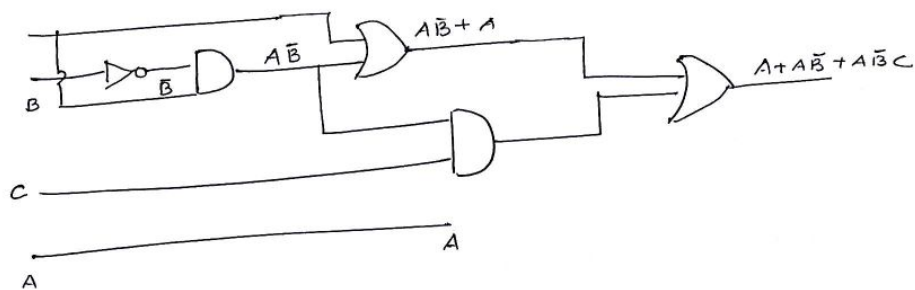
7. $\overline{AB} + AC + \overline{A} \overline{B} C$

$$\begin{aligned} &\Rightarrow \overline{AB} \cdot \overline{A} C + \overline{A} \overline{B} C \\ &\Rightarrow A \cdot \overline{A} + \overline{A} \overline{C} + \overline{B} \overline{A} + \overline{B} \overline{C} + \overline{A} \overline{B} C \\ &\Rightarrow \overline{A} + \overline{A} \overline{C} + \overline{A} \overline{B} + \overline{B} \overline{C} + \overline{A} \overline{B} C \\ &\Rightarrow \overline{A} C (1 + \overline{C}) + \overline{A} \overline{B} C (1 + \overline{C}) + \overline{B} \overline{C} \\ &\Rightarrow \overline{A} + \overline{A} \overline{B} + \overline{B} \overline{C} \\ &\Rightarrow \overline{A} (1 + \overline{B}) + \overline{B} \overline{C} \\ &= \underline{\underline{\overline{A} + \overline{B} \overline{C}}} \end{aligned}$$

Q. Simplify the following boolean expression and implement each expression by logic gates. Then implement the simplified expression & compare the no. of gates.

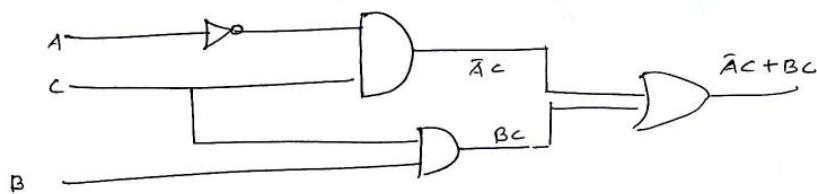
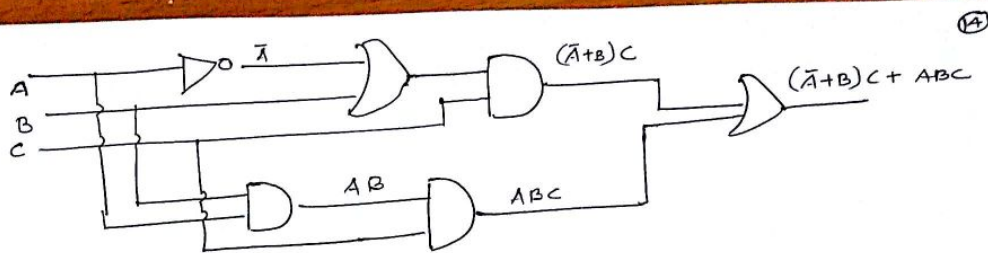
a) $A + A\overline{B} + A\overline{B}C$

$$\begin{aligned} &\Rightarrow A(1 + \overline{B}) + A\overline{B}C \\ &\Rightarrow A + A\overline{B}C = A(1 + \overline{B}C) = A. \end{aligned}$$



b) $(\overline{A} + B)C + ABC$

$$\begin{aligned} &= \overline{A}C + BC + ABC \\ &= \overline{A}C + (1 + A)BC \\ &= \overline{A}C + BC \\ &= \underline{\underline{\overline{A}C + BC}} \end{aligned}$$



PRODUCT OF SUMS FORM

This form is also called conjunctive canonical form. It is also called Expanded product of sums form or Canonical Product of sums form. This is derived by considering the combinations for which $f = 0$. Each term is a sum of all the variables. A variable appears in uncomplemented form if it has a value of 1 in the combination & appears in complemented form if it has a value of 0 in the combination. For n variables, each term which contains each of the n variables is either complemented or uncomplemented form is called a max-term. A max-term assumes the value 0 only for one combination of the variables. For all other combinations it will be 1. There will be 2^n max-terms. The product of max-terms corresponding to the rows for which $f = 0$ is the standard or canonical product of sums form of the function.

Q. Find the canonical pos form for the following expression

$$\begin{aligned}
 1. \quad f(A, B, C) &= \bar{A}(B + \bar{C}) \\
 &= \bar{A}B + \bar{A}\bar{C} \\
 &= \bar{A}B(C + \bar{C}) + \bar{A}(B + \bar{B})\bar{C} \\
 &= \bar{A}BC + \bar{A}B\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C} \\
 &= \bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C} \\
 &= \sum m(3, 2, 0) \\
 &= \sum m(0, 2, 3) \\
 &= \pi M(1, 4, 5, 6, 7) \\
 &= (A + B + \bar{C})(\bar{A} + B + C)(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + \bar{C})
 \end{aligned}$$

$$\begin{aligned}
 2. \quad f(A, B, C) &= \bar{A}C + A\bar{C} \\
 &= \bar{A}(B + \bar{B})C + A(\bar{B} + B)\bar{C} \\
 &= \bar{A}BC + \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}\bar{C} \\
 &= \sum m(3, 1, 6, 4) \\
 &= \sum m(1, 3, 4, 6) \\
 &= \pi M(0, 2, 5, 7) \\
 &= (A + B + C)(\bar{A} + \bar{B} + C)(\bar{A} + B + \bar{C})(\bar{A} + \bar{B} + \bar{C})
 \end{aligned}$$

$$\begin{aligned}
 3. \quad f(A, B, C, D) &= AC \\
 &= A(B + \bar{B})C(D + \bar{D}) \\
 &= ABC(D + \bar{D}) + A\bar{B}C(D + \bar{D}) \\
 &= ABCD + ABC\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} \\
 &= \sum m(15, 14, 11, 10) \\
 &= \sum m(10, 11, 14, 15) \\
 &= \pi M(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13)
 \end{aligned}$$

$$\begin{aligned}
 & (A+B+C+D)(A+B+C+D)(A+B+C+D)(A+B+C+D) \\
 & (A+B+C+D)(A+B+C+D)(A+B+C+D)(A+B+C+D) \\
 & (A+B+C+D)(A+B+C+D)(A+B+C+D)(A+B+C+D)
 \end{aligned}$$

$$4. \quad f(A, B, C, D) = AB + \bar{A}B + \bar{C}D$$

$$= AB(C+\bar{C}) + \bar{A}B(C+\bar{C}) + (\bar{A}+\bar{A})C(\bar{C}+C)$$

$$\begin{aligned}
 & = ABC + AB\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + \bar{A}BC + \bar{A}B\bar{C} \\
 & \quad + \bar{A}BC + \bar{A}B\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + \bar{A}BC + \bar{A}B\bar{C}
 \end{aligned}$$

$$\begin{aligned}
 & = ABC + AB\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + \bar{A}BC + \bar{A}B\bar{C} \\
 & \quad + \bar{A}BC + \bar{A}B\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + \bar{A}BC + \bar{A}B\bar{C}
 \end{aligned}$$

$$\sum m(15, 14, 13, 12, 7, 6, 9, 5, 1)$$

$$\sum m(1, 5, 6, 7, 9, 12, 13, 14, 15)$$

$$\pi M(0, 2, 3, 4, 8, 10, 11)$$

$$\begin{aligned}
 & (A+B+C+D)(A+B+C+D)(A+B+C+D)(A+B+C+D) \\
 & (A+B+C+D)(A+B+C+D)(A+B+C+D)(A+B+C+D)
 \end{aligned}$$

$$5. \quad f(A, B, C) = AB + \bar{A}\bar{B} + AC + \bar{A}\bar{C}$$

$$= AB(C+\bar{C}) + \bar{A}\bar{B}(C+\bar{C}) + A(B+\bar{B})C + \bar{A}(\bar{B}+\bar{\bar{B}})\bar{C}$$

$$= ABC + AB\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + ABC + AB\bar{C} + \bar{A}BC + \bar{A}B\bar{C}$$

$$= ABC + AB\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + ABC + AB\bar{C} + \bar{A}BC + \bar{A}B\bar{C}$$

$$= \sum m(0, 1, 2, 5, 6, 7)$$

$$\pi M(3, 4)$$

$$= (A+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)$$

K- MAP

The Karnaugh map is a systematic method of simplifying the Boolean expressions. The K-map is a chart or a group composed of an arrangement of adjacent cells, each representing a particular combination of variables in sum or product form.

TWO VARIABLE K- MAP

A two variable expression can have $2^2 = 4$ possible combinations. Each of these combinations are $\bar{A}\bar{B}$, $\bar{A}B$, $A\bar{B}$ and AB (in the SOP form) is called a minterm. Instead of representing the minterms in terms of the input variable using short hand notation the minterms may be represented in terms of their decimal designation m_0 — for $\bar{A}\bar{B}$ m_1 for $\bar{A}B$ m_2 for $A\bar{B}$ & m_3 for AB .

MAPPING OF SOP EXPRESSIONS

A two variable K map has $2^2 = 4$ squares. These squares are called cells. Each square on the K map represents a unique minterm. A '1' placed in any square indicates that the corresponding minterm is included in the output expression & a '0' or no entry in a square indicates that the corresponding minterms does not appear in the expression for input.

MAP FORMATa) Two variable

If there are n variables 2^n combinations are there and 2^n cells are there.

	B	\bar{B}	B
A	m_0	m_1	
\bar{A}	m_2	m_3	

m_0, m_1, m_2 & m_3 are called min terms

b) Three variable

consider three variables A B & C

	\bar{C}	C
AB	m_0	m_1
$\bar{A}\bar{B}$	m_2	m_3
$\bar{A}B$	m_4	m_5
AB	m_6	m_7
$A\bar{B}$	m_8	m_9

3 variable - 3rd row change.

c) four variable

consider 4 variables A, B, C & D

	CD	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$
AB	m_0	m_1	m_2	m_3
$\bar{A}\bar{B}$	m_4	m_5	m_6	m_7
$\bar{A}B$	m_8	m_9	m_{10}	m_{11}
AB	m_{12}	m_{13}	m_{14}	m_{15}
$A\bar{B}$	m_{16}	m_{17}	m_{18}	m_{19}

PLOTTING THE BOOLEAN EXPRESSION

Once the Boolean expression is in sum of the product form. Plot each on the K-map by placing a 1 in each cell corresponding to the terms in SOP expression

i) $AB + \bar{A}\bar{B}$

	A	B	\bar{B}	B
\bar{A}		1	0	
A		0	1	

ii) $\bar{A}BC + AB\bar{C} + ABC$

	\bar{B}	C
$\bar{A}\bar{B}$	0	0
$\bar{A}B$	0	1
AB	1	1
$A\bar{B}$	0	0

iii) $\bar{A}\bar{B}CD + \bar{A}B\bar{C}D + A\bar{B}C\bar{D} + A\bar{B}CD$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	0	1	0
$\bar{A}B$	0	0	1	0
AB	0	0	0	0
$A\bar{B}$	0	0	1	1

GROUPING OF CELLS

Group the 1's that are in adjacent cell. Adjacent cells are cells that differ by only a single variable.

	A	B	\bar{B}	B
\bar{A}		1	1	
A		1	1	

Adjacent cells of $\bar{A}\bar{B}$ are $\bar{A}B$ & $A\bar{B}$

The 1's in adjacent cells must be combined in groups of 1, 2, 4, 8, 16 & so on.

SIMPLIFICATION

Each group of 1 composed of all variable that appears in one form with in group. The variables that appear both complemented & uncomplemented are eliminated. The final simplified expression is formed by summing the part terms of all groups.

2 Variable K-map

1. $AB + A\bar{B}$

$$S = \sum (m_0, m_2)$$

	\bar{B}	B
\bar{A}	1	0
A	1	0

A & \bar{A} are eliminated

$$AB + A\bar{B} = \bar{B}$$

2. $\bar{A}\bar{B} + \bar{A}B$, $S = \sum (m_0, m_1)$

	\bar{B}	B
\bar{A}	1	1
A	0	0

$$\bar{A}\bar{B} + \bar{A}B = \bar{A}$$

3. $\bar{A}\bar{B} + \bar{A}B + A\bar{B} + AB$

	\bar{B}	B
\bar{A}	1	1
A	1	1

$$= 1$$

4. $S = \sum (m_0, m_1, m_2)$

	\bar{B}	B
\bar{A}	1	1
A	1	0

$$= \bar{A} + \bar{B}$$

3 VARIABLE

1. $S = \sum (m_0, m_1, m_2, m_4, m_7)$

	\bar{E}	C
$\bar{A}\bar{B}$	1	1
$\bar{A}B$	1	0
AB	0	1
$A\bar{B}$	1	0

$\Rightarrow \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}\bar{C} + ABC$

2. $A\bar{B}C + \bar{A}B\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$

	\bar{E}	C
$\bar{A}\bar{B}$	1	1
$\bar{A}B$	0	1
AB	0	0
$A\bar{B}$	1	1

$\Rightarrow \bar{B} + \bar{A}C$

3. $S = \sum (m_0, m_1, m_4, m_5, m_6, m_7, m_8, m_9, m_{12}, m_{13}, m_{14}, m_{15})$

	$\bar{E}\bar{D}$	$\bar{E}D$	ED	ED
$\bar{A}\bar{B}$	1	1	0	0
$\bar{A}B$	1	1	1	1
AB	1	1	1	1
$A\bar{B}$	1	1	0	0

$\Rightarrow \bar{E} + B$

4. $S = \sum (3, 3, 4, 5, 10, 11, 14, 15)$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	CD
$\bar{A}\bar{B}$	0	0	1	1
$\bar{A}B$	1	0	0	1
AB	0	0	1	1
$A\bar{B}$	0	0	1	1

$\Rightarrow AC + \bar{B}C + \bar{A}B\bar{D} + C\bar{D}$

K- MAP INCLUDING DONT CARE

Q. $S = \sum (0, 13, 14, 15) + \sum d(1, 2, 3, 9, 10, 11)$

↓
min terms

↓
don't care term

(can assume values 0 or 1)

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	d	d	d
$\bar{A}B$	0	0	0	0
AB	0	1	1	1
$A\bar{B}$	0	d	d	d

⇒

1	1	1	1
0	0	0	0
0	1	1	1
0	1	1	1

$$S = \bar{A}B + AD + AC$$

Q. $S = \sum (0, 1, 2, 3, 8, 9, 10, 13, 15) + \sum d(4, 5, 11, 14)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	1	1
$\bar{A}B$	d	d	0	0
AB	0	1	1	d
$A\bar{B}$	1	1	d	1

⇒

1	1	1	1
0	0	0	0
0	1	1	0
1	1	1	1

$$S = AD + \bar{B}$$

Q. Simplify the following expression using k-map approach.

$$S = \sum (1, 5, 7, 11, 15) + \sum d(0, 3, 5)$$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	1	1
$\bar{A}B$	0	1	1	0
AB	0	0	1	0
$A\bar{B}$	0	0	1	0

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	1	1
$\bar{A}B$	0	0	1	0
AB	0	0	1	0
$A\bar{B}$	0	0	1	0

$$S = \underline{CD + A\bar{B}}$$

II not completed.
 25/11/17

MODULE-3

COMBINATIONAL CIRCUITS

XOR (Exclusive OR)

Same i/p \Rightarrow o/p = 0
 diff i/p \Rightarrow o/p = 1

} Also known as COMPARATOR

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

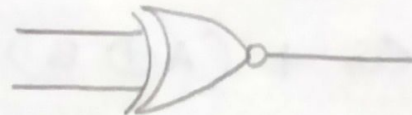
$$Y = \bar{A}B + A\bar{B}$$



XNOR (Exclusive NOR)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

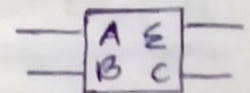
$$Y = \bar{A}\bar{B} + AB$$



ADDER CIRCUITS

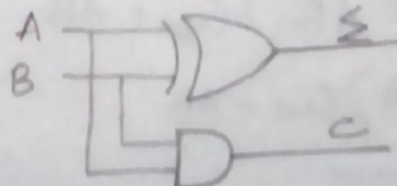
①. Half adder \rightarrow Only two bits

A	B	Σ	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



$$\Sigma = A \oplus B$$

Carry: AB



② Full adder

A	B	C _{in}	Σ	C _{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$\bar{A}\bar{B}C_{in} + \bar{A}B\bar{C}_{in} + A\bar{B}\bar{C}_{in} + ABC_{in}$$

$$(\bar{A}\bar{B} + AB)C_{in} + (\bar{A}B + A\bar{B})C_{in}$$

$$= \overline{A \oplus B} C_{in} + (A \oplus B)C_{in}$$

$$A \oplus B = Y$$

$$= \bar{Y}C_{in} + YC_{in}$$

$$= Y \oplus C_{in}$$

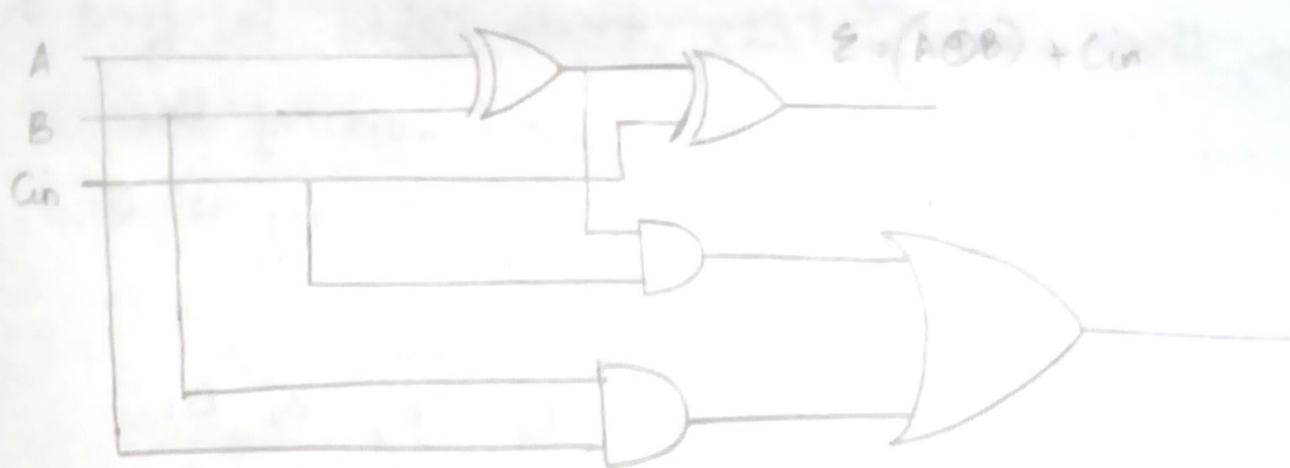
$$= \underline{\underline{[A \oplus B] \oplus C_{in}}}$$

$$C_{out} = \bar{A}BC_{in} + A\bar{B}C_{in} + AB\bar{C}_{in} + ABC_{in}$$

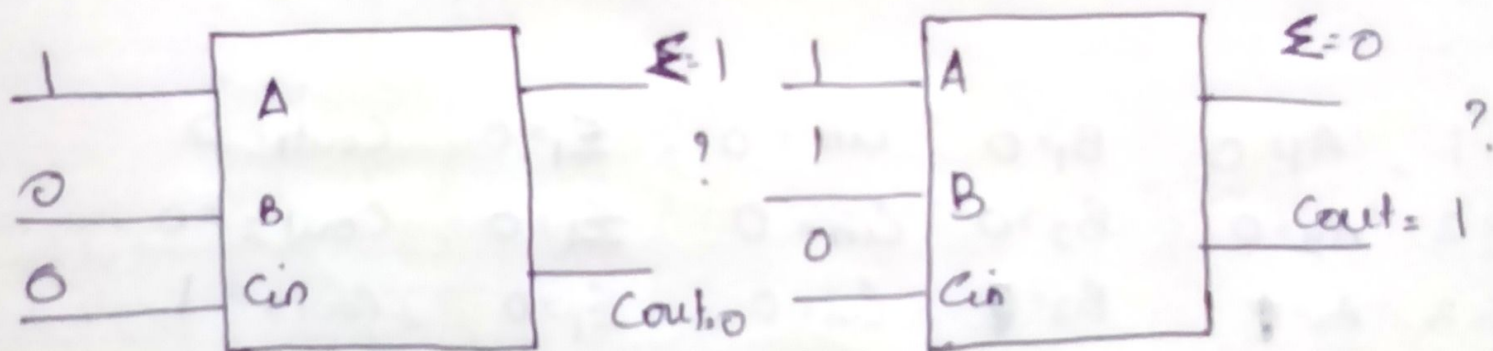
$$= \bar{A}BC_{in} + A\bar{B}C_{in} + AB$$

$$= (\bar{A}B + A\bar{B})C_{in} + AB$$

$$= (A \oplus B)C_{in} + AB$$



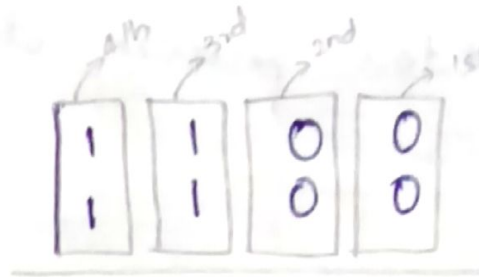
For the full adder shown, determine the outputs:



Q. Use the 4 bit ^{full} adder truth table to find the sum and o/p carry for the addition of the following two 4 bit nos. if i/p carry is 0.

A₄ A₃ A₂ A₁
1 1 0 0

B₄ B₃ B₂ B₁
1 1 0 0



n=1, A₁=0

B₁=0 C_{in}=0

Σ₁=0 Cout₁=0

n=2, A₂=0

B₂=0 C_{in}=0

Σ₁=0 Cout₂=0

n=3, A₃=0

B₃=0 C_{in}=0

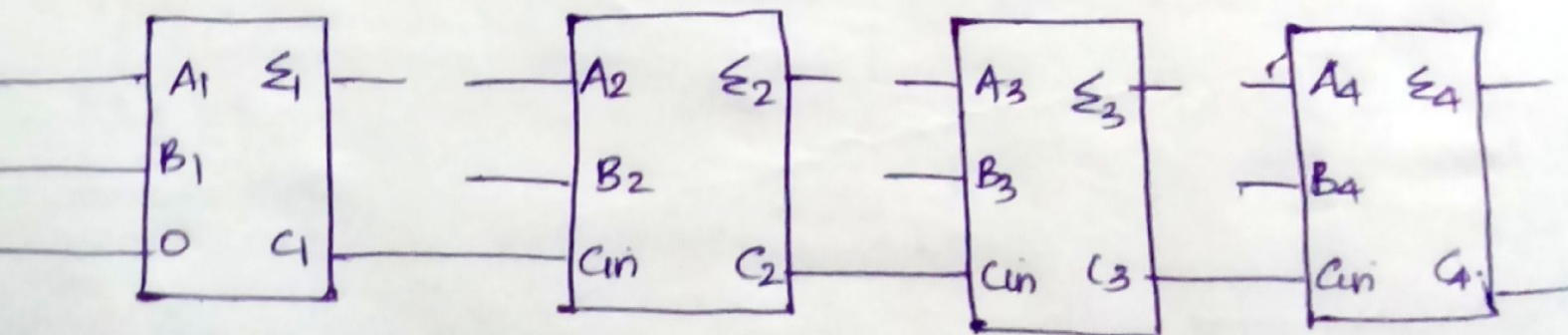
Σ₁=0 Cout₃=1

n=4, A₄=1

B₄=1 C_{in}=1

Σ₁=1 Cout₄=1

= 11000₂



Q. Use the truth table to find the result of adding the binary number 1011 & 1010 using full ladder.

$$n=1, A_1=1, B_1=0, C_1=0, \Sigma_1=1, \text{Carry}=0$$

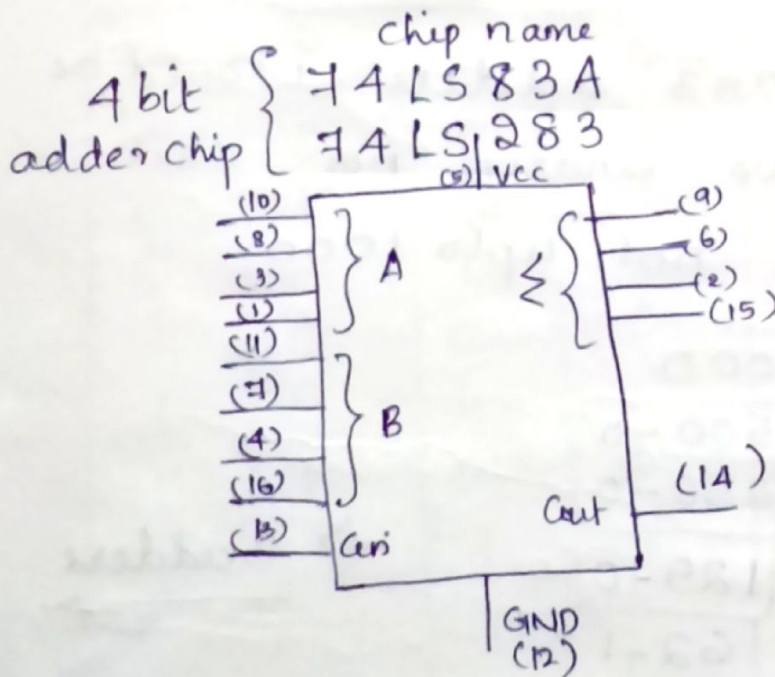
$$n=2, A_2=1, B_2=1, C_2=0, \Sigma_2=0, \text{Carry}=1$$

$$n=3, A_3=0, B_3=0, C_3=1, \Sigma_3=1, \text{Carry}=0$$

$$n=4, A_4=1, B_4=1, C_4=0, \Sigma_4=0, \text{Carry}=1$$

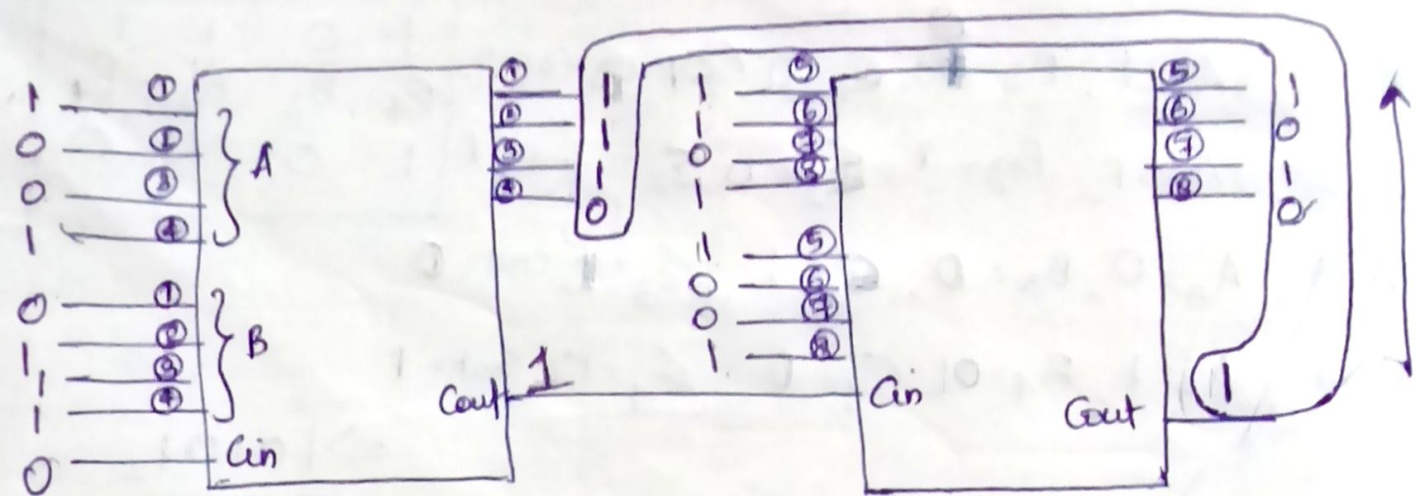
A_4	A_3	A_2	A_1
1	0	1	1
B_4	B_3	B_2	B_1
1	0	1	0

$\Rightarrow 10101_2$



Q. Show how two 74LS83A adders can be connected to form a 8-bit adder to add the two eight bits 10111001 & 10011110

A_8 A_7 A_6 A_5 A_4 A_3 A_2 A_1
 1 0 1 1 1 0 0 1
 +
 B_8 B_7 B_6 B_5 B_4 B_3 B_2 B_1
 1 0 0 1 1 1 1 0



$$= \underline{\underline{10101011_2}}$$

2. How many 74LS 283 adders would be required to add two binary nos. each representing decimal nos upto 1000.

2	1000
2	500-0
2	250-0
2	125-0
2	62-1
2	31-0
2	15-1
2	7-1
2	3-1
2	1-1

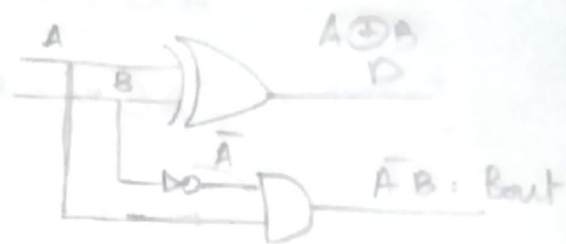
$\Rightarrow \underline{\underline{3 \text{ adders}}}$

HALF SUBTRACTOR

A	B	D	Bout
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$$D = \bar{A}B + A\bar{B} = A \oplus B$$

$$Bout = \bar{A}B$$



FULL SUBTRACTOR

A	B	Bin	Bout	D
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

$$\begin{aligned} D &= \bar{A} \bar{B} Bin + \bar{A} B \bar{Bin} + A \bar{B} \bar{Bin} + A B Bin \\ &= (\bar{A} \bar{B} + AB) Bin + (\bar{A} B + A \bar{B}) \bar{Bin} \\ &= (\overline{A \oplus B}) Bin + (A \oplus B) \bar{Bin} \\ &= \underline{\underline{(A \oplus B) \oplus Bin}} \end{aligned}$$

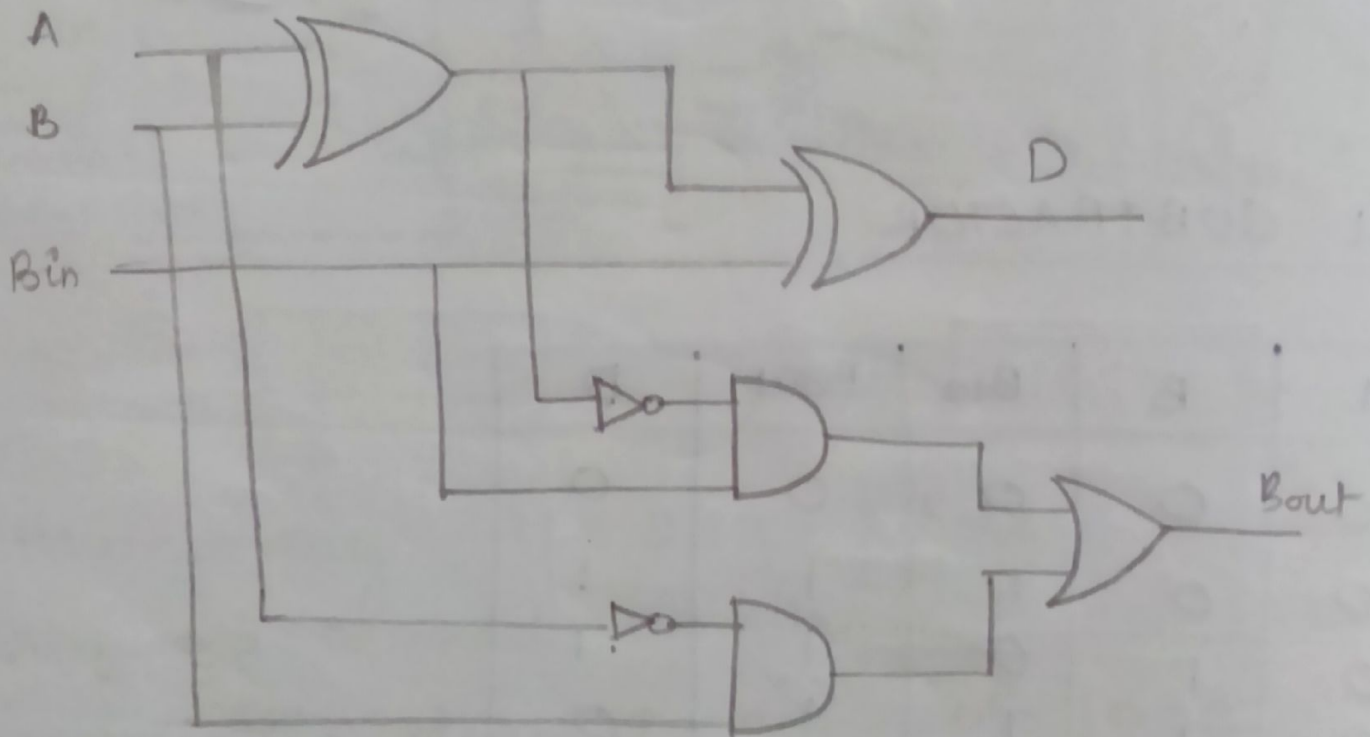
$$B_{out} = \bar{A} \bar{B} B_{in} + \bar{A} B \bar{B}_{in} + \bar{A} B B_{in} + A B B_{in}$$

$$= \bar{A} \bar{B} B_{in} + \bar{A} B + A B B_{in}$$

$$= (\bar{A} \bar{B} + A B) B_{in} + \bar{A} B$$

$$= (\overline{A \oplus B}) B_{in} + \bar{A} B$$

$$[\overline{A \oplus B} = \bar{A} \bar{B} + A B]$$



2. Explain ALU using block diagram.

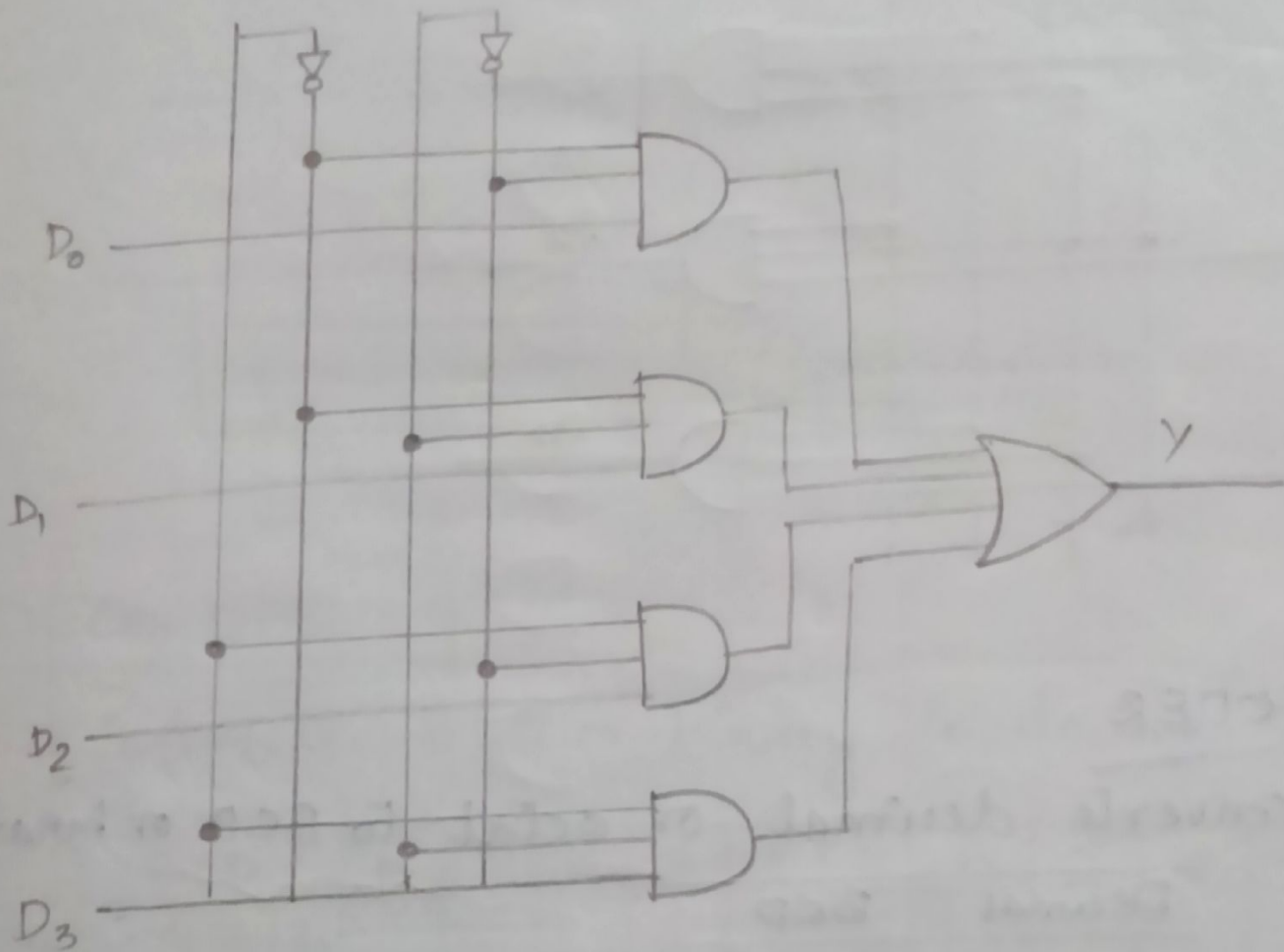
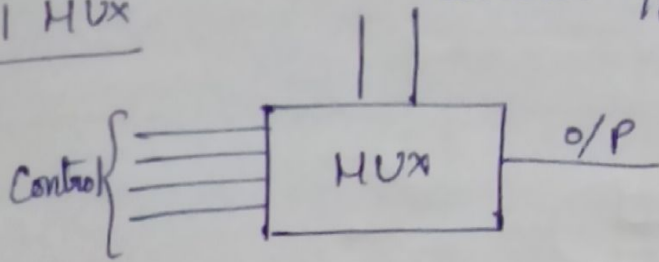
3. Implement a full subtractor using a full adder.

MULTIPLEXER (MUX)

⇒ 4 to 1 MUX

control i/p's

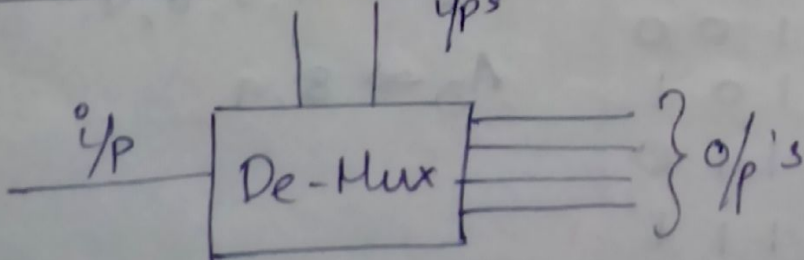
4 i/p's ; 1 o/p

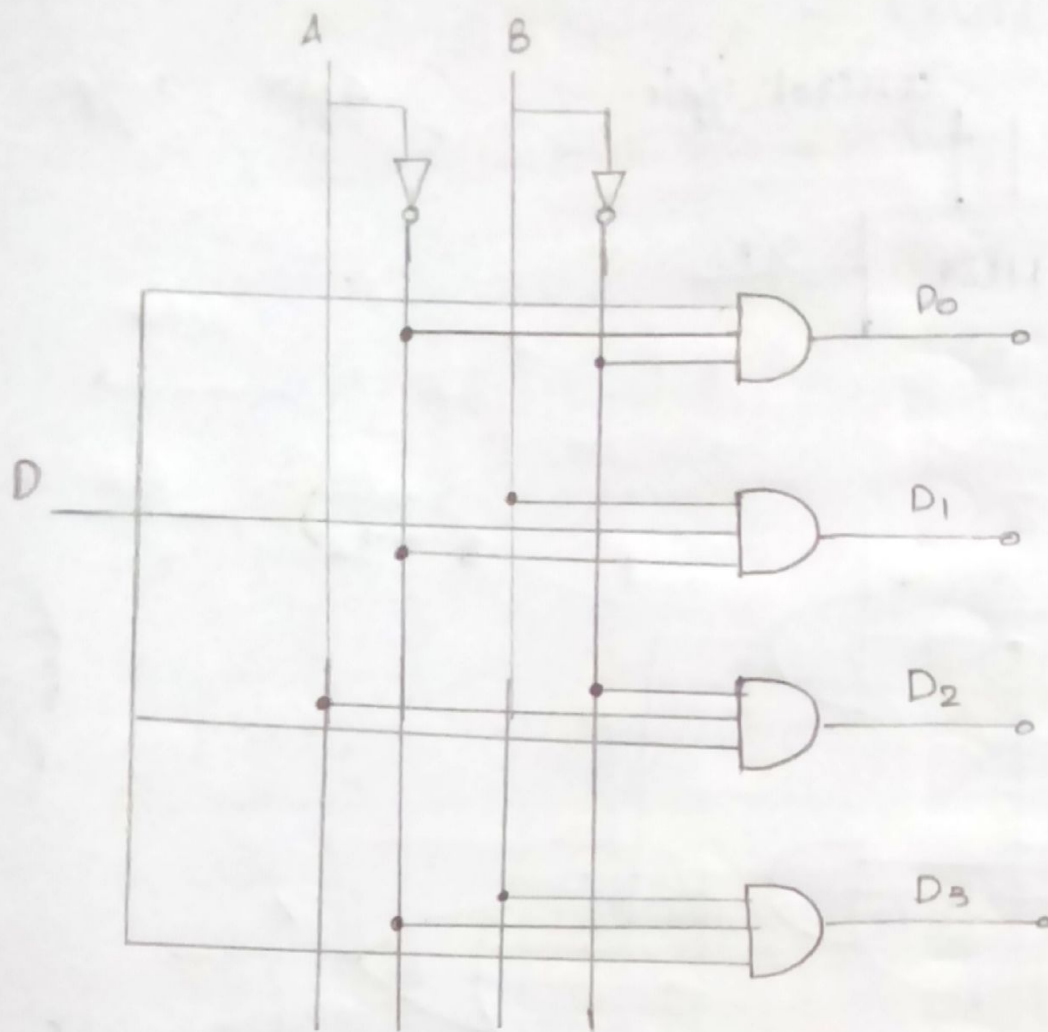


* when $A = 0$, $B = 0$, $Y = D_0$

DE-MUX

control i/p's





ENCODER

converts decimal or octal to BCD or binary

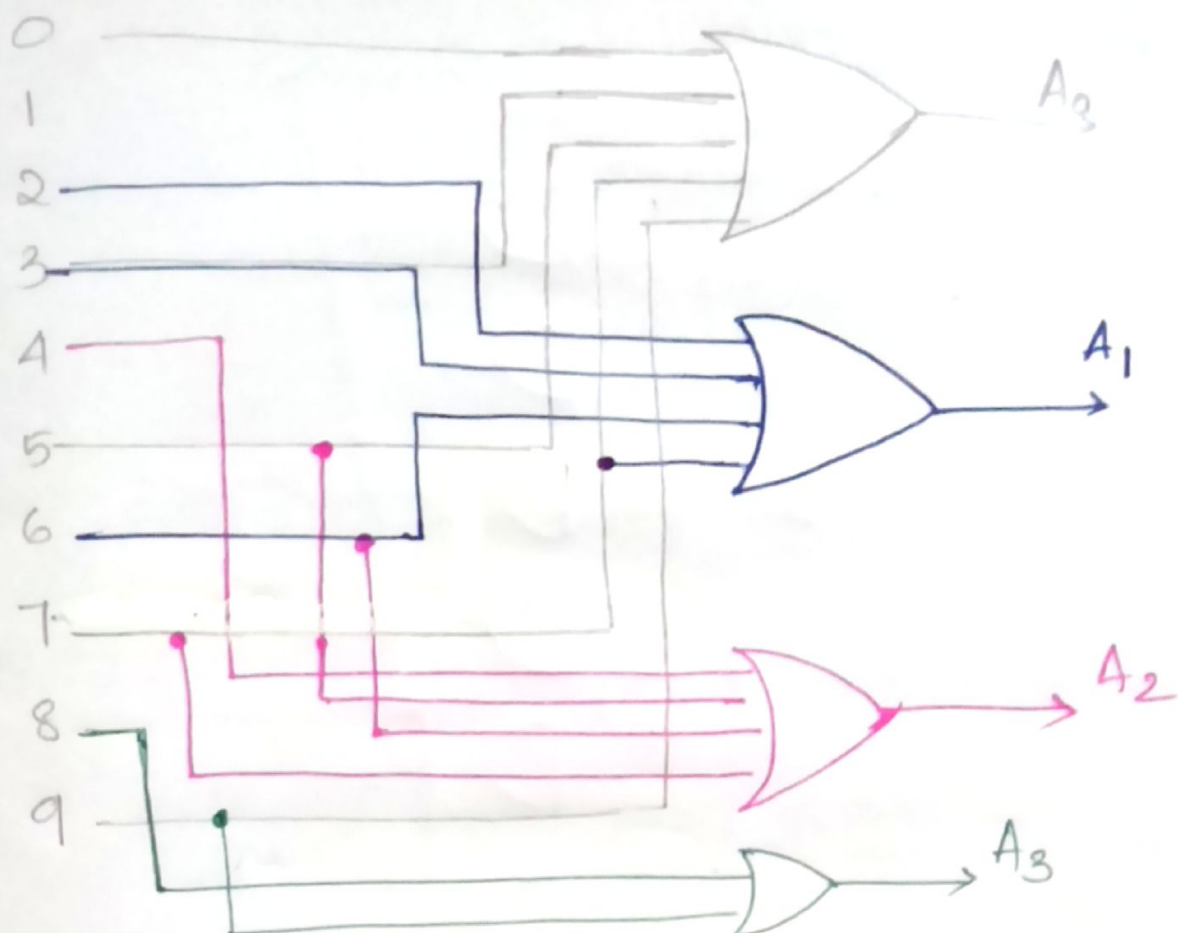
Decimal	BCD			
	A_3	A_2	A_1	A_0
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1

$A_0 \Rightarrow 1, 3, 5, 7, 9$

$A_1 \Rightarrow 2, 3, 6, 7$

$A_2 \Rightarrow 4, 5, 6, 7$

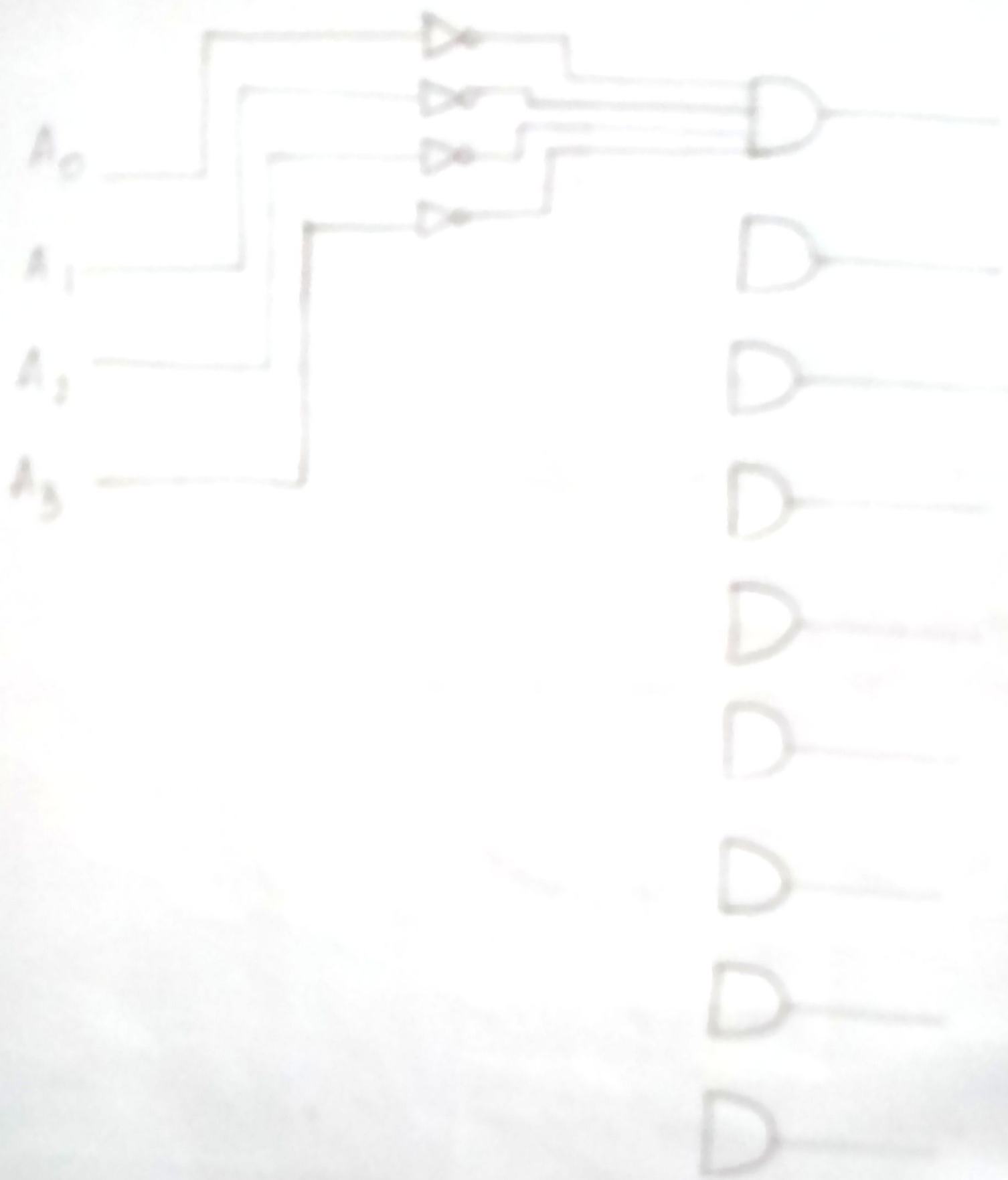
$A_3 \Rightarrow 8, 9$



DECODER

Converts BCD or binary to decimal or octal.

BCD				0	1	2	3	4	5	6	7	8	9
A ₃	A ₂	A ₁	A ₀										
0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	1	0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0	0	0	0	0	0	0
0	0	1	1	0	0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	1	0	0	0	0	0
0	1	0	1	0	0	0	0	0	1	0	0	0	0
0	1	1	0	0	0	0	0	0	0	1	0	0	0
0	1	1	1	0	0	0	0	0	0	0	1	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	1
1	0	0	1	0	0	0	0	0	0	0	0	0	1



MODULE - 4

FLIP FLOP

→ Logic gates - Combinational circuits
- because it does not care about the previous i/p & it does not have a memory.

Sequential circuit → It has an i/p & o/p.
→ It has a memory sequence and is given as input through a feedback.

FLIP - FLOPS

* Printed notes.

CONVERSION OF FLIP-FLOP

Excitation table

Q_t (present)	Q_{t+1} (desired)	S	R	J	K	D	T
0	0	0	1	0	0	0	0
0	1	1	0	0	1	1	1
1	0	0	1	1	0	0	1
1	1	0	0	1	1	1	0

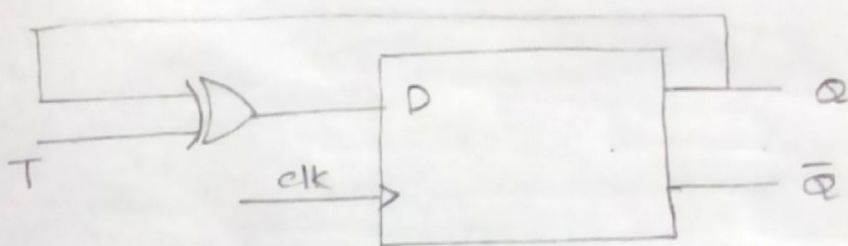
Q. Convert DFF to TFF

From the excitation table:

T	Q	D
0	0	0
1	0	1
1	1	0
0	1	1

$$D = T\bar{Q} + Q\bar{T}$$

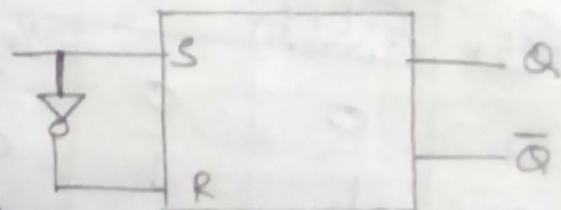
$$= \underline{\underline{T \oplus Q}}$$



Q. Convert SRFF to DFF

From the excitation table:

S	R	D	Q
0	X	0	0
1	0	1	0
0	1	0	1
X	0	1	1



0	1
1	0

$R = \bar{D}$

0	0
1	X

$S = D$

$$R = \bar{D}$$

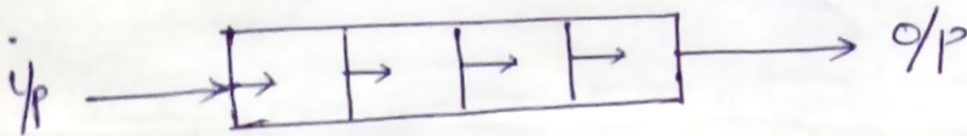
$$S = D$$

REGISTERS

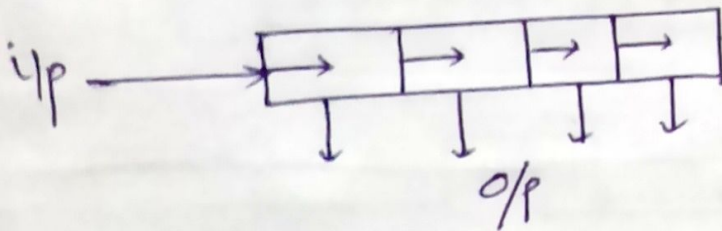
↳ store values

↳ shifts values.

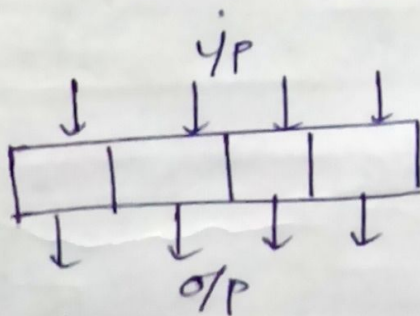
1) Serial i/p Serial o/p (SISO)



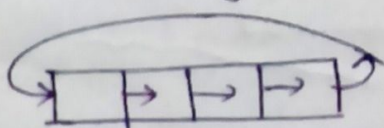
2) Serial input Parallel o/p (SIPO)



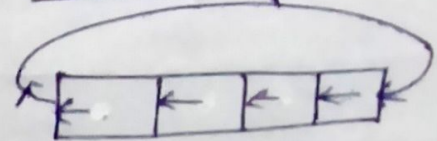
3) Parallel i/p Parallel o/p (PIPO)



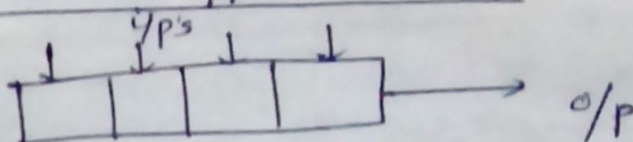
Rotate right:



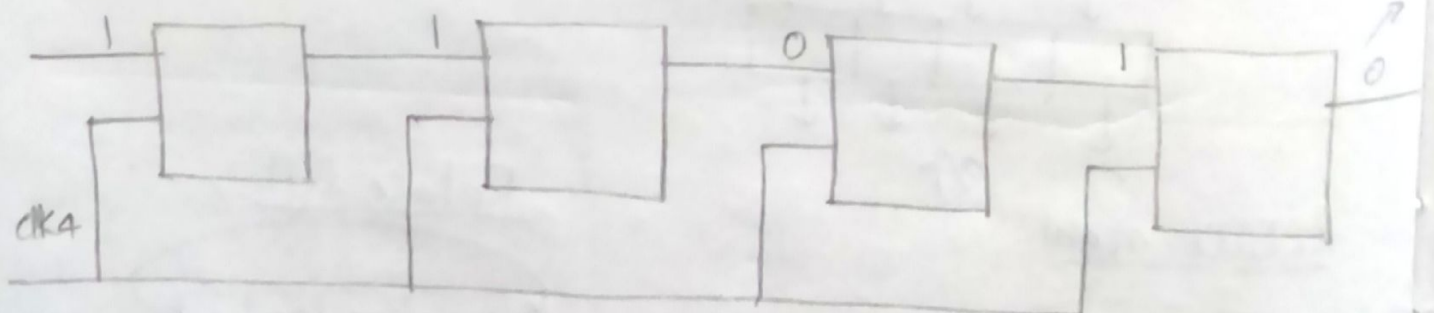
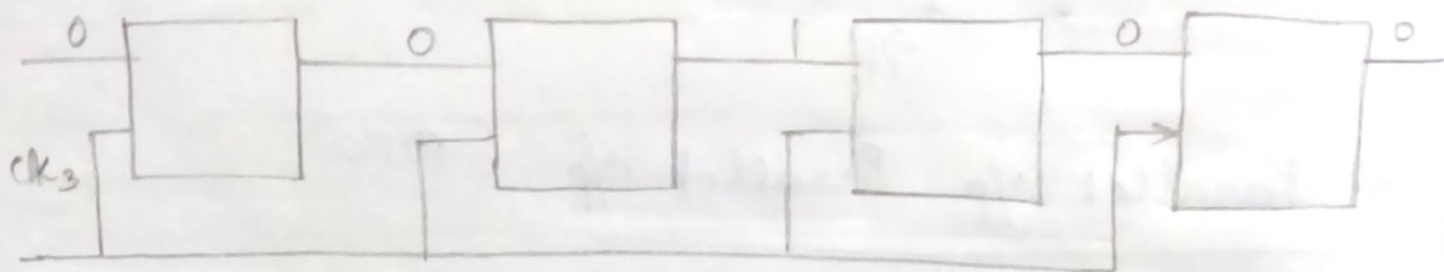
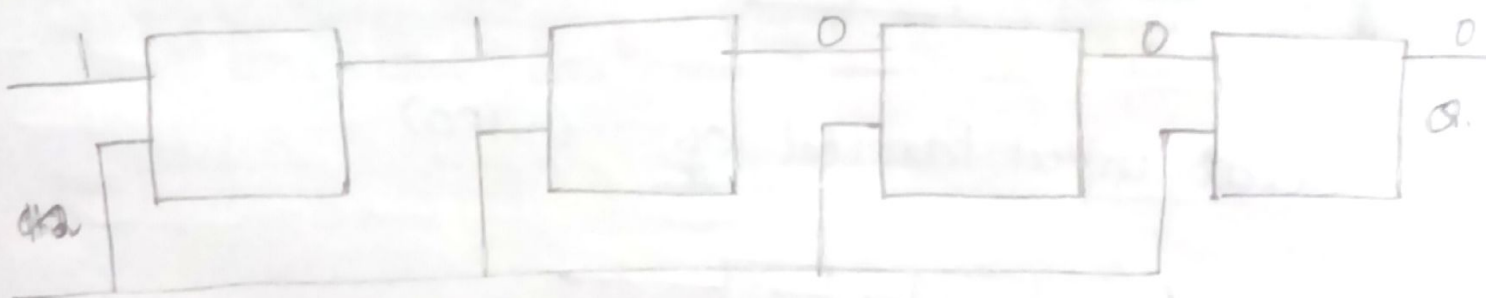
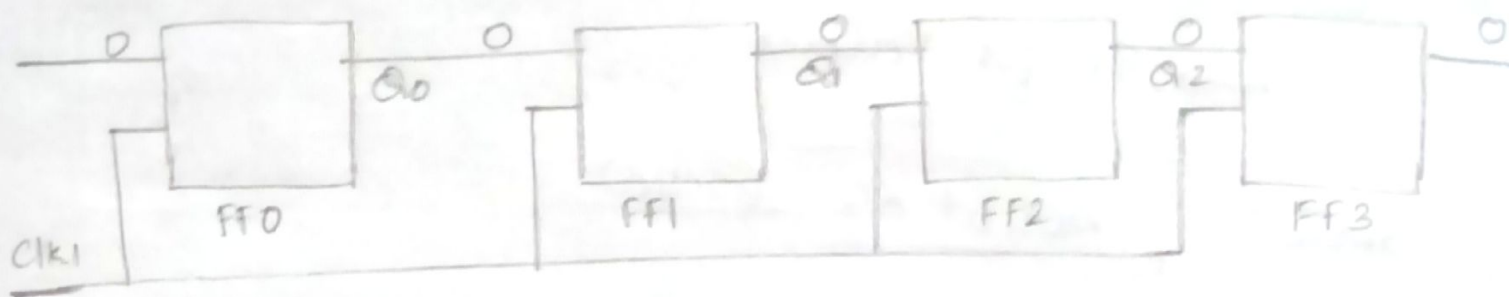
Rotate left:



4) Parallel i/p Serial o/p (PISO)



SERIAL i/p SERIAL o/p (SISO)



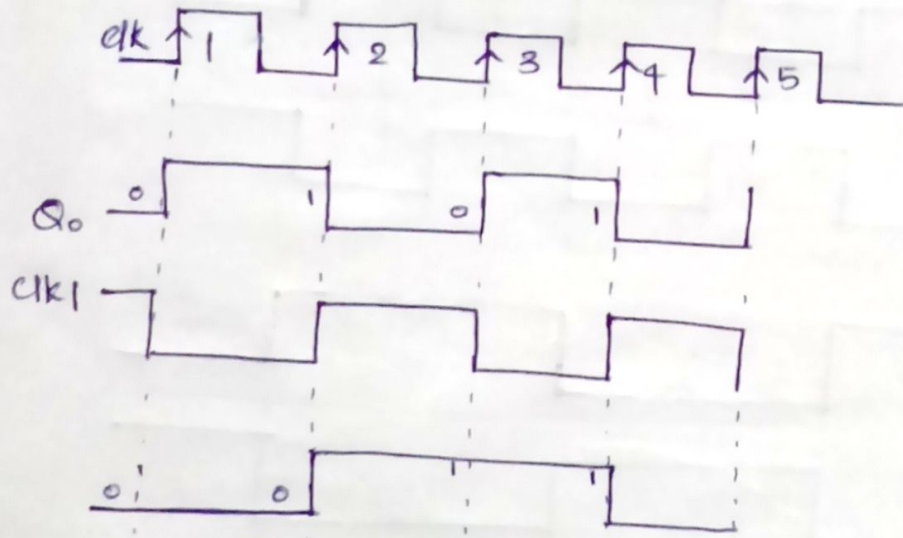
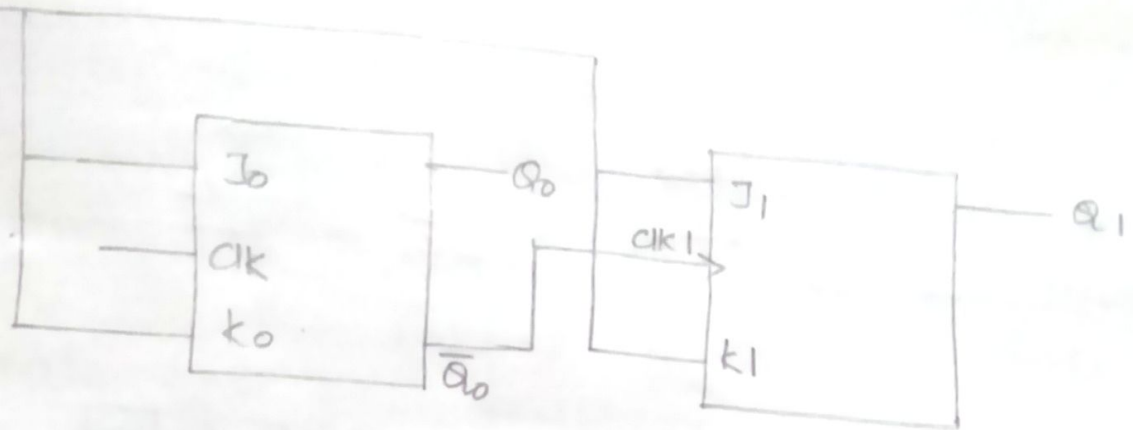
CLK 5 \rightarrow Q_2 as o/p
 CLK 6 \rightarrow Q_1 as o/p
 CLK 7 \rightarrow Q_0 as o/p
 CLK 8 \rightarrow 0

Flip Flop clear

ASYNCHRONOUS COUNTER (RIPPLE COUNTER)

0-3 COUNTER

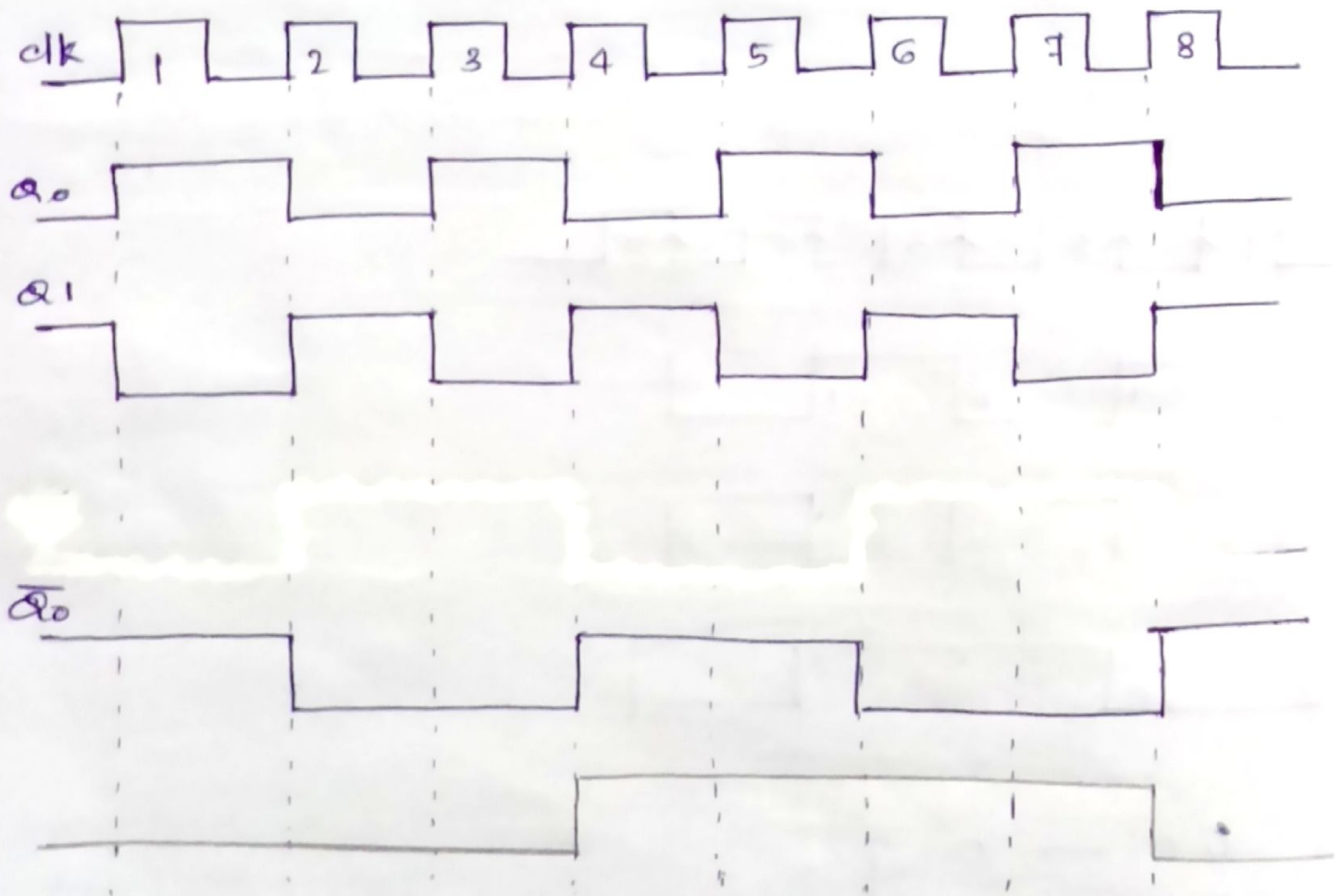
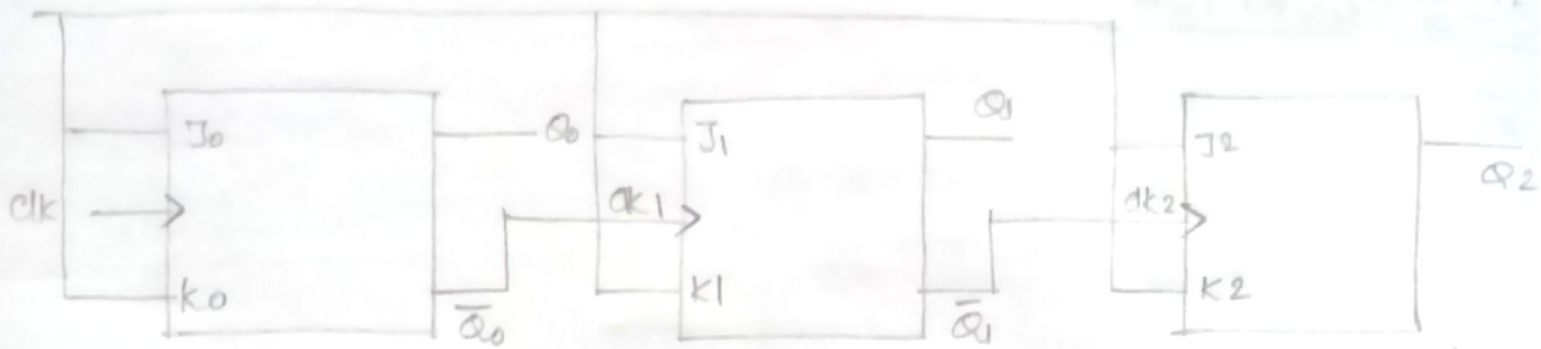
High



0 0	→	0	} 0-3 counter.
0 1	→	1	
1 0	→	2	
1 1	→	3	

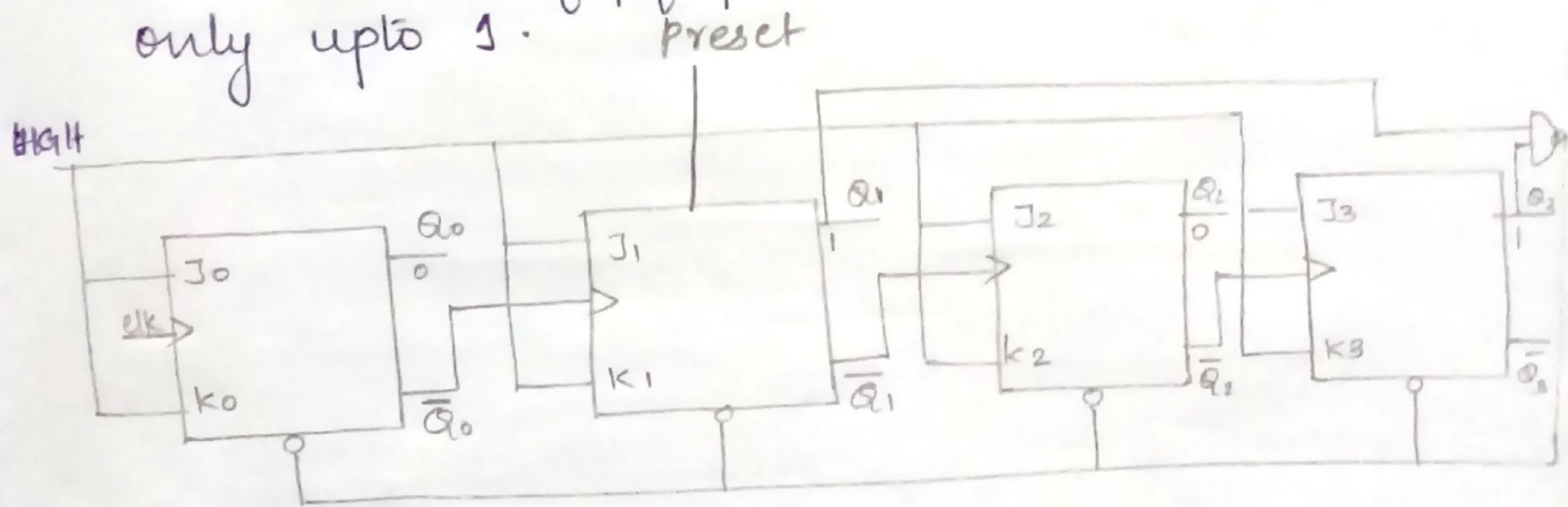
→ 1st flip flop is clocked by external clock pulse & then each successive flip-flop is clocked by the o/p of the preceding flip flop. An Asynchronous Counter is one in which the flip-flops within the counter do not change the states exactly at the same time because they do not have a common clock pulse.

0-7 COUNTER

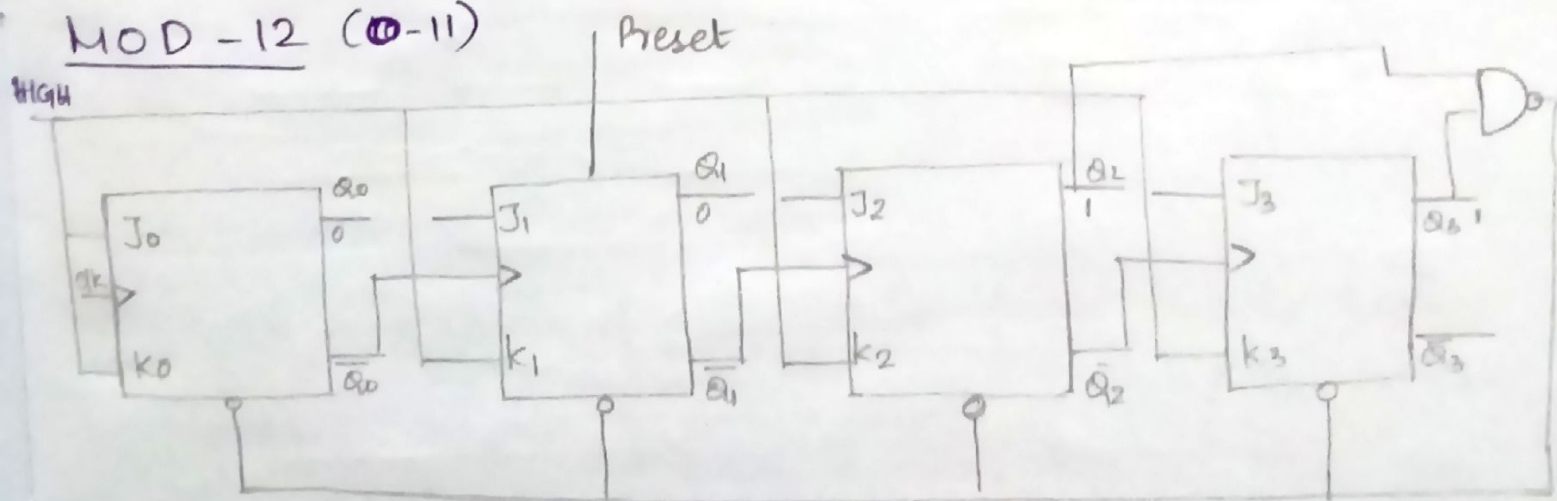


Mod-10 (0-9)

To count 10 Nos (0-9) we need 4 flip-flops.
But 4 flip-flops will count upto 0-15.
So use 'clear'. Here, when the 10th clock comes a high (1) will come to 'clear' and sets all the flipflops to 0. Thus containing only upto 9.



MOD-12 (0-11)

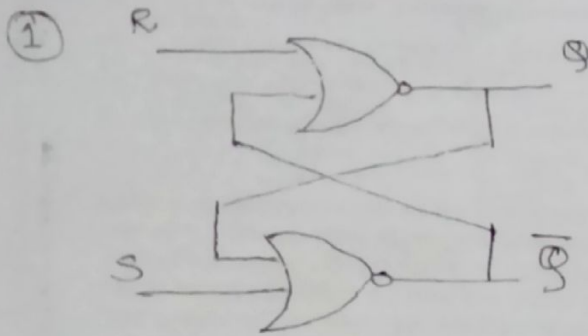


* with bubbles - Active Low

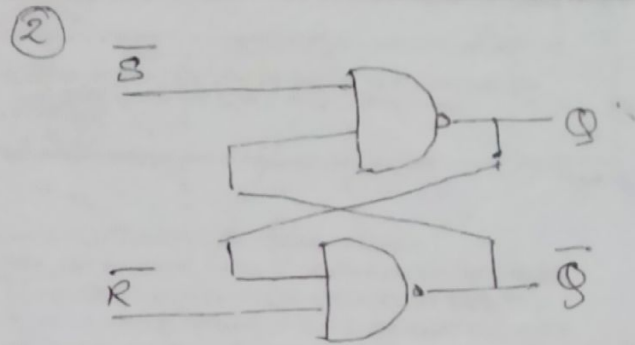
* without bubbles - Active high.

S-R FLIPFLOP

Two versions:



Active High i/p
SR Latch.



Active Low i/p
SR Latch.

TRUTH TABLE:-

①

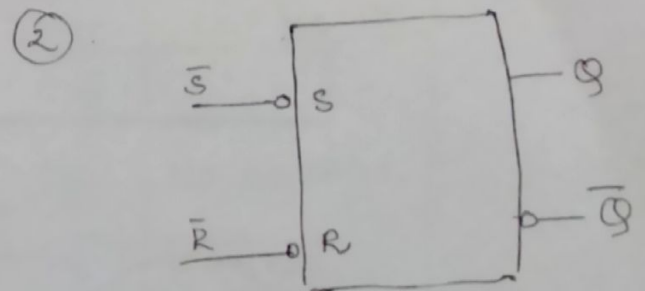
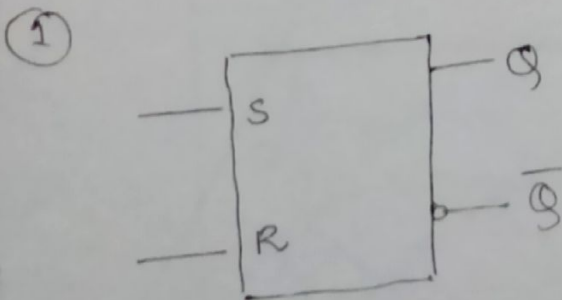
INPUTS		OUTPUTS		COMMENTS
S	R	Q	Q̄	
0	0	NC	NC	Remains same
0	1	0	1	Latch SET
1	0	1	0	Latch SET
1	1	0	0	Invalid

②

INPUTS		OUTPUTS		COMMENTS
S	R	Q	Q̄	
0	0	NC	NC	Remain same
0	1	0	1	Latch RESET
1	0	1	0	Latch SET
1	1	1	1	Invalid

* NC - No change (o/p as previous)

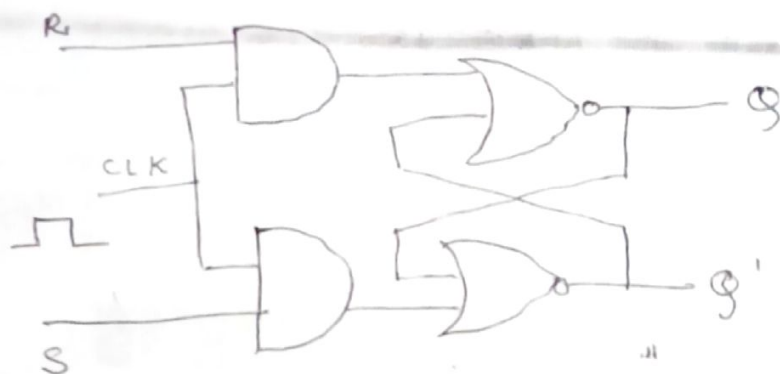
LOGIC SYMBOL:-



Active high i/p → indicates the circuit is activated when a HIGH signal is given.

Active low i/p → indicates the circuit is activated when a low signal is given.

(a)



(b) LOGIC SYMBOL



- O/P of the AND gates remains 0 as long as clock pulse is 0.
- With both $S=1$; $R=1$ and clock pulse \uparrow the o/p goes to 0, both Q & Q' . Then when pulse goes to Low the o/p of AND gates becomes 0, which causes it to choose a state $Q=0$ or $Q=1$ and thus it is called indeterminate.

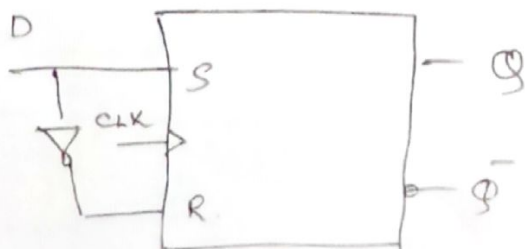
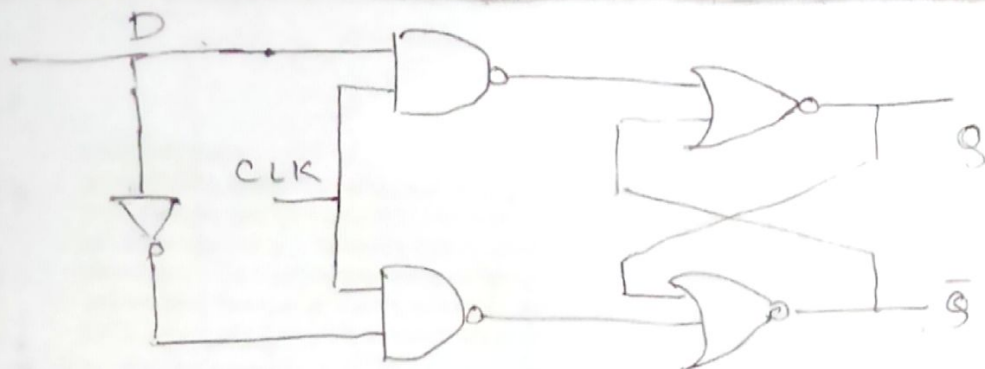
TRUTH TABLE OF POSITIVE EDGE TRIGGERED S-R FF :-

INPUTS			OUTPUTS		COMMENTS
S	R	CLK	Q	\bar{Q}	
0	0	X	Q_0	\bar{Q}_0	No change
0	1	\uparrow	0	1	RESET
1	0	\uparrow	1	0	SET
1	1	\uparrow	Invalid		

- * $\uparrow \rightarrow$ clock transition Low to HIGH
- X \rightarrow irrelevant ("don't care")
- $Q_0 \rightarrow$ previous state; prior to clock transition.

D FLIP FLOP :

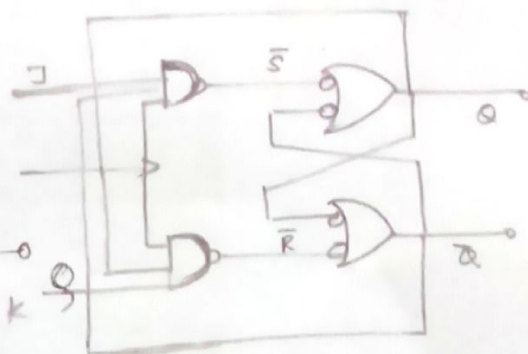
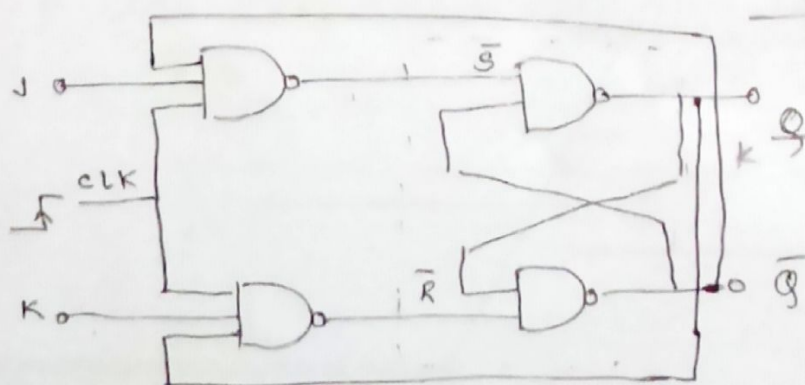
'D' — Data



operation of positive edge triggered.

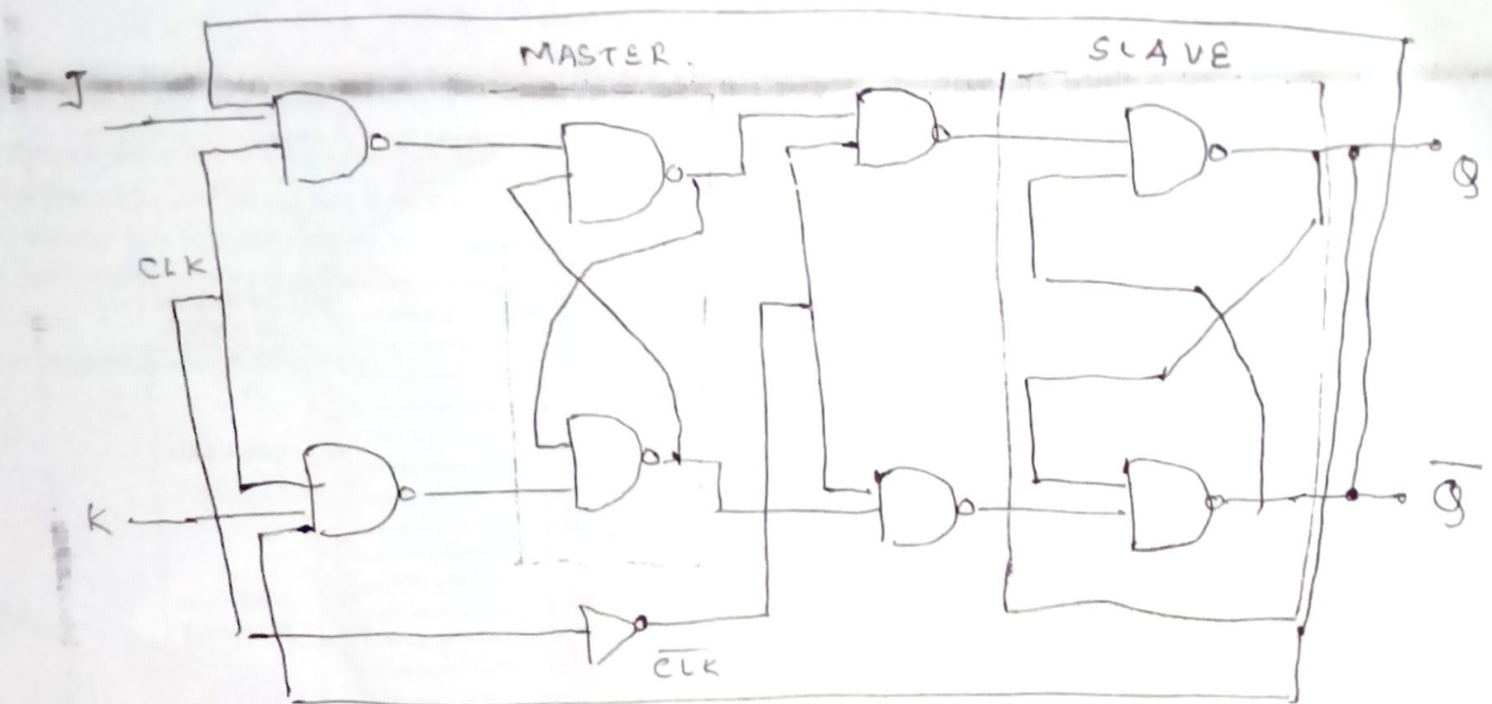
Q_0	D	Q
0	0	0
0	1	0
1	0	0
1	1	1

J K FLIP FLOP : -



INPUTS			OUTPUTS		COMMENTS
J	K	CLK	Q	\bar{Q}	
0	0	↑	Q_0	\bar{Q}_0	No change
0	1	↑	0	1	RESET
1	0	↑	1	0	SET
1	1	↑	\bar{Q}_0	Q_0	Toggle

MASTER SLAVE JK FLIP FLOP

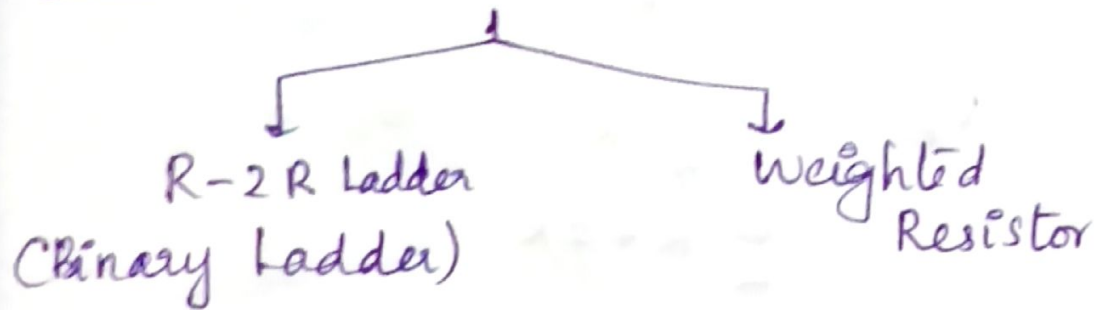


The input J K is connected to master SR.
 As clock to 'slave' FF is complement of master clock input, the slave SR flip flop does not toggle.

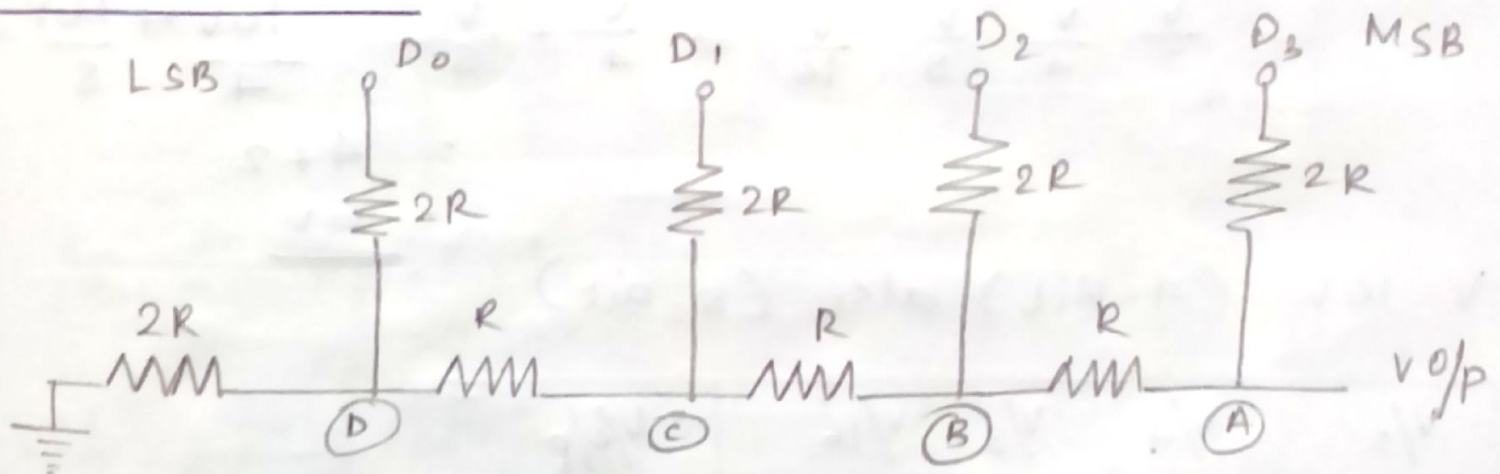
MODULE-6

DIGITAL TO ANALOG CONVERTOR (DAC)

DIGITAL TO ANALOG CONVERTER (DAC)



1) R-2R LADDER

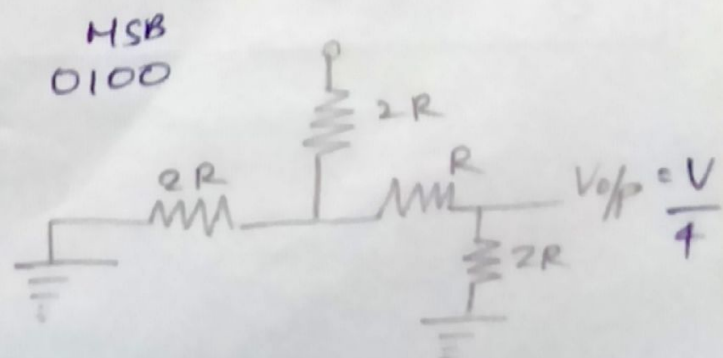
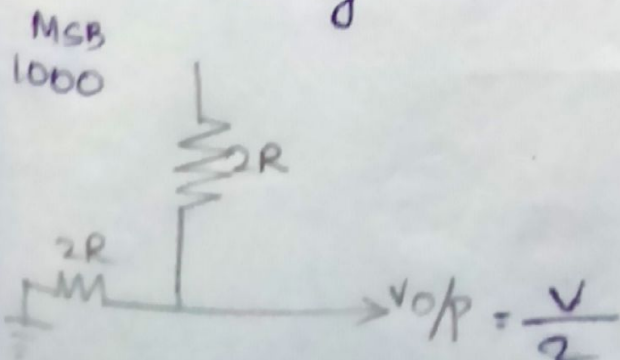


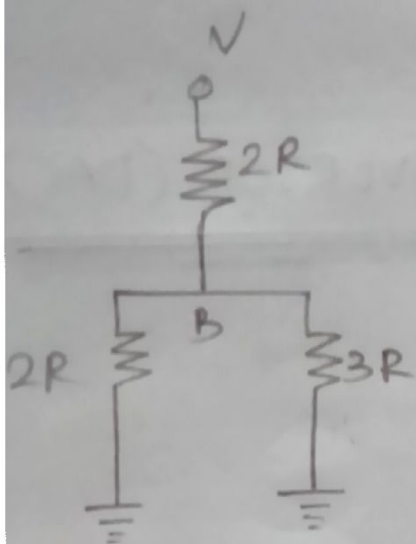
* R and 2R

* A, B, C, D \rightarrow R

* Voltage is weighted.

\rightarrow From any node to ground, resistance is 2R.





$$V_B = \frac{V \left(\frac{6}{5} \right) R}{\frac{16 \times R}{5}} = \underline{\underline{\frac{3}{8} V}}$$

$$V_B = \frac{V \times \frac{6}{5} R}{\frac{6}{5} R + 2R}$$

$$16V \rightarrow 0110$$

$$\frac{V}{2} \quad \frac{V}{4} \quad \frac{V}{8} \quad \frac{V}{16} \Rightarrow \frac{V}{4} + \frac{V}{8} = \frac{16V}{4} + \frac{16V}{8}$$

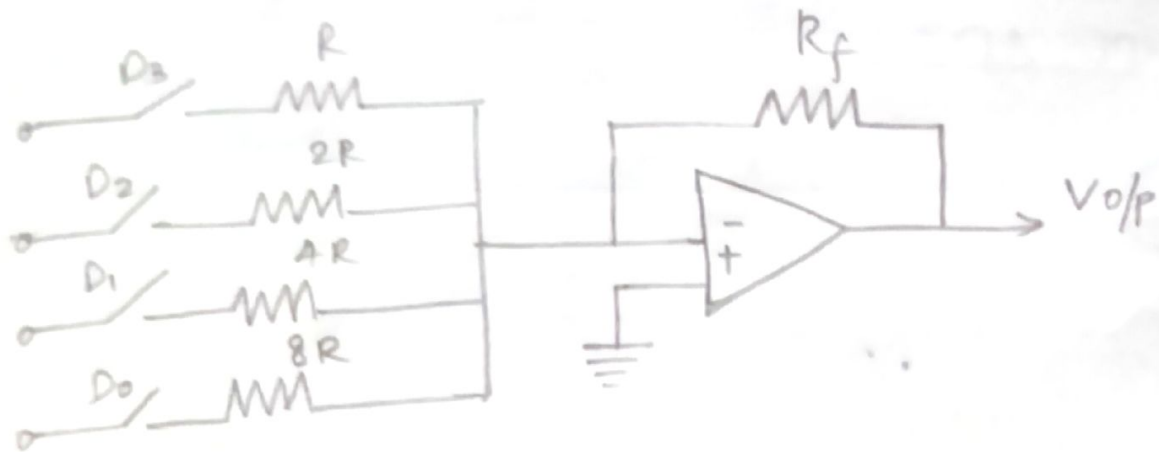
$$= 4 + 2$$

$$= \underline{\underline{6V}}$$

V = 16V (4-Bit) also (8-Bit)

$V/2$	$V/4$	$V/8$	$V/16$	$V_{o/p}$
D_3	D_2	D_1	D_0	
0	0	0	0	0
0	0	0	1	1V
0	0	1	0	2V
0	0	1	1	3V

4-BIT WEIGHTED RESISTOR (Check 8-bit weighted resistor)



$$V_{0/p} = -R_f \left[\frac{V}{R} + \frac{V}{2R} + \frac{V}{4R} + \frac{V}{8R} \right]$$
$$= -R_f \left[\frac{D_3 V}{R} + \frac{D_2 V}{2R} + \frac{D_1 V}{4R} + \frac{D_0 V}{8R} \right]$$

Assume $R_f = R = 1$

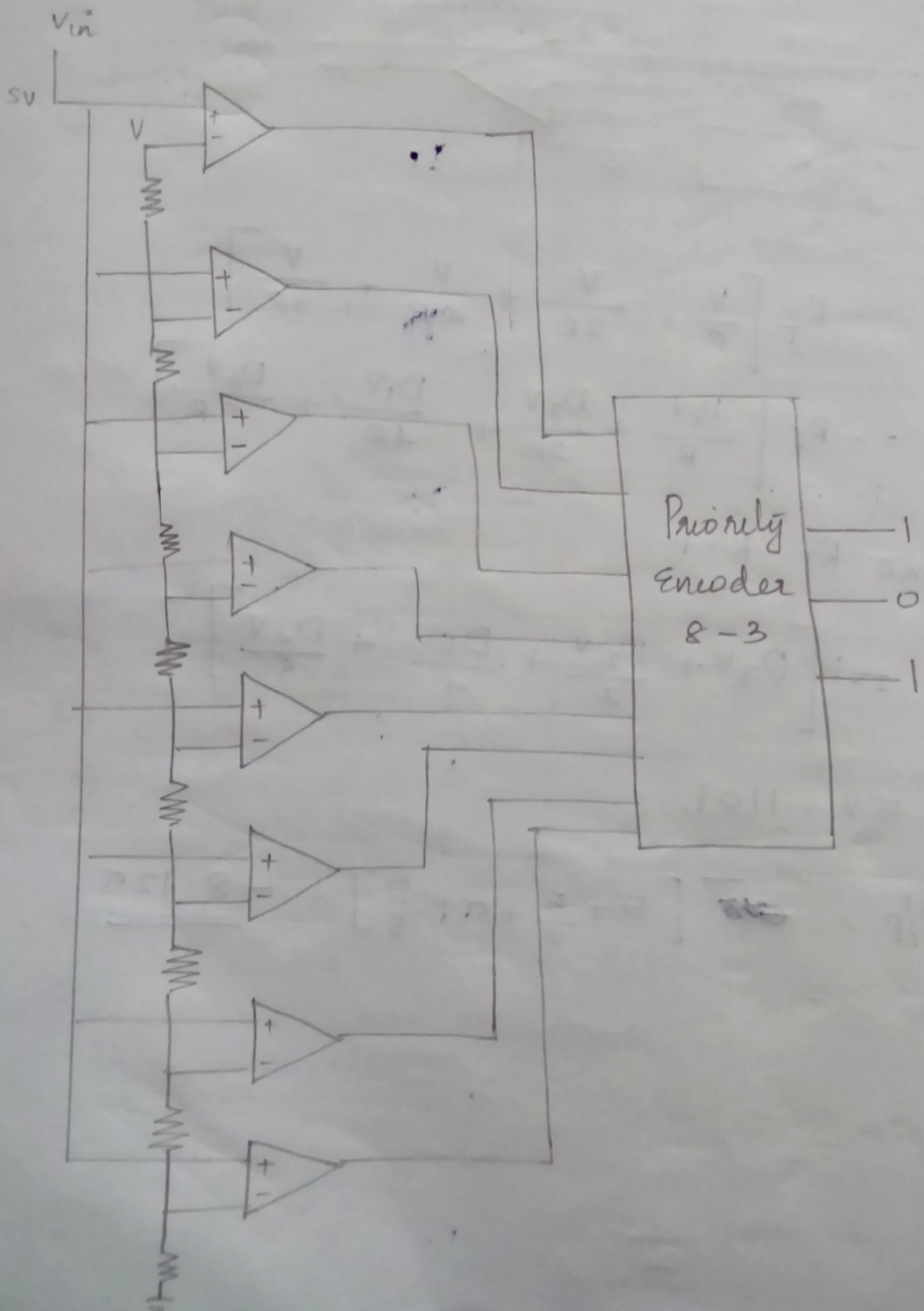
$$V_{0/p} = -1 \times \left[D_3 V + \frac{D_2 V}{2} + \frac{D_1 V}{4} + \frac{D_0 V}{8} \right]$$

$$V = 5V = 1101$$

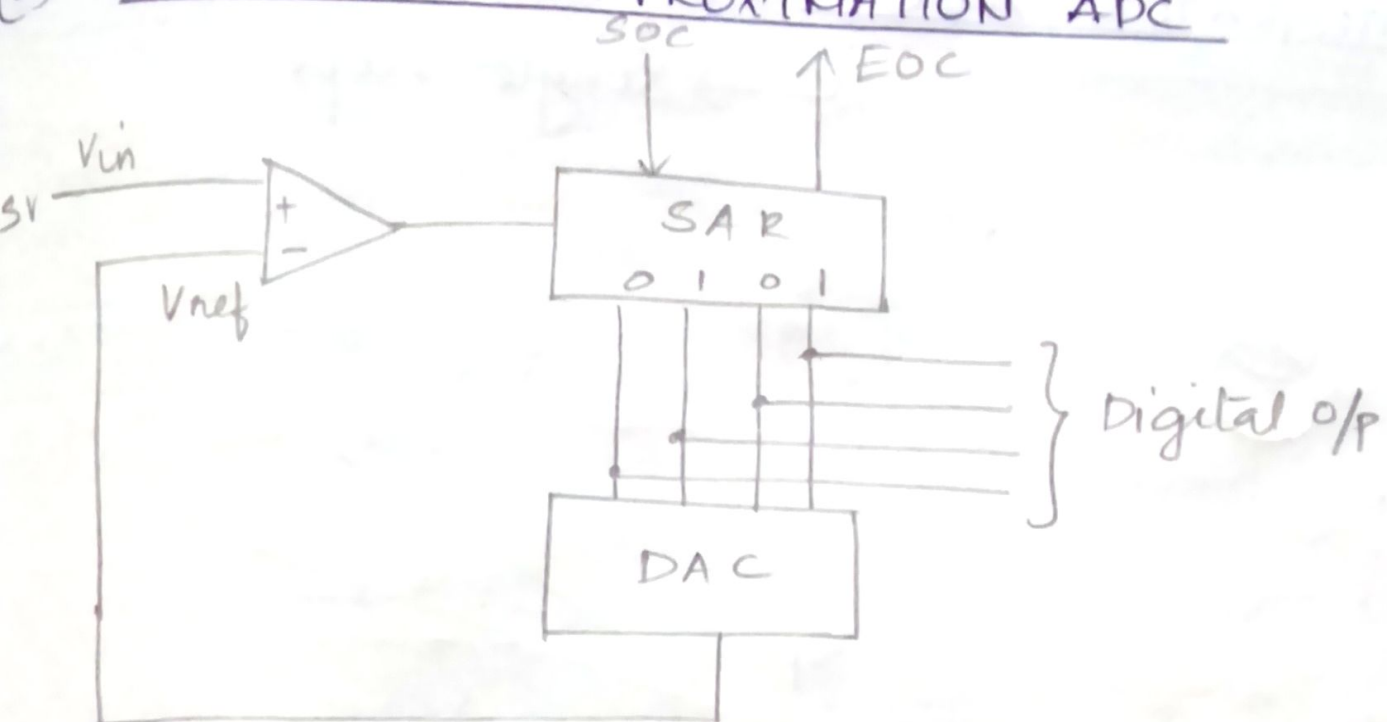
$$V_{0/p} = - \left[5 + \frac{5}{2} + 0 + \frac{5}{8} \right] = \underline{\underline{-8.125}}$$

ANALOG TO DIGITAL CONVERTER (ADC)

D. FLASH TYPE ADC



2) SUCCESSIVE APPROXIMATION ADC



$V_{in} = 5V$

V_{in}	V_{ref}	
5	0	0000
5V	8V	1000
5V	4V	0100
5V	6V	0110
5V	5V	0101

$V_{in} = 10V$

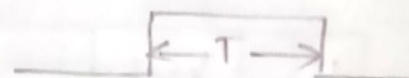
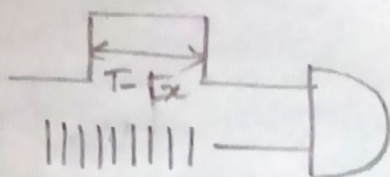
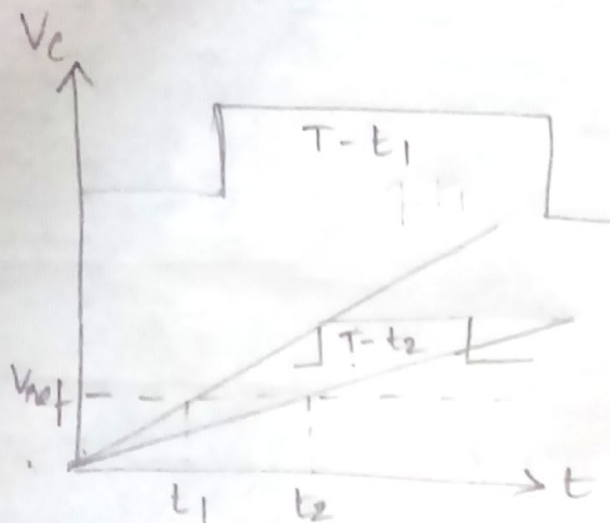
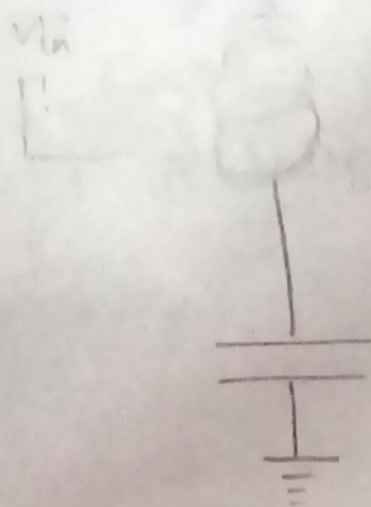
V_{in}	V_{ref}	
10V	0	0000
10V	8	1000
10V	12	1100
10V	10	1010

ANALOG TO DIGITAL

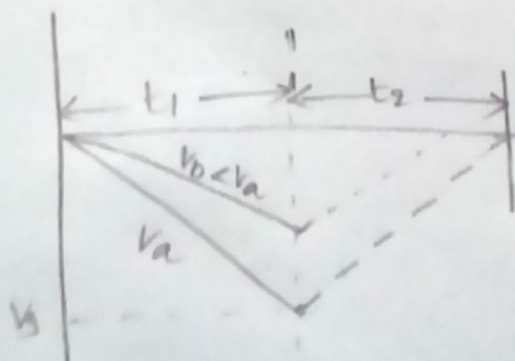
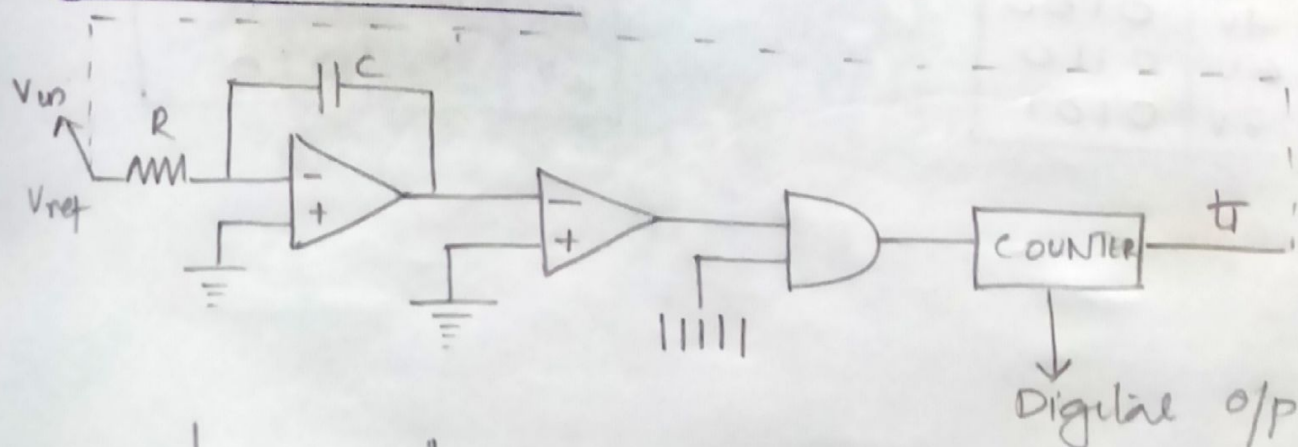
→ Dual Slope

→ Single Slope

D. FLASH TYPE A



DUAL SLOPE



$$V_0 = \frac{-V_{in}}{RC} \times t_1 = \frac{V_{ref}}{RC} \times t_2$$

constants

$$V_{in} \times t_1 = V_{ref} \times t_2$$

variable

$$V_{in} \propto t_2$$

VHDL

VHSIC

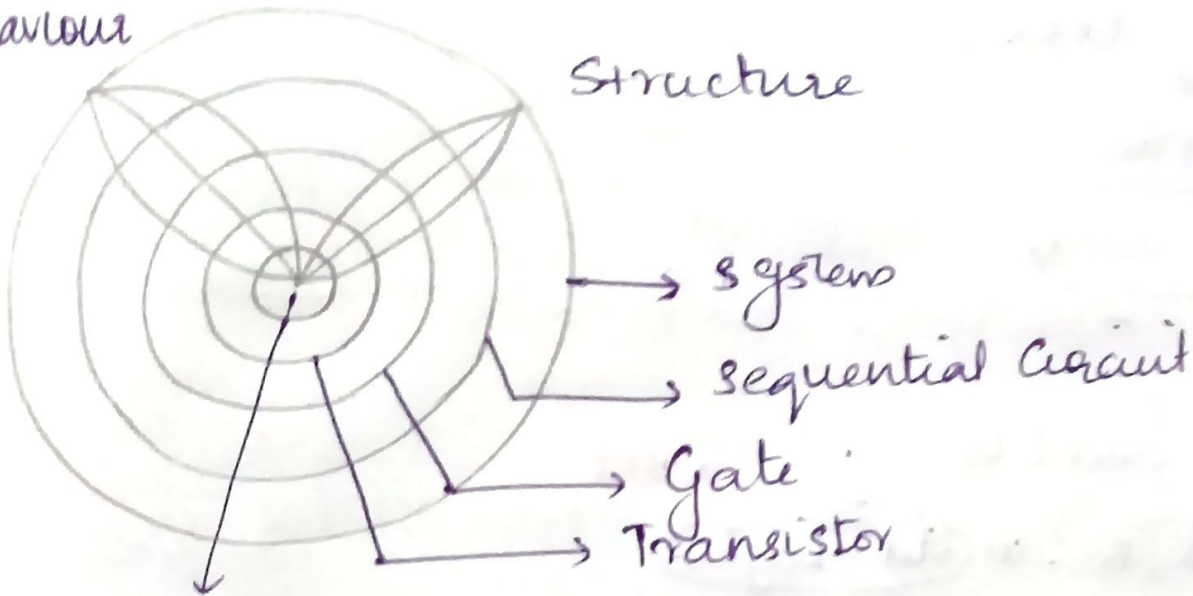
Hardware Design Language

Very High Scale Integrated Circuit

GAJSKI KUHN Y-CHART

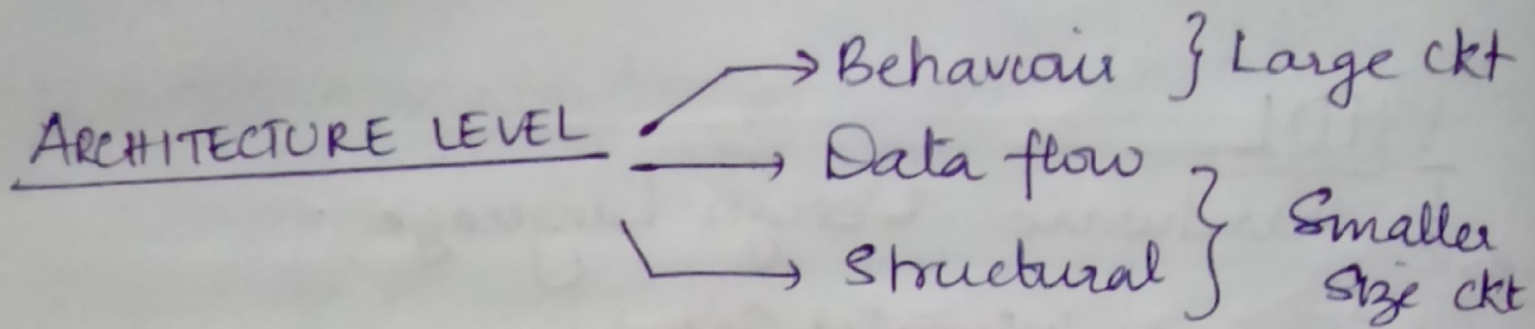
Behaviour

Structure



Physical

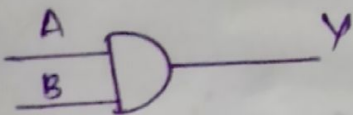
- 1). Entity level - Name plate i/p
o/p
- 2). Architecture level
- 3). Configuration
- 4). Package declaration
- 5). Package body.



CONFIGURATION

library ieee;

AND GATE



entity and1 is

Port (A, B : in std-Logic; ^{BIT}
Y : out std-Logic);

end and1;

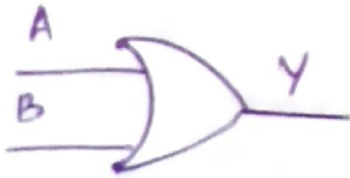
architecture behaviour1 is

begin

Y \leftarrow A and B

end behaviour1;

OR GATE



entity or1 is

Port (A, B : in Std-Logic;

Y : out Std-Logic;

end or1;

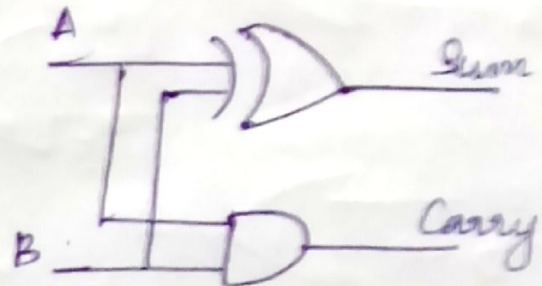
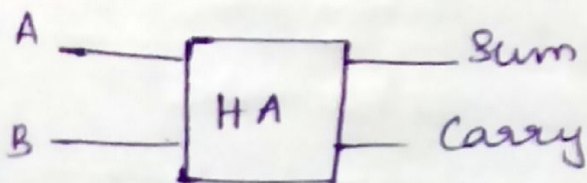
architecture behaviour1 is

begin

$Y \leftarrow A \text{ or } B$

end behaviour1;

Half adder



entity half adder1 is

Port (A, B : in BIT;

SUM, CARRY : out BIT);

end half-adder1;

architecture structural-HA of half adder1 is

Component XOR1 is

Port (A, B : in BIT;

Y : out BIT);

architecture structural -HA of halfadder is

Component and 1 is

port (A, B : in BIT;

Carry : out BIT);

end component;

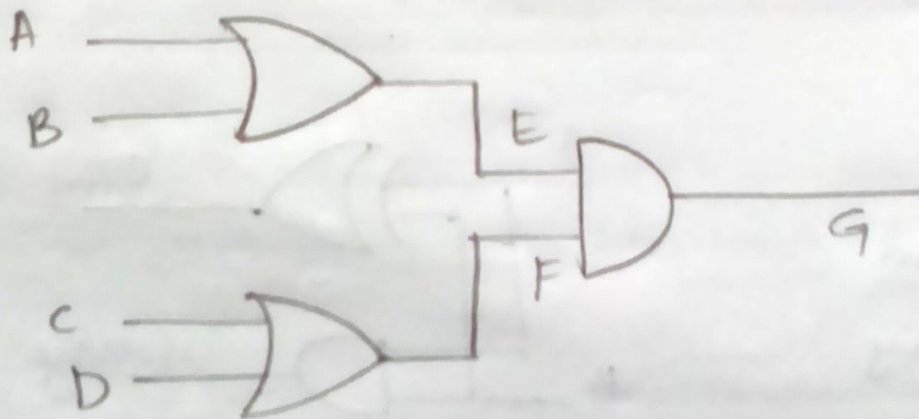
begin

x1 : XOR1 (A, B, Sum);

x2 : AND1 (A, B, Carry);

end structural HA.

Q - Write the VHDL code for :



Ans:

entity ckt1 is

Port (A, B, C, D: In BIT;
G: Out BIT);

end ckt1;

architecture structural1 of ckt1 is
component OR1 is

Port (X, Y: In BIT;
Z: Out BIT);

end component;

Component NAD1

Signal E, F: BIT;

begin (Instances).

X1: OR1 port map (A, B, E)

X2: OR1 port map (C, D, F)

X3: AND1 port map (E, F, G)

end structural1

PLD (PROGRAMMABLE LOGIC DEVICE)

↳ PAL (Programmable array Logic)
Program AND gate program OR gate

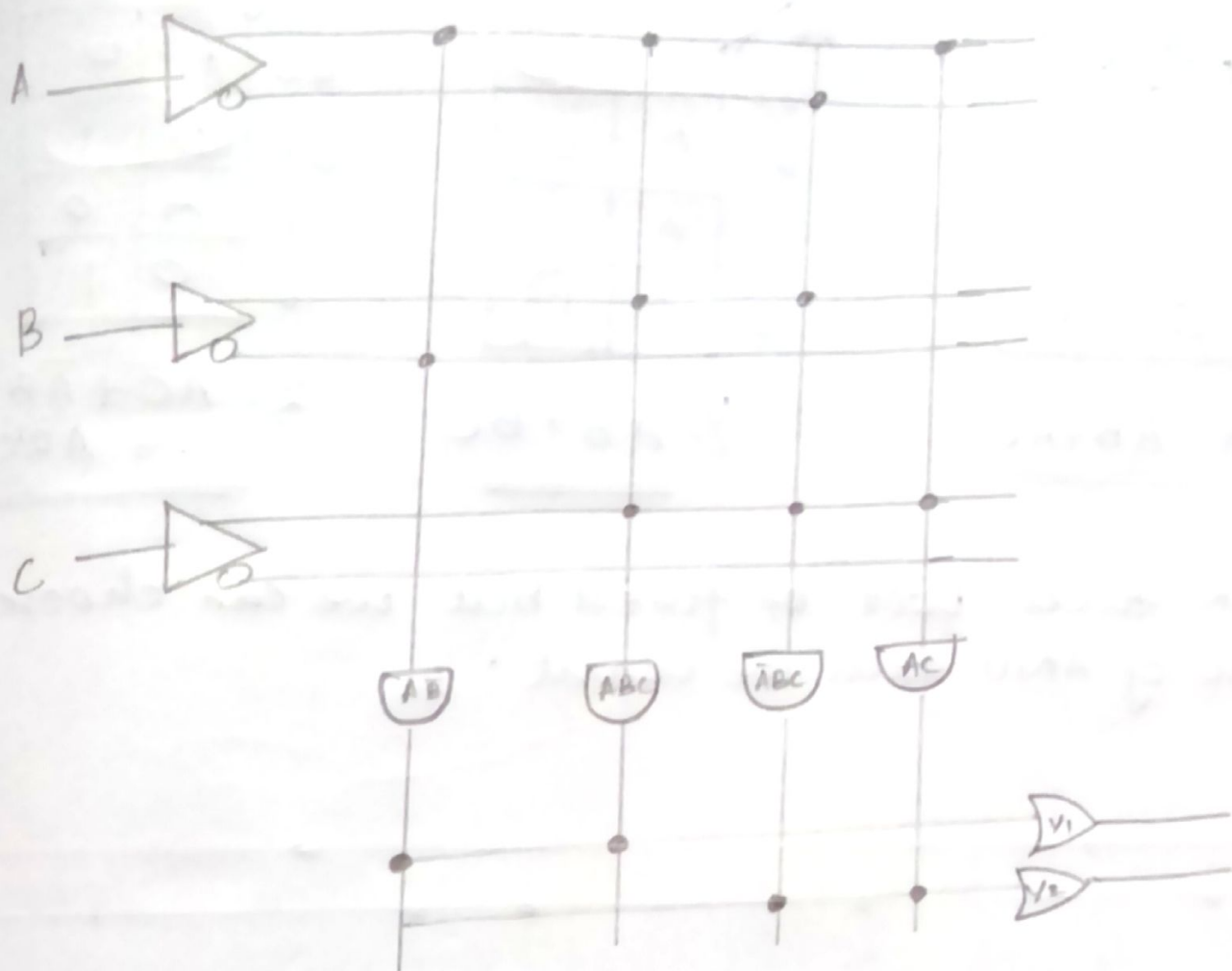
↳ PLA (Programmable Logic Array)
Program AND gate Fixed OR gate

PAL

A	B	C	Y ₁	Y ₂
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	0
1	0	1	1	1
1	1	0	0	0
1	1	1	1	1

$$Y_1 = AB + \bar{A}BC$$

$$Y_2 = \bar{A}BC + AC$$



PLA

$$X = \sum m(2, 3, 5, 7)$$

$$Y = \sum m(0, 1, 5)$$

$$Z = \sum m(0, 2, 3, 5)$$

		C	
		0	1
AB	00	0 ₀	0 ₁
	01	1 ₂	1 ₃
	11	0 ₆	1 ₇
	10	0 ₄	1 ₅

$$X = \bar{A}B + AC$$

		C	
		0	1
AB	00	1	1
	01	0	0
	11	0	0
	10	0	1

$$Y = \bar{A}\bar{B} + \bar{B}C$$

		C	
		0	1
AB	00	1	0
	01	1	1
	11	0	0
	10	0	1

$$Z = \bar{A}\bar{C} + \bar{A}B + A\bar{B}C$$

→ The OR gates will be fixed but we can choose the no. of AND gates we want.

