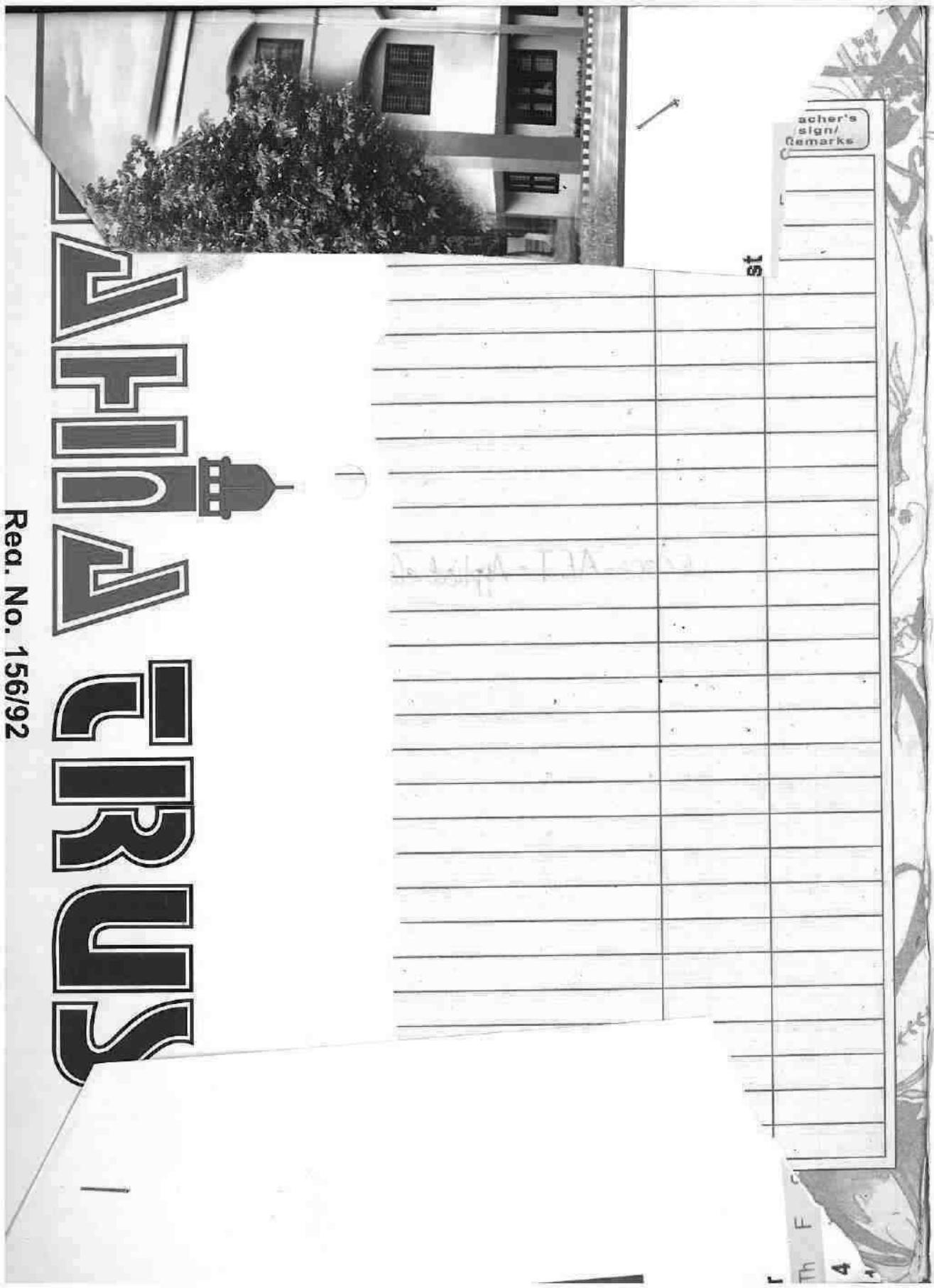


# APPLIED PHYSICS



01/08/2018

## Module - I

ElectroMagnetic - is the branch of physics or Electrical engineering in which electric and magnetic phenomena are studied. It has many applications such as .., mobile communication, satellite communication, wireless communication, induction heaters for melting ramming, etc.

### Review of Vectors

Scalar - quantity with only magnitude, A, B.

Vector - measures both magnitude and direction  $\vec{A}, \vec{B}$ .

Field - is a region of space in which every point in the region associated with both scalars or vector. Two types of field; scalar field & vector field.

Scalar field: If the influence of physical function is felt at each point of a region is a scalar field at each point of a region is a scalar field.

eg: temperature distribution in a building

sound intensity inside a theatre.

### vector field

eg: velocity of raindrop .., gravitational force in earth.

### Unit vector

Represented as  $a_A$ . A unit vector along a vector  $A$  ( $\vec{A}$ ) is defined as  $\frac{\vec{A}}{|\vec{A}|}$ . Magnitude is one.  $|\vec{A}|/|\vec{a}_A|$  is a scalar quantity, represented by  $A$ .  $|\vec{A}| = A a_A$ .

### Vector algebra

Mathematical operation done on vectors. Addition, subtraction, Multiplication.

$$\vec{A} + \vec{B} = \vec{C}$$

$$\vec{B} + \vec{A} = \vec{C} \rightarrow \text{Commutative}$$

$$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C} \rightarrow \text{Associative}$$

Multiplication of vector by a scalar :  $k \cdot \vec{A} = k\vec{A}$ .

Then Magnitude of the vector changes. The direction depend on the value of the scalar multiplied.

If scalar is +ve then direction remain the same

If scalar is -ve then direction is reversed.

Two vectors are said to be equal. ( $\vec{A} = \vec{B}$ )

$$\text{if } \vec{A} - \vec{B} = 0$$

### vector components and unit vector

1. Rectangular coordinate system / cartesian ( $x, y, z$ )

$$\vec{A} = A_x \vec{a}_x + A_y \vec{a}_y + A_z \vec{a}_z$$

$A_x, A_y, A_z \rightarrow$  Vector components in  $x, y, z$  directions respectively.  
 $\vec{a}_x, \vec{a}_y, \vec{a}_z \rightarrow$  Unit vector along  $x, y, z$  axis respectively.

$$\text{Magnitude } |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$a_A = \frac{\vec{A}}{|\vec{A}|}$$

$\vec{A}$  in a rectangular coordinate sys can be represented by  $A_x, A_y, A_z$ . Can be represented along with unit vector. And also vectors being sum of these components gives the vector.  $\vec{a}_x, \vec{a}_y, \vec{a}_z$  are unit vectors.

Q. A given vector  $\vec{A} = a_x + 3a_z$

$$\vec{B} = 5a_x + 2a_y - 6a_z$$

determine (i)  $|\vec{A} + \vec{B}|$

(ii)  $5\vec{A} - \vec{B}$

(iii) Component of  $\vec{A}$  along  $a_y$

(iv) Unit vector ||el to  $3\vec{A} + \vec{B}$

(i)  $\vec{A} + \vec{B}$ ,  $\vec{A} = a_x + 3a_z$   $\vec{B} = 5a_x + 2a_y - 6a_z$

$$\vec{A} + \vec{B} = 6a_x + 2a_y - 3a_z$$

$$|\vec{A} + \vec{B}| = \sqrt{6^2 + 2^2 + (-3)^2} = \underline{\underline{7}}$$

H.W Q Find the Unit vector of  $\vec{G}$ .

$$a_3 = \frac{\vec{G}}{|\vec{G}|} \Rightarrow 0.66\vec{a}_x - 0.66\vec{a}_y \\ - 0.33\vec{a}_z$$

(ii)  $5\vec{A} - \vec{B}$

$$5\vec{A} = 5\vec{a}_x + 15\vec{a}_z$$

$$5\vec{A} - \vec{B} = 5\vec{a}_x + 15\vec{a}_z - (5\vec{a}_x + 2\vec{a}_y - 6\vec{a}_z)$$

$$= 5\vec{a}_x + 15\vec{a}_z - 5\vec{a}_x - 2\vec{a}_y + 6\vec{a}_z$$

$$= -2\vec{a}_y + 21\vec{a}_z$$

(iii) Component of  $\vec{A}$  along  $\vec{a}_y$   
= 0

(iv) Unit vector  $\parallel$  el to  $3\vec{A} + \vec{B}$

$$\vec{C} = 3\vec{A} + \vec{B} = 3\vec{a}_x + 9\vec{a}_z + 5\vec{a}_x + 2\vec{a}_y - 6\vec{a}_z$$

$$\vec{C} = 8\vec{a}_x + 2\vec{a}_y + 3\vec{a}_z$$

Unit vector  $a_C = \frac{8\vec{a}_x + 2\vec{a}_y + 3\vec{a}_z}{|8\vec{a}_x + 2\vec{a}_y + 3\vec{a}_z|}$

$$a_C = \frac{8\vec{a}_x + 2\vec{a}_y + 3\vec{a}_z}{\sqrt{8^2 + 2^2 + 3^2}} = \frac{8\vec{a}_x + 2\vec{a}_y + 3\vec{a}_z}{\sqrt{77}}$$

$$a_C = \frac{8\vec{a}_x + 2\vec{a}_y + 3\vec{a}_z}{8.77} = \underline{0.912\vec{a}_x + 0.22\vec{a}_y + 0.33\vec{a}_z}$$

Q.  $\vec{A} = 10\vec{a}_x - 4\vec{a}_y + 6\vec{a}_z$

$$\vec{B} = 2\vec{a}_x + \vec{a}_y$$

a) Find component of  $\vec{A}$  along  $\vec{a}_y$ .

b) Magnitude of  $3\vec{A} - \vec{B}$

c) Unit vector along  $\vec{A} + 2\vec{B}$

### POSITION AND DISTANCE VECTOR

\* Position vector  $\vec{r}_p = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z$

Point p in a cartesian coordinate system may be represented by  $(x, y, z)$ . The position vector  $\vec{r}_p$  or radius vector of point p as the direct distance from the origin O to p.

e.g.: if  $P(3, 4, 5)$  then position vector  $\vec{r}_p = 3\vec{a}_x + 4\vec{a}_y + 5\vec{a}_z$

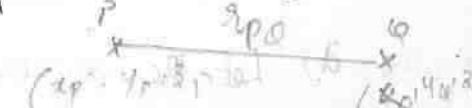
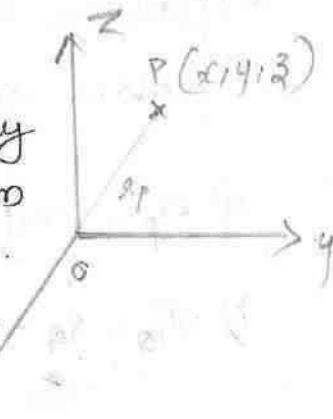
\* Distance vector  $\vec{r}_{pq}$

If two points  $P(x_p, y_p, z_p)$  &  $Q(x_q, y_q, z_q)$  then the distance vector  $\vec{r}_{pq}$  is position vector  $\vec{r}_q - \vec{r}_p$ .

$$\vec{r}_{pq} = \vec{r}_q - \vec{r}_p$$

$$\vec{r}_{pq} = (x_q - x_p)\vec{a}_x + (y_q - y_p)\vec{a}_y + (z_q - z_p)\vec{a}_z$$

$$\text{distance} = |\vec{r}_{pq}|$$



$$\text{eg if } p(2, -1, 4), \vec{A} = 2xy\mathbf{a}_x + y^2\mathbf{a}_y - x^2\mathbf{a}_z$$

$\vec{A}$  at point p.

$$\begin{aligned}\vec{A} &= 2 \times 2x - 1 \mathbf{a}_x + (-1)^2 \mathbf{a}_y - 2x^2 \mathbf{a}_z \\ &= -4\mathbf{a}_x + \mathbf{a}_y - 2x^2 \mathbf{a}_z\end{aligned}$$

Q. Given point  $p(0, 2, 4)$ ,  $q(-3, 1, 5)$ . Calculate

(i) position vector  $\vec{r}_p$

(ii) Distance vector from p to q ie  $\vec{r}_{pq}$ .

(iii) Distance b/w p and q.

(iv) draw a vector  $\parallel$  to  $pq$  with magnitude of 10.

$$a) \vec{r}_p = 2\mathbf{a}_y + 4\mathbf{a}_z$$

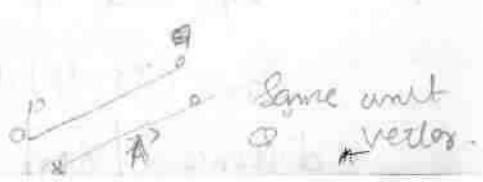
$$\begin{aligned}b) \vec{r}_{pq} &= \vec{r}_q - \vec{r}_p = -3\mathbf{a}_x + 1\mathbf{a}_y + 5\mathbf{a}_z - 2\mathbf{a}_y - 4\mathbf{a}_z \\ &= -3\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z\end{aligned}$$

$$c) \text{Distance} = |\vec{r}_{pq}| = \sqrt{(-3)^2 + (-1)^2 + (1)^2} = \sqrt{11} = 3.31\text{ft}$$

d) Let the required vector  $\vec{A} = A\vec{r}_{pq}$

$$|\vec{A}| = 10 = A$$

$$A = \vec{r}_{pq} = \frac{\vec{r}_{pq}}{|\vec{r}_{pq}|}$$



$$= -3\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z$$

$$\frac{1}{3.31\text{ft}}$$

$$\therefore \vec{A} = 10 \times \frac{-3\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z}{3.31\text{ft}}$$

Q. Given points  $p(1, -3, 5)$ ,  $q(2, 4, 6)$ ,  $r(0, 3, 8)$

(i) find the position vector of p & r

(ii) distance  $\vec{r}_{pr}$ .

$$a) \vec{r}_p = \left\{ \left[ 125 / (1-2)^2 + (4-3)^2 + (6-5)^2 \right] \right\} \{ (x-1)\mathbf{a}_x + (y-2)\mathbf{a}_y + (z+1)\mathbf{a}_z \}$$

(i) Evaluate  $\vec{r}_p$  at  $p(2, 4, 3)$

(ii) Unit vector that gives the direction of  $\vec{r}_p$  at p

Ans:

$$(i) \vec{r}_p = \left\{ \left[ 125 / (2-1)^2 + (4-3)^2 + (3-5)^2 \right] \right\} \{ (x-1)\mathbf{a}_x + (y-2)\mathbf{a}_y + (z+1)\mathbf{a}_z \}$$

$$= \left\{ \left[ 125 / 1 + 2^2 + 4^2 \right] \right\} \{ x\mathbf{a}_x + 2\mathbf{a}_y + 4\mathbf{a}_z \}$$

$$= \frac{125}{21} \mathbf{a}_x [x\mathbf{a}_x + 2\mathbf{a}_y + 4\mathbf{a}_z] = \frac{125}{21} (x\mathbf{a}_x + 2\mathbf{a}_y + 4\mathbf{a}_z)$$

$$= \frac{125}{21} \mathbf{a}_x [145\mathbf{a}_x + 290\mathbf{a}_y + 580\mathbf{a}_z] = 5.95\mathbf{a}_x + 11.9\mathbf{a}_y + 23.8\mathbf{a}_z$$

$$= 1450x + 290ay + 580az$$

(ii)  $a_s = \frac{\vec{s}}{|\vec{s}|} = \frac{1450x + 290ay + 580az}{\sqrt{(145)^2 + (290)^2 + (580)^2}}$

$$= \underline{5.95ax + 11.9ay + 23.8az}$$

$$\underline{\sqrt{(5.95)^2 + (11.9)^2 + (23.8)^2}}$$

$$= \underline{5.95ax + 11.9ay + 23.8az}$$

$\underline{27.26}$

$$= \underline{0.218ax + 0.436ay + 0.873az}$$

### VECTOR MULTIPLICATION

\* Dot product (or) scalar product  $= \vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta_{AB}$ .

$\theta_{AB}$   $\rightarrow$  smallest angle b/w A and B.

When two vectors are multiplied the result is either a scalar or vector depending on how they are multiplied.

1. Dot product (or) scalar product
2. Cross Product (or) Vector product
3. Scalar Triple product
4. Vector Triple product.

1. Dot product  $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta_{AB}$

Product of Magnitude of A, B and cosine of smaller angle b/w them.

$$\vec{A} = A_x a_x + A_y a_y + A_z a_z$$

$$\vec{B} = B_x a_x + B_y a_y + B_z a_z$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$a_x \cdot a_y = a_y \cdot a_z = a_z \cdot a_x = 0$$

$$a_x \cdot a_x = a_y \cdot a_y = a_z \cdot a_z = 1 \quad [\text{eg: } a_x \cdot a_x = |a_x| |a_x| \cos 0^\circ]$$

$$* \underline{a_A \cdot a_A = 1}$$

$$* \underline{\vec{A} \cdot \vec{A} = A^2}$$

Q. Consider a vector field  $\vec{G} = 2ax - 2.5ay + 3az$ .

D)  $G_A @ Q(4, 5, 2)$ .

1)  $G_A @ Q$  (dot prod)  $\rightarrow$  dot prod

2)  $a_N = 1/3(2ax + ay - 2az)$  scalar component?

3) vector component of  $G_A @ Q$  is the direction of  $a_N$ .

4) Angle b/w  $G_A @ Q$  and  $a_N$

Mis  $\vec{G} = 2ax - 2.5ay + 3az$

$$1) \vec{G}_A @ Q = 5ax - 2.5 \times 4ay + 3 \times 2az$$

$$= \underline{5ax - 10ay + 6az}$$

$$2) a_N \cdot \vec{G}_A = |\vec{G}_A| |a_N| \cos \frac{10 \times 4}{3} + \frac{10 \times 2}{3} + \frac{6}{3}$$

$$= 3.3 \times 3.3 - 3.3 - 12/3 = 3.3 - 3.3 - 4 = -4 //$$

3)

$\rightarrow$  scalar comp  $\Rightarrow$  dot prod

Vector comp  $\Rightarrow$  scalar comp  $\times$  unit vector.

H.W

9

Ans

$$\vec{G} = 2ax - 2ay - a_3$$

$$a_G = \frac{\vec{G}}{|\vec{G}|}$$

$$= \frac{2ax - 2ay - a_3}{|2ax - 2ay - a_3|} = \frac{2ax - 2ay - a_3}{\sqrt{a^2 + (-2)^2 + (-1)^2}}$$

$$= \frac{2ax - 2ay - a_3}{\sqrt{9}} = \frac{2ax - 2ay - a_3}{3} \\ = \underline{-0.66ax - 0.66ay - 0.33a_3}$$

$$9) \vec{A} = 10ax - 4ay + 6a_3 \quad \vec{B} = 2ax + ay$$

$$a) -4$$

$$b) 3\vec{A} - \vec{B} = 30ax - 12ay + 18a_3 - 2ax + ay \\ = \underline{28ax - 13ay + 18a_3} = \sqrt{28^2 + (-13)^2 + 18^2} = \underline{85.44}$$

$$c) \vec{A} + 2\vec{B} = 10ax - 4ay + 6a_3 + 4ax + 2ay \\ = \underline{14ax - 2ay + 6a_3} \quad a_E = \frac{14ax - 2ay + 6a_3}{\sqrt{14^2 + 2^2 + 6^2}} \\ = \underline{-0.911a_x - 0.130a_y + 0.390a_3}$$

3) Vector component = scalar component  $\times$  unit vector.

$$\text{Scalar component} = -4 \quad \text{unit vector } \vec{a}_N = \frac{1}{\sqrt{3}}(2ax + ay - a_3)$$

$$\text{vector component} = \frac{-8ax + 4ay + 8a_3}{3}$$

$$= \underline{-2.66ax - 1.33ay + 2.66a_3}$$

$$1) \vec{G}\vec{a}_Q \cdot \vec{a}_N = |\vec{G}\vec{a}_Q| |\vec{a}_N| \cos \theta$$

$$-4 = \sqrt{5^2 + 10^2 + 6^2} \cdot \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2} \cos \theta$$

$$-4 = \sqrt{161} \sqrt{1} \cos \theta$$

$$-4 = \sqrt{161} \cos \theta$$

$$\frac{-4}{\sqrt{161}} = \cos \theta, \cos \theta = -0.31524$$

$$\theta = \underline{108.3^\circ}$$

$$9) P(1, -3, 5) \quad Q(2, 4, 6) \quad R(0, 3, 8)$$

$$(i) \vec{r}_P = ax - 3ay + 5a_3 \quad \vec{r}_Q = 2ax + 4ay + 6a_3, \vec{r}_R = 3ay + 8a_3$$

$$(ii) \vec{r}_{PQ} = \vec{r}_Q - \vec{r}_P = 2ax + 4ay + 6a_3 - ax + 3ay - 5a_3 \\ = \underline{ax + 7ay + a_3}$$

$$\therefore |\vec{r}_{PQ}| = \sqrt{1^2 + 7^2 + 1^2} = \sqrt{51} = \underline{7.14}$$

$$\vec{r}_{RP} = ax - 3ay + 5a_3 - 3ay - 8a_3 = ax - 6ay - 3a_3, |\vec{r}_{RP}| = \sqrt{1^2 + 6^2 + 3^2} \\ = \sqrt{46}, \underline{6.782}$$

$$\vec{r}_{PQ} = \underline{13.92331}$$

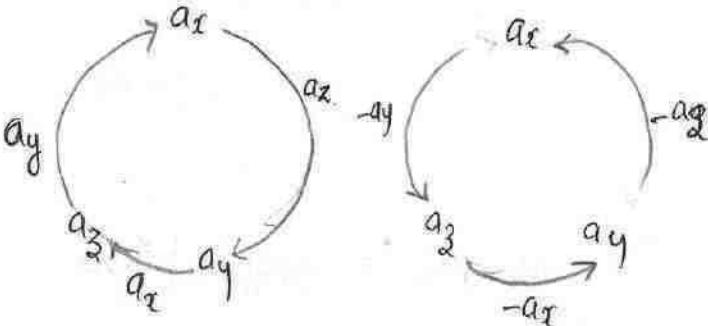
### Cross product

\*  $\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta_{AB} \cdot a_N$

$$a_x \times a_y = a_z$$

$$a_y \times a_z = a_x$$

$$a_z \times a_x = a_y$$



$$\vec{A} \times \vec{B} = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

\* Area of a triangle =  $\frac{1}{2} |\vec{A} \times \vec{B}|$

\* Unit vector  $a_N = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$

- Q. The three vertices of a triangle are located at  
 A(6, -1, 2), B(-2, 3, -4), C(-3, 1, 5).

a)  $s_{AB}, s_{AC}$

b) area of  $\triangle ABC$

c) Unit vector  $a_N$

anticlock -  
clock

Ans,

i)  $s_{AB} = s_B - s_A$

$$= -2a_x + 3a_y - 4a_z - 6a_x + a_y - 2a_z$$

$$= -8a_x + \underline{4a_y} - 6a_z$$

$s_{AC} = s_C - s_A$

$$= -3a_x + a_y + 5a_z - 6a_x + a_y - 2a_z$$

$$= -9a_x + 2a_y + 3a_z$$

b)

$$s_{AB} \times s_{AC} = |s_{AC}| |s_{AB}| \sin \theta_{ACB} a_N$$

$$= \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} a_x & a_y & a_z \\ -8 & 1 & -6 \\ -9 & 2 & 3 \end{vmatrix}$$

$$= a_x(4x3 - (-6x2)) - a_y(-8x3 - (-9x-6)) + a_z(-8x2 - (4x-9))$$

$$= 24a_x + 48a_y + 20a_z //$$

$$\text{Area of } \triangle = \frac{|\vec{A} \times \vec{B}|}{2} = \frac{\sqrt{24^2 + 48^2 + 20^2}}{2} = \frac{42.01}{2}$$

C) Unit vector  $a_N = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$

$$= \frac{24a_x + 18a_y + 20a_z}{84.02}$$

$$= 0.285a_x + 0.928a_y + 0.238a_z$$

<sup>base linear.</sup>  
General Orthogonal coordinate S/m

3 planes.

$u_1, u_2, u_3 \rightarrow$  in rectangular -  $x, y, z$ .

In order to represent a vector we need magnitude and direction. To represent the direction  $\Rightarrow$  co-ordinates needed.

Electromagnetics deals with 3 dimensional quantity so we use 3-co-ordinate S/m.

To describe co-ordinate S/m  $\rightarrow$  we need  
 (i) axis (rep. to latm)  
 (ii) unit vectors (directing)

Any co-ordinate S/m has 3 surfaces which are 1<sup>st</sup> to each other. that is why it is called orthogonal.

Generally 3 Surfaces -  $u_1, u_2, u_3$   
Unit vectors -  $a_{u_1}, a_{u_2}, a_{u_3}$

$$\text{Cross pdt} \Rightarrow a_{u_1} \times a_{u_2} = a_{u_3}$$

$$a_{u_2} \times a_{u_3} = a_{u_1}$$

$$a_{u_3} \times a_{u_1} = a_{u_2}$$

$$\text{dot pdt} \Rightarrow a_{u_1} \cdot a_{u_2} = a_{u_2} \cdot a_{u_3} = a_{u_3} \cdot a_{u_1} = 0$$

<sup>General rep of</sup>  
Any vector rep in orthogonal sym.

$$\vec{A} = A_{u_1} a_{u_1} + A_{u_2} a_{u_2} + A_{u_3} a_{u_3}$$

Differential length, Differential Area, Differential volume.

Differential length  $\rightarrow dl_i = h_i du_i$

$h_i \rightarrow$  Matrix coefficient

$$dl = h_1 a_{u_1} du_1 + h_2 a_{u_2} du_2 + h_3 a_{u_3} du_3$$

Differential area,  $ds$

$$ds_1 = dl_2 dl_3 h_2 h_3$$

$$ds_2 = dl_1 dl_3 h_1 h_3$$

$$ds_3 = dl_1 dl_2 h_1 h_2$$

Differential volume,  $dv$

$$dv = dl_1 dl_2 dl_3 h_1 h_2 h_3$$

→ Cartesian coordinate system or rectangular.

$$u_1, u_2, u_3 \Rightarrow x, y, z.$$

$$a_1, a_2, a_3 \Rightarrow a_x, a_y, a_z.$$

$$\vec{A} = A_x a_x + A_y a_y + A_z a_z$$

$$a_x \cdot a_y = a_y \cdot a_z = a_z \cdot a_x = 0$$

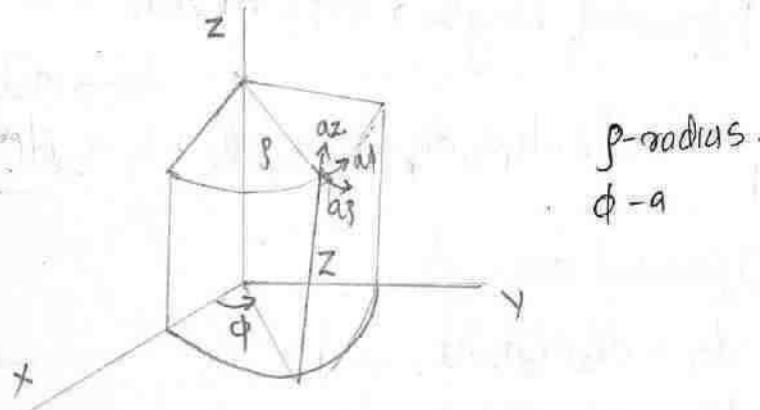
$$a_x \times a_y = a_z, a_y \times a_z = a_x, a_z \times a_x = a_y.$$

$$\text{Differential Area} = dx dy, dy dz, dz dx = ds.$$

$$\text{Volume} = dx dy dz = dv$$

$$\text{Length} = a_x dx + a_y dy + a_z dz.$$

### Circular cylindrical coordinate system.



$r$  - radius of the cylinder

$\phi$  - azimuthal angle

$$u_1, u_2, u_3 \Rightarrow r, \phi, z$$

$$\vec{A} = A_r a_r + A_\phi a_\phi + A_z a_z$$

$$a_r \times a_\phi = a_z$$

$$a_\phi \times a_z = a_r$$

$$a_z \times a_r = a_\phi$$

$$a_r \cdot a_\phi = a_\phi \cdot a_z = a_z \cdot a_r = 0$$

$$a_r \cdot a_r = a_\phi \cdot a_\phi = a_z \cdot a_z = 1.$$

$$\text{Differential length } dl = d_r a_r + d_\phi a_\phi$$

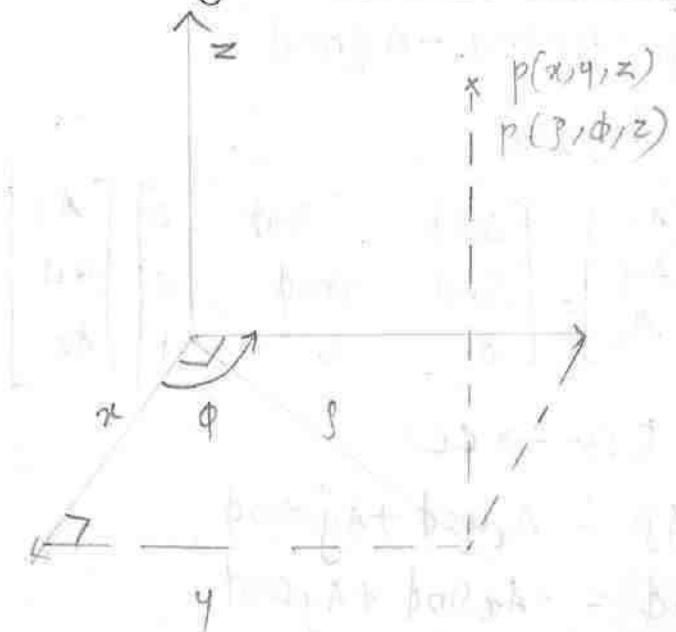
$$\text{Differential volume } dv = d_r d_\phi dz.$$

$$\text{Differential area. } ds_r = r d\phi dz \quad r = p$$

$$ds_\phi = dr dz.$$

$$ds_z = pd\phi dz$$

### Relation b/w rectangular and circular c.s.



Relation b/w rectangular coordinate S/m and circular coordinate systems

$$x = p \cos\phi$$

$$y = p \sin\phi$$

$$z = z$$

$$p = \sqrt{x^2 + y^2}$$

$$\phi = \tan^{-1} y/x$$

$$z = z$$

CSC  $\rightarrow$  RCS

$$A_x = A_p \cos\phi - A_\phi \sin\phi$$

$$A_y = A_p \sin\phi - A_\phi \cos\phi$$

$$A_z = A_z$$

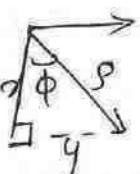
$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_p \\ A_\phi \\ A_z \end{bmatrix}$$

RCS  $\rightarrow$  CCS

$$A_p = A_x \cos\phi + A_y \sin\phi$$

$$A_\phi = -A_x \sin\phi + A_y \cos\phi$$

$$A_z = A_z$$



$$\cos\phi = x/p$$

$$x = p \cos\phi$$

$$\sin\phi = y/p$$

$$y = p \sin\phi$$

$$\begin{bmatrix} A_p \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Dot prod of unit vector in cylindrical and cartesian coordinates.

$$\begin{array}{c|ccc} & a_p & a_\phi & a_z \\ \hline a_x & \cos\phi & -\sin\phi & 0 \\ a_y & \sin\phi & \cos\phi & 0 \\ a_z & 0 & 0 & 1 \end{array}$$

(i) Transform the vector  $\vec{B} = yax - xay - za z$  into cylindrical coordinate S/m.

$$\vec{B} = yax - xay - za z$$

$$B_p$$

$$B_\phi$$

$$B_z$$

vector component  
= scalar compnt  
unit vector

$$B_p = B \cdot a_p$$

$$\begin{aligned} B_p &= B \cdot a_p \\ &= (yax - xay - za z) \cdot a_p \\ &= yax \cdot a_p - xay \cdot a_p - za z \cdot a_p \\ &= y \cos\phi - x \sin\phi + 0 \\ &= p \sin\phi \cos\phi - p \cos\phi \sin\phi \\ &= \underline{\underline{0}} \end{aligned}$$

$$\begin{aligned} x &= p \cos\phi \\ y &= p \sin\phi \end{aligned}$$

$$B\phi = B \cdot a\phi$$

$$= (yax - xay - zaz) a\phi$$

$$= yax a\phi - xay a\phi - zaz a\phi$$

$$= yx \sin\phi - x \cos\phi - zx 0$$

$$= -yx \sin\phi - x \cos\phi$$

$$= -p \sin\phi \sin\phi - p \cos\phi \cos\phi$$

$$= -p \sin^2\phi - p \cos^2\phi$$

$$= -p(\sin^2\phi + \cos^2\phi)$$

$$= -p //$$

$$B_x = B_z$$

$$\text{Vector in CS} = \vec{B} = -p a\phi + B_z a_z$$

Q) Give the cartesian coordinate of vector.

$$\vec{H} = 20ap - 10a\phi + 3az \text{ at a point } P(5, 2, 1)$$

$$H_x = H \cdot a_x$$

$$= (20ap - 10a\phi + 3az) a_x$$

$$= 20apax - 10a\phi ax + 3az \cdot ax$$

$$= 20x \cos\phi - 10x - \sin\phi + 0$$

$$= 20 \cos\phi + 10 \sin\phi$$

$$H_y = H \cdot a_y$$

$$= 20apay - 10a\phi ay + 3azay$$

$$= 20 \sin\phi - 10 \cos\phi + 3x0$$

$$= 20 \sin\phi - 10 \cos\phi$$

$$H_z = H \cdot a_z$$

$$\phi = \tan^{-1} y/x$$

$$= \tan^{-1}(2/5) = 21.8^\circ$$

$$H_x = 20 \cos(21.8^\circ) + 10 \sin(21.8^\circ)$$

$$H_y = 20 \sin(21.8^\circ) - 10 \cos(21.8^\circ)$$

Spherical-coordinate system.

Sphere of radius  $r$ .

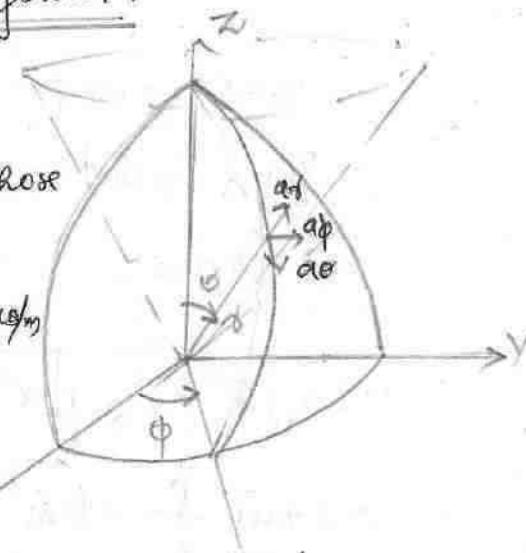
$\theta \rightarrow$  half angle of cone whose apex is at origin

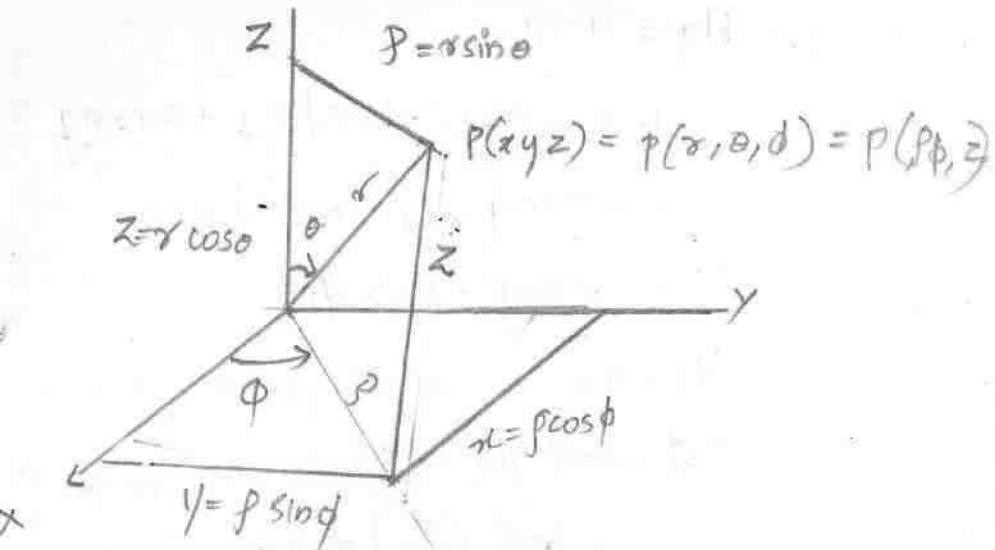
$\phi \rightarrow$  angle as in cylindrical  $(\rho, \phi)$

$$u_1, u_2, u_3 = r, \theta, \phi$$

$$\vec{A} = A_r a_r + A_\theta a_\theta + A_\phi a_\phi$$

Transformation from Cartesian - Spherical





cylindrical spherical.  
 $x = r \cos \theta = r \sin \phi \cos \phi$

$$y = r \sin \theta = r \sin \phi \sin \phi$$

$$z = r \cos \theta$$

$$\boxed{x = r \sin \theta \cos \phi}$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta.$$

$$r = \sqrt{\phi^2 + z^2} = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1} \frac{f}{z} = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z}$$

$$\phi = \tan^{-1} \frac{y}{x}.$$

$$\rho = r \sin \theta$$

consider  $r$  as hypotenuse  
 $\therefore$  applying  $b^2 + a^2 = h^2$ .

$$\boxed{f = x^2 + y^2}$$

Dot prod of unit vectors.

Spherical to Cartesian C.S.

$$\begin{matrix} a_x & a_\theta & a_\phi \\ \end{matrix}$$

$$a_x \quad \sin \theta \cos \phi \quad \cos \theta \cos \phi \quad -\sin \phi$$

$$a_y \quad \sin \theta \sin \phi \quad \cos \theta \sin \phi \quad \cos \phi$$

$$a_z \quad \cos \theta \quad -\sin \theta \quad 0$$

RCS  $\rightarrow$  SCS

$$(A_x, A_y, A_z) \rightarrow (A_\theta, A_\phi, A_\phi).$$

\* In matrix form.

$$\begin{bmatrix} A_\theta \\ A_\phi \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

SCS  $\rightarrow$  RCS.

$$(A_\theta, A_\phi, A_\phi) \rightarrow (A_x, A_y, A_z).$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \cos \theta \cos \phi & \sin \phi \\ \sin \theta \sin \phi & \cos \theta \sin \phi & \cos \phi \\ \cos \theta & -\sin \theta & 0 \end{bmatrix} \begin{bmatrix} A_\theta \\ A_\phi \\ A_\phi \end{bmatrix}$$

SCS  $\rightarrow$  CCS.

$$(A_\theta, A_\phi, A_\phi) \rightarrow (A_\rho, A_\phi, A_\phi)$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \\ \cos\phi & -\sin\phi & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

CCCS  $\rightarrow$  SCS.

$$(A_x, A_y, A_z) \rightarrow (A_r, A_\theta, A_\phi)$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\phi & 0 & \cos\phi \\ \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

RCS  $\rightarrow$  CCCS.

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

CCCS  $\rightarrow$  RCS.

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

Dot product of unit vectors in CCCS & RCS.

$$\begin{array}{ccc|c} & a_r & a_\theta & a_\phi \\ \hline a_r & \cos\phi & -\sin\phi & 0 \\ a_\theta & \sin\phi & \cos\phi & 0 \\ a_\phi & 0 & 0 & 1 \end{array}$$

Dot prod of unit vectors in SCS & RCS.

$$\begin{array}{ccc|c} & a_r & a_\theta & a_\phi \\ \hline a_r & \sin\phi & \cos\phi & -\sin\phi & \sin\phi \cos\phi & \cos\phi \cos\phi & -\sin\phi \\ a_\theta & \cos\phi & -\sin\phi & 0 & \sin\phi \sin\phi & \cos\phi \sin\phi & \cos\phi \\ a_\phi & 0 & 0 & 1 & \cos\phi & -\sin\phi & 0 \end{array}$$

Transformation from RCS to CCS / CCS to RCS.

$$\begin{aligned} x &= r\cos\phi & p &= \sqrt{x^2 + y^2} \\ y &= r\sin\phi & \phi &= \tan^{-1} y/x \\ z &= z \end{aligned}$$

Transformation from RCS to SCS & SCS to RCS.

$$\begin{aligned} x &= r\cos\phi \sin\theta & \rho &= \sqrt{x^2 + y^2 + z^2} & \theta &= \tan^{-1} \sqrt{x^2 + y^2} \\ y &= r\sin\phi \sin\theta & \phi &= \tan^{-1} y/x \\ z &= r\cos\theta \end{aligned}$$

## Vector representation in 3-coordinate sys.

$$RCS: A_x a_x + A_y a_y + A_z a_z$$

$$CCS: A_\rho a_\rho + A_\phi a_\phi + A_z a_z$$

$$SCS: A_\theta a_\theta + A_\phi a_\phi + A_\rho a_\rho$$

Q. Express the vector  $\vec{B} = \frac{10}{\rho} a_\rho + \rho \cos \theta a_\theta + \rho \phi a_\phi$  to RCS/CCS.

$$\text{Find } B(-3, 4, 0) \text{ & } B(5, \pi/2, -2)$$

$$\text{component of a vector } B_x = \vec{B} \cdot a_x$$

$$= \left( \frac{10}{\rho} a_\rho + \rho \cos \theta a_\theta + \rho \phi a_\phi \right) \cdot a_x$$

$$= \frac{10}{\rho} \sin \theta \cos \phi + \rho \cos \theta \cos \theta \cos \phi + \rho \sin \phi$$

$$B_x = \frac{10 \sin \theta \cos \phi}{\rho} + \rho \cos \theta \cos \phi - \rho \sin \phi$$

$$B_y = \vec{B} \cdot a_y = \left( \frac{10}{\rho} a_\rho + \rho \cos \theta a_\theta + \rho \phi a_\phi \right) \cdot a_y$$

$$= \frac{10}{\rho} \sin \theta \sin \phi + \rho \cos \theta \cos \theta \sin \phi + \rho \cos \phi$$

$$= \frac{10}{\rho} \sin \theta \sin \phi + \rho \cos \theta \sin \phi + \rho \cos \phi$$

$$B_z = \vec{B} \cdot a_z = \frac{10}{\rho} \cos \theta - \rho \cos \theta \sin \phi$$

$$\rho = \sqrt{x^2 + y^2 + z^2}$$

$$B(-3, 4, 0)$$

$$\rho = \sqrt{(-3)^2 + 4^2 + 0^2} = \sqrt{25} = 5$$

$$\theta = \tan^{-1} \frac{\sqrt{x^2 + y^2}}{z} = \tan^{-1}(0) = 90^\circ \quad (\text{to get the } +180^\circ \text{ case})$$

$$\phi = \tan^{-1} \frac{y}{x} = \tan^{-1} \left( -\frac{4}{3} \right) = -0.92 \quad -53.13^\circ \quad 126.87^\circ$$

$$B_x = \frac{10}{5} \sin 90 \cos(126.87^\circ) + 5 \cos^2(90) - \sin 90 \sin(126.87^\circ)$$

$$= -1.2000 + -0.9999 = -1.9999 \approx -2$$

$$= -2$$

$$B_y = \frac{10}{5} \sin 90 \sin(126.87^\circ) + 5 \cos^2(90) \sin(126.87^\circ) + \cos(126.87^\circ)$$

$$= -1.2000 + -0.5997$$

$$B_z = \frac{10}{5} \cos 90 - 5 \cos 90 \sin 90 = 0$$

SCS  $\rightarrow$  CCS,  $B_\rho, B_\phi, B_\theta$

$$B_\rho = \left( \frac{10}{\rho} a_\rho + \rho \cos \theta a_\theta + \rho \phi a_\phi \right) \cdot a_\rho = \frac{10}{\rho} \sin \theta \cos \phi \sin \theta$$

$$B_\phi = \left( \frac{10}{\rho} a_\rho + \rho \cos \theta a_\theta + \rho \phi a_\phi \right) \cdot a_\phi$$

$$B_\theta = 10/\rho \cos \theta + \rho \sin \theta \sin \theta$$

Q. Given point  $P(-2, 6, 3)$  and;

$\vec{A} = yax + (x+3)ay$ . Express  $\vec{P}$  and  $\vec{A}$  in cylindrical and spherical coordinate system.

Evaluate  $a@P$  in the cartesian, cylindrical & spherical cs.

?

$$P(-2, 6, 3), \quad \vec{A} = yax + (x+3)ay$$

$$\rho = \sqrt{x^2 + y^2} = \sqrt{4 + 36} = 6.32 //.$$

(so Ans.)

$$\phi = \tan^{-1} y/x = \tan^{-1}(6/-2) = 108.431$$

$$\underline{\underline{\lambda = z = 3}}$$

$$Ap = \vec{A} \cdot ap$$

$$= [yax + (x+3)ay] a_3 p$$

$$= y \cos\phi + (x+3) \sin\phi //$$

$$= y \cos\phi + (x+y) \sin\phi .$$

$$A\phi = \vec{A} \cdot a\phi$$

$$= [yax + (x+3)ay] \cdot a\phi$$

$$= y \sin\phi + (x+y) \cos\phi$$

$$A_3 = \vec{A} \cdot a_3$$

$$= [yax + (x+3)a_3] \cdot a_3$$

$$= 0 .$$

	Cartesian	Cylindrical	Spherical	
Diff length $dl$	$dl = dx \hat{i} + dy \hat{j} + dz \hat{k}$	$dl = r d\phi \hat{\theta} + rdz \hat{k}$	$dl = dr \hat{r} + r d\theta \hat{\theta} + r \sin\theta d\phi \hat{k}$	$d\phi \hat{\phi}$
Diff volume $dv$	$dv = dx dy dz$	$dv = r^2 \sin\theta dr d\theta d\phi$	$dv = r^2 \sin\theta dr d\theta d\phi$	
Diff. area $ds$ .	$ds_x = dy dz \hat{x}$	$ds = r d\phi d\theta \hat{r}$	$ds_r = r^2 \sin\theta d\theta d\phi \hat{r}$	
	$ds_y = dx dz \hat{y}$	$\hat{\theta} = r d\theta d\phi \hat{r}$	$ds_\theta = r \sin\theta dr d\phi \hat{\theta}$	
	$ds_z = dx dy \hat{z}$	$= r d\phi d\theta \hat{r}$	$ds_\phi = r dr d\theta \hat{\phi}$	
	$u_1 = x, u_2 = y, u_3 = z$	$u_1 = r, u_2 = \theta, u_3 = \phi$	$u_1 = r, u_2 = \theta, u_3 = \phi$	
	$b_1 = b_2 = b_3 = 1$	$b_1 = 1, b_2 = r, b_3 = 1$	$b_1 = 1, b_2 = r \sin\theta, b_3 = r$	

## Vector Integration

- Line Integral  $\int S$
- Surface Integral  $\iint S$
- Volume Integral  $\iiint S$

### 1. Line Integral

line integral of vector field  $\vec{A}$

$$\int \vec{A} \cdot d\vec{l} = \int \vec{A} \cdot d\vec{l} - \text{Curly bracket of } \vec{A}$$

If path is closed

$d\vec{l} \rightarrow$  small elemental length



A line may be a curved path / contour.

### 2. Surface Integral

$$\int_S \vec{A} \cdot d\vec{s} \Rightarrow \psi \text{ flux}$$

consider a surface  $S$  placed in vector  $\vec{A}$   
then surface  $\int_S \vec{A} \cdot d\vec{s}$  is, if closed surface.  $\vec{A} \cdot d\vec{s}$  = net outward flux of  $\vec{A}$

### 3. Volume Integral

$$\int_V \vec{A} \cdot d\vec{v}$$

flux - total no. of field lines passing through the surface in given time.

The volume integral of vector field  $A$  is represented as

$$\int_V \vec{A} \cdot d\vec{v}$$

### DEL operator $\nabla$ form

Also called as gradient operator.

$\nabla \Rightarrow$  differentiator operator.

$$\nabla = a_{u_1} \frac{1}{h_1} \frac{\partial}{\partial u_1} + a_{u_2} \frac{1}{h_2} \frac{\partial}{\partial u_2} + a_{u_3} \frac{1}{h_3} \frac{\partial}{\partial u_3}$$

### R.C.S.

$$\nabla = a_x \frac{\partial}{\partial x} + a_y \frac{\partial}{\partial y} + a_z \frac{\partial}{\partial z}$$

### cyl C.C.S.

$$\nabla = a_\rho \frac{\partial}{\partial \rho} + a_\theta \frac{1}{\rho} \frac{\partial}{\partial \theta} + a_z \frac{\partial}{\partial z}$$

### S.C.S.

$$\nabla = a_\phi \frac{\partial}{\partial \phi} + a_\theta \frac{1}{r} \frac{\partial}{\partial \theta} + a_r \frac{1}{r \sin\theta} \frac{\partial}{\partial \phi}$$

$x, \theta, \phi$

gradient of a scalar. (vector quantity)

Scalar  $V'$

$$\nabla V \Rightarrow \text{grad } V = a_n \frac{dV}{dn}$$

R.C.S.

$$\nabla V = \text{grad } V = a_x \frac{\partial V}{\partial x} + a_y \frac{\partial V}{\partial y} + a_z \frac{\partial V}{\partial z}$$

Matrix coefficient

C.C.S.

$$\nabla V = a_\rho \frac{dV}{d\rho} + a_\theta \frac{1}{\rho} \frac{\partial V}{\partial \theta} + a_\phi \frac{1}{\rho \sin \theta} \frac{\partial V}{\partial \phi}$$

S.C.S.

$$\nabla V = a_r \frac{\partial V}{\partial r} + a_\theta \frac{1}{r} \frac{\partial V}{\partial \theta} + a_\phi \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi}$$

Divergence of a vector.  $\text{div } \vec{A} = \nabla \cdot \vec{A}$

$$\text{div } \vec{A} = \nabla \cdot \vec{A} = \lim_{\Delta V \rightarrow 0} \frac{\oint_S \vec{A} \cdot d\vec{s}}{\Delta V} = \underset{\Delta V \rightarrow 0}{\text{lim}} \text{ Net outward flux}$$

In general;

$$\text{div } \vec{A} = \nabla \cdot \vec{A} = \frac{1}{b_1 b_2 b_3} \left[ \frac{\partial}{\partial x_1} b_2 b_3 A_{x_1} + \frac{\partial}{\partial x_2} b_1 b_3 A_{x_2} + \frac{\partial}{\partial x_3} b_1 b_2 A_{x_3} \right]$$

R.C.S.

$$\nabla \cdot \vec{A} = \frac{1}{1} \left[ \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z} \right]$$

C.C.S.

$$\nabla \cdot \vec{A} = \frac{1}{\rho} \left[ \frac{\partial a_\rho}{\partial \rho} + \frac{\partial a_\theta}{\partial \theta} + \frac{\partial a_\phi}{\partial \phi} + \frac{\partial}{\partial z} p a_z \right]$$

S.C.S.

$$\nabla \cdot \vec{A} = \frac{1}{r^2 \sin \theta} \left[ \frac{\partial a_r}{\partial r} + \frac{\partial}{\partial \theta} r \sin \theta a_\theta + \frac{\partial}{\partial \phi} r a_\phi \right]$$

- Q. Divergence of  $\vec{A}$  is the outward flux per unit volume as the volume shrinks to zero.

Curl of a vector

Curl  $\vec{A}$   $\Rightarrow \nabla \times \vec{A}$

$$\text{curl } \vec{A} = \nabla \times \vec{A} = \left( \lim_{\Delta S \rightarrow 0} \frac{1}{\Delta S} \left[ a_n \oint_C \vec{A} \cdot d\vec{l} \right] \right)_{\max}$$

In general:

$$\nabla \times \vec{A} = \frac{1}{b_1 b_2 b_3} \begin{vmatrix} a_{11} b_{11} & a_{12} b_{12} & a_{13} b_{13} \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} \\ b_{11} a_{21} & b_{12} a_{22} & b_{13} a_{23} \end{vmatrix}_{\text{anti}}$$

Curl of  $\vec{A}$  is the net circulation of vector whose magnitude is the maximum circulation of vector  $\vec{A}$  per unit area as area tends to zero. whose direction is normal direction when net circulation is Max.

### Coulomb's Law

Consider two point charges  $q_1$  and  $q_2$ , Coulomb's law states that the force  $f$  b/w 2 point charges  $q_1$  and  $q_2$  along the line joining them, directly proportional to the product of  $q_1$  and  $q_2$  inversely proportional to the square of the distance between them.

$$|\vec{F}| = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

where  $\epsilon_0$  is the permittivity of free space  $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$ .

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \vec{a}_r$$

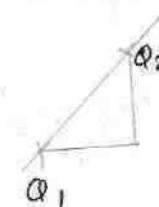
If  $q_1$  and  $q_2$  are of the same sign, then the force is repulsive. If  $q_1$  and  $q_2$  are of opposite sign then the force = attractive.

$$\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \vec{a}_{r_{12}}$$

$$\vec{F}_2 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \vec{a}_{r_{21}}$$

where  $\vec{a}_{r_{12}} = \vec{r}_{12} - \vec{r}_{21}$

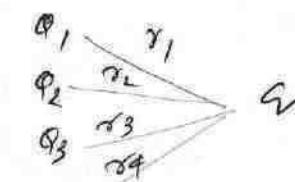
$$|\vec{r}_{12}|/r_{12}$$



$$\vec{F}_{\text{net}} = \frac{1}{4\pi\epsilon_0} q \left[ \frac{q_1}{r_{12}^2} \vec{a}_{r_{12}} + \frac{q_2}{r_{21}^2} \vec{a}_{r_{21}} + \frac{q_3}{r_{31}^2} \vec{a}_{r_{31}} + \dots \right]$$

$$\vec{F}_{\text{net}} = \frac{1}{4\pi\epsilon_0} q \sum_{i=1} \frac{q_i}{r_{i1}^2} \vec{a}_{ri}$$

$$\text{or } \sum \frac{q_i}{r_{i1}^2} \vec{a}_{ri}$$



This is superposition principle.

- ③ A charge  $q_A = -20 \mu\text{C}$  is located at a point  $(-6, 4, 1)$  and a charge  $q_B = 50 \mu\text{C}$  at a point  $(5, 8, -2)$  in free space. If the directions are given in meters. And  $\vec{a}_{AB}$  is determined in vector space, exerted on  $q_A$  by  $q_B$ .

$$\cdot \epsilon_0 = \frac{10^9}{36\pi}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C/N} \cdot \text{m}$$



Ans

$$\begin{aligned} \mathbf{R}_{AB} &= (5 - -6)\mathbf{a}_x + (8 - 4)\mathbf{a}_y + (-2 - 1)\mathbf{a}_z \\ &= 11\mathbf{a}_x + 4\mathbf{a}_y - 3\mathbf{a}_z \end{aligned}$$

$$|\mathbf{R}_{AB}| = \sqrt{11^2 + 4^2 + 3^2} = 14.46$$

$$\mathbf{q}_{AB} = \frac{11\mathbf{a}_x + 4\mathbf{a}_y - 3\mathbf{a}_z}{14.46} = 0.75\mathbf{a}_x + 0.27\mathbf{a}_y + 0.61\mathbf{a}_z$$

$$\begin{aligned} \mathbf{F}_{AB} &= \frac{1}{4\pi\epsilon_0} \frac{q_A q_B}{r_{AB}^2} \mathbf{a}_{r_{AB}} \\ &= \frac{1}{4\pi \frac{10^{-9}}{36\pi}} \cdot \frac{-20 \times 10^{-6} \times 50 \times 10^{-6}}{14.46^3} \times [0.75\mathbf{a}_x + 0.27\mathbf{a}_y + 0.61\mathbf{a}_z] \\ &= -2.8 \times 10^3 [0.75\mathbf{a}_x + 0.27\mathbf{a}_y - 0.61\mathbf{a}_z] \\ &= -2.1 \times 10^3 \mathbf{a}_x - 7.56 \times 10^2 \mathbf{a}_y + 1.408 \times 10^3 \mathbf{a}_z \end{aligned}$$

Electric field Intensity

Represented as  $\vec{E}$

$$\vec{E} = \frac{\vec{F}}{q}$$

It is defined as force per unit charge when placed in the electric field.

$$\vec{E} = \frac{\vec{F}}{q} = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} \mathbf{a}_{r_{qq}} = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} \mathbf{a}_{qq}$$

$\vec{E}$  is proportional to and in the direction of  $\vec{F}$ . Unit is N/C or V/m.

Electric field.

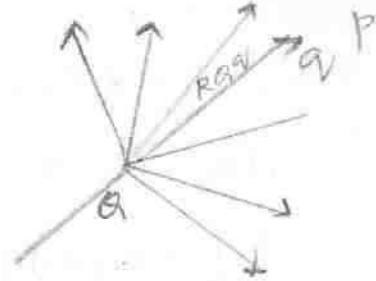
Electric charge has a region of space around it. It is called electric field.

Consider a point charge  $q$  placed as shown in fig. It has an electric field around it. In order to find electric field, first we need to find the force  $\vec{F}$ . For that, place a test charge  $q_2$  placed at a point  $P$  from  $q$ . Distance between them is  $r_{qq}$ . The force  $\vec{F}$  on the test charge due to  $q$  is

$$\frac{1}{4\pi\epsilon_0} \frac{q_2}{r_{qq}^2} \mathbf{a}_{qq}$$

Electric field intensity,  $\vec{E}$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} \mathbf{a}_{qq}$$



159. Determine the electric field intensity at  $P[0.2, 0, -2]$  due to a point charge  $5\text{nC}$ . At  $Q[+0.2, 0.1, -2.5]$  in air. All dimensions are in meters.

$$(i) R_{Qq} = R_q - R_Q \\ = 0.4a_x + 0.1a_y - 0.3a_z$$

$$|R_{Qq}| = \sqrt{0.4^2 + 0.1^2 + (-0.3)^2} \\ = 0.51$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q_q}{R_{Qq}^2} a_{R_{Qq}}$$

$$a_{R_{Qq}} = \frac{0.4a_x + 0.1a_y - 0.3a_z}{0.51}$$

$$= 0.78a_x + 0.2a_y - 0.59a_z$$

$$\vec{E} = 9 \times 10^9 \times \frac{-9}{0.51^2} [0.78a_x + 0.2a_y - 0.59a_z]$$

$$= 143.01 [0.78a_x + 0.2a_y - 0.59a_z]$$

$$= 134.94a_x + 36.33a_y - 102.08a_z$$

160. Two charges  $1\mu\text{C}$  and  $-2\mu\text{C}$  are located at  $[3, 2, -1]$  and  $[-1, -1, 4]$  respectively. Calculate the electric force  $10\text{nC}$  charge located at  $[0, 3, 1]$  and electric field at that point.

$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

$$\vec{F}_1 = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_3}{r_{13}^2} a_{r_{13}}$$

$$\vec{r}_{13} = \vec{r}_3 - \vec{r}_1 \\ = -3a_x + a_y + 2a_z$$

$$|\vec{r}_{13}| = \sqrt{9+1+4} = \sqrt{14} = 3.74$$

$$a_{r_{13}} = \frac{-3a_x + a_y + 2a_z}{3.74}$$

$$\vec{F}_1 = \frac{9 \times 10^9 \times 1 \times 10^{-6} \times -10 \times 10^{-9}}{3.74^2} \times a_{r_{13}} = \frac{6.43 \times 10^6}{3.74} [-3a_x + a_y + 2a_z]$$

$$= 6.43 \times 10^6 [-0.802a_x + 0.27a_y + 0.53a_z]$$

$$= [-5.17a_x + 1.74a_y + 3.40a_z]$$

$$\vec{F}_2 = \frac{1}{4\pi\epsilon_0} \frac{Q_2 Q_3}{r_{23}^2} a_{r_{23}}$$

$$\vec{r}_{23} = a_x + 4a_y - 3a_z$$

$$|\vec{r}_{23}| = \sqrt{1+16+9} = 5.09$$

$$a_{r_{23}} = \frac{a_x + 4a_y - 3a_z}{5.09}$$

$$= \underline{(-1.36ax + -5.46ay + 4.09az) \times 10^6 N}$$

Q. point charge  $5\text{nc}$  and  $-2\text{nc}$  are located at  $[2, 0, 4]$  and  $[-3, 0, 5]$  respectively.

b) determine the force on A:  $1\text{nc}$  point-charge located at  $[1, -3, 7]$ . find the electric field  $E$  at  $[1, -3, 7]$ .

$$\vec{F}_{\text{net}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{(R_{Qq})^3} \vec{R}_{Qq}$$

$$f = f_1 + f_2$$

$$f_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{(\sigma_{13})^3} a_{\sigma_{13}}$$

$$\sigma_{13} = \sigma_3 - \sigma_1 = -ax - 3ay + 2az.$$

$$|\sigma_{13}| = \sqrt{1^2 + 3^2 + 3^2} = \sqrt{1 + 9 + 9} = \sqrt{19} = 4.358$$

$$f_1 = \frac{9 \times 10^9 \times 5 \times 10^9 \times 1 \times 10^9}{(4.358)^3} \times [-ax - 3ay + 2az]$$

$$= \frac{4.5 \times 10^8}{(4.358)^3} \times (-ax - 3ay + 2az)$$

$$= 5.4369 \times 10^{10} [-ax - 3ay + 2az]$$

$$= \underline{5.4369ax - 16.31ay + 16.407az}$$

$$\sigma_{23} = \sigma_3 - \sigma_2$$

$$= 4az - 3ay + 2az$$

$$|\sigma_{23}| = \sqrt{4^2 + 3^2 + 2^2} = \sqrt{16 + 9 + 4} = \underline{5.385}$$

$$\vec{f}_2 = \frac{9 \times 10^9 \times 2 \times 10^9 \times 1 \times 10^9}{(5.385)^3} x + ax - 3ay + 2az$$

$$= -1.15269 \times [4az - 3ay + 2az]$$

$$= -4.61ax + 3.458ay + 2.305az$$

$$f = f_1 + f_2$$

$$= -1.003ax - 1.28ay + 1.992az$$

$$E = \frac{f}{q} = \frac{-1.003ax - 1.28ay + 1.992az}{1 \times 10^9}$$

$$= \underline{-1.003ax - 1.28ay + 1.992az}$$

Electric field for infinite line charge

$$\vec{E} = \frac{\rho L}{2\pi\epsilon_0 r} \hat{a}_r$$

Surface charge  $\vec{E} = \frac{\rho_s}{2\epsilon_0} \hat{a}_n$

(infinite sheet of charge)

Volume charge  $\vec{E} = \frac{\rho}{4\pi\epsilon_0 r^2} \hat{a}_r$

$\vec{E}$  due to point charge  $\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2} \hat{a}_r$

Line  $\Rightarrow Q = \int \rho_L dz$

Surface  $\Rightarrow Q = \int \rho_s dS$

Volume  $\Rightarrow Q = \int \rho_v dv$

Flux and flux density ( $\Phi$ )

flux  $\rightarrow \Psi$



Measure of flow of electric field lines is called flux denoted by  $\Psi$

flux density

flux  $\Psi$  = charge enclosed  $\frac{q}{\text{area}}$

flux per unit area

$$\vec{D} = \frac{\Psi}{A} \hat{a}_n = \frac{\text{flux}}{\text{unit Area}} = \frac{Q}{4\pi r^2}$$

\* Electric flux is the measure of flow of electric field lines  
Area through a given area.

flux  $\Rightarrow \Psi = Q$ .

\* flux density  $\vec{D}$

Electric flux density or displacement ( $\vec{D}$ ) flux per unit area

$$\vec{D} = \frac{\Psi}{A} \hat{a}_n$$

Unit is Coulomb/m<sup>2</sup> C/m<sup>2</sup>

$$= \frac{Q}{A} \hat{a}_n$$

$\rightarrow$  Relation b/w  $\vec{E}$  and  $\vec{D}$ .

Consider a point charge  $Q$  in a sphere of radius  $R$ .



Volume  $\frac{4\pi R^3}{3}$

$$\vec{D} = \frac{\Psi}{A} \hat{a}_n = \frac{Q}{4\pi R^2} \hat{a}_r$$

We know  $\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2} \hat{a}_r$   $\therefore \vec{E} = \frac{\vec{D}}{\epsilon_0}$

$$\boxed{\vec{D} = \epsilon_0 \vec{E}}$$

$$\vec{D} = \frac{\rho_L}{2\pi R} \hat{a}_R^1 \rightarrow \text{Infinite line of charge}$$

$$\vec{D} = \frac{\rho_S}{2} \hat{a}_n^1 \rightarrow \text{Infinite sheet of charge / surface charge distribution}$$

$$\vec{D}_V = \int_V \frac{\rho_V dv}{4\pi R^2} \hat{a}_R^1 \rightarrow \text{Volume charge distribution}$$

$$\vec{D}_p = \epsilon_0 \vec{E} \rightarrow \text{point charge}$$

$$\boxed{\vec{D}_L = \frac{Q}{4\pi R^2} \hat{a}_R^1 = \int \frac{\rho_L dl}{4\pi R^2} \hat{a}_R^1}$$

→ line charge

$$Q = \rho_L L$$

$$dQ = \rho_L dl$$

- Q. Find the  $\vec{D}$  in the region about a uniform line charge  $8 \text{ nC/m}$  line along  $x$ -axis in the free space given  $R = 3 \text{ meters}$ .

$$D_L = \frac{\rho_L}{2\pi R} \hat{a}_R^1$$

$$R \gg p$$

$$\vec{E} \text{ for line charge} = \vec{E} = \frac{\rho_L}{2\pi \epsilon_0 R} \hat{a}_R^1$$

$$\vec{D} = \epsilon_0 \vec{E} = \frac{\epsilon_0 \times \rho_L}{2\pi \epsilon_0 R} \hat{a}_R^1$$

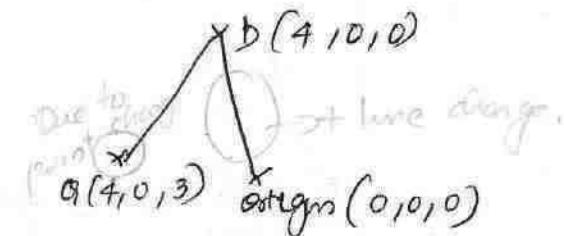
$$= \frac{8 \times 10^{-1}}{2\pi \times 3 \times 10^9} \hat{a}_R^1$$

$$= 0.42 \hat{a}_R^1 \text{ nC/m}$$

- Q. Determine  $\vec{D}$  at  $(4, 0, 3)$ . If, there is a point charge  $Q = -5\pi \text{ mC}$  @  $(4, 0, 0)$  and a line charge  $= 3\pi \text{ mC/m}$  along the  $y$  axis.

$$\vec{D}(4, 0, 3)$$

$$\text{line charge} = 3\pi \text{ mC/m}$$



$$\vec{D} = \vec{D}_q + \vec{D}_L$$

$$\vec{D}_L = \epsilon_0 \vec{E} = \frac{Q}{2\pi R^2} \hat{a}_R^1$$

$$\vec{D}_q = \epsilon_0 \vec{E} = \frac{Q}{4\pi R^2} \hat{a}_R^1$$

$$\vec{D} = \frac{Q}{4\pi R^2} \hat{a}_R^1 + \frac{-\rho_L}{2\pi R} \hat{a}_R^1$$

$$\hat{a}_R^1 = (4, 0, 3) - (4, 0, 0)$$

$$= \frac{(0, 0, 3)}{\sqrt{0+0+9}} = \frac{3a_3}{\sqrt{9}} = \hat{a}_2^1 //$$

$$D_q = \frac{Q}{4\pi R^2} \hat{a}_R^1$$

$$\vec{D} = \frac{-5\pi \times 10^{-6}}{4\pi \times 3^2} \hat{a}_2 + \frac{3\pi \times 10^{-6}}{2\pi \times 4} \hat{a}_3$$

$$\Rightarrow -1.388 \times 10^4 \hat{a}_2 + 6 \times 10^4 \hat{a}_3$$

$$= 3.612 \times 10^4 \hat{a}_2 - 2.362 \times 10^4 \hat{a}_3$$

$$\hat{a}_p = \frac{(4, 0, 3) - (0, 0, 0)}{\sqrt{4^2 + 0^2 + 3^2}} = \frac{4\hat{a}_x + 3\hat{a}_z}{5}$$

$$a_p^2 = \frac{4^2 + 3^2}{5^2}$$

$$\theta = 5^\circ$$

$$\vec{D}_L = \frac{Q}{2\pi P} \hat{a}_p = \frac{3\pi \times 10^{-3}}{2\pi \times 5} \times \hat{a}_p$$

$$= \frac{3\pi \times 10^{-3}}{2\pi \times 5} \times \frac{4\hat{a}_x + 3\hat{a}_z}{5}$$

$$= 0.24 \hat{a}_x + 0.18 \hat{a}_z \text{ NC/m}^2$$

$$\vec{D} = \vec{D}_q + \vec{D}_L = 0.24 \hat{a}_x + 0.42 \hat{a}_z \text{ NC/m}^2$$

Q.3 A <sup>surface point</sup> charge of  $+30\text{NC}$  is located at origin while plane  $y=3$  carries a <sup>surface</sup> charge  $10\text{NC/m}^2$ . Find  $\vec{D}$  at  $(0, 4, 3)$ .

$$\vec{D}_S = \frac{Ps}{2} \hat{a}_n$$

$$\vec{B} = \vec{D}_Q + \vec{D}_S$$

$$\vec{D}_S = \frac{Q}{4\pi R^2} \hat{a}_R$$

$$= \frac{30\text{NC}}{4\pi R^2} \hat{a}_R$$

$$\hat{a}_R = \frac{(0, 4, 3) - (0, 0, 0)}{\sqrt{4^2 + 3^2}} = \frac{4\hat{a}_y + 3\hat{a}_z}{5}$$

$$\vec{D}_S = \frac{30\text{NC}}{4\pi R^2} \times \frac{4\hat{a}_y + 3\hat{a}_z}{5}$$

$$= \frac{30 \times 10^{-9}}{4\pi \times 5^2} \left( \frac{4\hat{a}_y + 3\hat{a}_z}{5} \right)$$

$$= \frac{30\text{NC}}{4\pi \times 5^2} (0.8\hat{a}_y + 0.6\hat{a}_z)$$

$$= \frac{24\hat{a}_y + 18\hat{a}_z}{4\pi \times 5^2}$$

$$= 0.0463\hat{a}_y + 0.054\hat{a}_z$$

$$\vec{D}_S = \frac{Ps}{2} \hat{a}_n = \frac{10\text{NC}}{2} \hat{a}_y \quad (\hat{a}_n = \hat{a}_y = \hat{a}_y \text{ NC})$$

$$\vec{D} = \vec{D}_q + \vec{D}_S = 5.046\hat{a}_y + 0.054\hat{a}_z$$

$[y=3 \text{ [unit vector, } \hat{a}_y]$

\* Electric flux  $\Phi = Q = \int \vec{D} \cdot d\vec{s}$ . [Total no. of any particular line of force in electric field]

\* flux density  $D = \frac{q_{in}}{A} = \frac{Q_{in}}{A}$

\* Relation between  $E$  &  $D$ ,  $\vec{D} = \epsilon_0 \vec{E}$

\*  $\vec{D}$  due to point charge  $D_p = \frac{Q}{4\pi r^2} \hat{a}_r$

\*  $\vec{D}$  due to infinite line of charge.

$$D_L = \frac{\rho L}{4\pi r} \hat{a}_r$$

\*  $\vec{D}$  due to line of charge

$$D_L = \frac{Q}{4\pi r^2} \hat{a}_r = \frac{\rho L dr}{4\pi r^2} \hat{a}_r$$

$$\hat{a}_n \rightarrow \hat{a}_r$$

\*  $\vec{D}$  due to surface charge  $D_s = \frac{\int s \rho_s ds}{4\pi r^2} \hat{a}_r$

\*  $\vec{D}$  due to infinite sheet of charge (plane)  $D_{inf} = \frac{\rho_s}{2} \hat{a}_r$

\*  $\vec{D}$  due to volume charge  $D_v = \frac{\int_V \rho dv}{4\pi r^2} \hat{a}_r$

### Gauss's law

Gauss's law states that the total electric flux  $\Phi$  through any enclosed surface is equal to the total charge enclosed by that surface.

$$\Phi = \oint \vec{D} \cdot d\vec{s} = Q. \quad Q \rightarrow \text{charge enclosed.}$$

→ Integral form.

$$\nabla \cdot \vec{D} = \rho_v \rightarrow \text{Differential form.}$$

(Q1) The flux density  $D = \frac{8}{3} \hat{a}_r \text{nC/m}^2$  is in the free space find  $\vec{E}$  at  $r = 0.2 \text{ m}$ . find  $\Phi$  leaving the sphere of radius  $R = 0.2 \text{ m}$ . find the total charge within the sphere of  $R = 0.3 \text{ m}$ .  $\rho = \frac{4}{3}\pi r^3 n$

ans,  $\vec{D} = \epsilon_0 \vec{E}$

$$\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{8/3 \hat{a}_r \text{nC/m}^2}{8.82 \times 10^{-12}} = \frac{0.2/3 \hat{a}_r}{8.82 \times 10^{-12}} \text{nC/m}^2$$

$$= \frac{0.2 \times 10^{-9}}{8 \times 8.82 \times 10^{-12}}$$

$$= 4.53 \hat{a}_r$$

$$= 6.455 \hat{a}_r \quad \boxed{7.5 \times 10^3 \hat{a}_r}$$

$$(b) \quad \Psi = \oint_S \vec{E} \cdot d\vec{s}$$

$S \rightarrow S \rightarrow \theta \rightarrow \phi$

$$d\vec{s} = \sigma^2 \sin\theta d\theta d\phi \hat{a}_z. \quad \hat{a}_\theta = 1$$

$$\Psi = \int_{\theta=0}^{2\pi} \int_{\phi=0}^{\pi} \frac{\sigma^3}{3} \sin\theta d\theta d\phi \quad nC = 0.183 nC.$$

$$= \frac{\sigma^3}{3} \left[ [-\cos\theta]_0^{\pi} (\phi)_0^{2\pi} \right].$$

$$= \frac{\sigma^3}{3} \left[ -\cos\pi - \cos 0 \right] \times [2\pi - 0]$$

$$= \frac{\sigma^3}{3} [2 \times 2\pi]$$

$$\Rightarrow \frac{4\pi\sigma^3}{3} nC$$

$$\text{At } \sigma = 0.2 \text{ m} \rightarrow Q = 0.033 nC$$

$$\text{At } \sigma = 0.3 \text{ m} \rightarrow Q = \frac{4\pi}{3} (0.3)^3 nC \\ = 0.19 nC$$

- 5) Find the volume charge density in the field  $\vec{D} = [y + e^x] \hat{a}_x - y e^{-y} \hat{a}_y + \frac{3}{2} \hat{a}_z$  C/m<sup>3</sup>.

$$\nabla \cdot \vec{D} = f$$

$$\begin{aligned} \nabla \cdot \vec{D} &= \frac{\partial}{\partial x} [y + e^x] + \frac{\partial}{\partial y} [-y e^{-y}] + \frac{\partial}{\partial z} [3/2] \\ &= -e^x + y \cdot e^{-y} x - 1 + e^{-y} - 1 + \\ &= -e^x + y e^{-y} - e^{-y} + 1 \end{aligned}$$

## Electric potential V

\* scalar quantity.

$$\vec{E} = -\nabla V$$

workdone in moving a point charge  $q$  by a distance  $d\vec{l}$   $\vec{F}_c$ ;  $d\vec{w} = \vec{F} \cdot d\vec{l}$

$$\text{Total workdone } W_{AB} = \int_B^A \vec{F} \cdot d\vec{l}$$

$\therefore q$  gives the potential energy.

$$\frac{W_{AB}}{q} = V_{AB} = - \int_B^A \frac{q \vec{E} \cdot d\vec{l}}{q} = - \int_B^A \vec{E} \cdot d\vec{l}$$

$$V_{AB} = - \int_B^A \vec{E} \cdot d\vec{l}, \text{ potential drift.}$$



Electric potential or scalar potential.

E can be obtained from V by using the relation  $\vec{E} = -\nabla V$

Consider a point charge  $q$  moves from B to A in an electric field  $\vec{E}$ . The workdone in moving  $q$  by a distance  $d\vec{l}$  is given by i.e.,  $d\vec{w} = \vec{F} \cdot d\vec{l}$

$$\text{Total workdone } W_{AB} = - \int_B^A \vec{F} \cdot d\vec{l}$$

If  $q$  is a unit charge then, dividing  $W$  by  $q$  gives the p.e per unit charge which is the potential difference  $V_{AB}$  and is denoted by  $V_{AB}$ .

$$\frac{W_{AB}}{q} = V_{AB} = - \int_B^A q \vec{E} \cdot d\vec{l} = - \int_B^A \vec{E} \cdot d\vec{l}$$

$V_{AB} = \int_B^A \vec{E} \cdot d\vec{l}$  potential drift.

- \* Potential is defined as workdone per unit charge in moving a unit charge from A to B. in the field  $\vec{E}$  is called potential difference b/w A & B. potential  $-V$ .

$$\vec{E} = -\nabla V$$

\* potential difference  $V_{AB} = - \int_B^A \vec{E} \cdot d\vec{l}$

\* potential difference due to point charge

$$V_{AB} = \frac{\sigma}{4\pi\epsilon_0} \left[ \frac{1}{r_A} - \frac{1}{r_B} \right]$$

Absolute potential at point A  
OR from B to point A

$$\left. \begin{aligned} V_A &= \frac{\phi}{4\pi\epsilon_0 r_A} \\ V_A &= \frac{\phi}{4\pi\epsilon_0 r_A} + C \end{aligned} \right\}$$

\* If reference is other than  $\infty$   $V_A = \frac{q}{4\pi\epsilon_0 r_A} + C$

\* Absolute potential due to line charge

$$V_A = \int_C \frac{\lambda l \cdot d\vec{l}}{4\pi\epsilon_0 r} \rightarrow \text{distance of point A from } q$$

- \* Surface charge ;  $V_A = \int_S \frac{\rho_s \cdot ds}{4\pi\epsilon_0 r}$
- \* Volume charge ,  $V_A = \int_V \frac{\rho_v \cdot dv}{4\pi\epsilon_0 r}$

Relation b/w  $\vec{E}$  &  $V$

$$V = - \int \vec{E} \cdot d\vec{l}$$

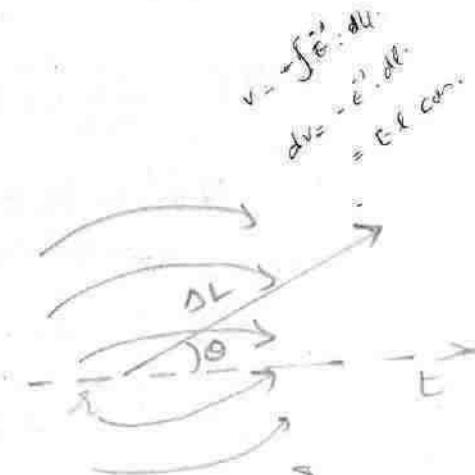
$$dV = - \vec{E} \cdot d\vec{l} \\ = E dl \cos\theta$$

$$\frac{dV}{dl} = - \vec{E} \cos\theta.$$

$$\left( \frac{dV}{dl} \right)_{\max} = -E$$

$$\text{grad } V = -\vec{E}$$

$$\underline{\underline{\vec{E}}} = -\nabla V$$



$$\begin{aligned} V &= - \int \vec{E} \cdot d\vec{l} \\ dV &= \vec{E} \cdot d\vec{l} \\ dV &= E dl \cos\theta \\ \frac{dV}{dl} &= -E \cos\theta \\ \left( \frac{dV}{dl} \right)_{\max} &= -E \\ \underline{\underline{\vec{E}}} &= -\nabla V \end{aligned}$$

ie conti  $\rightarrow \vec{E} \text{ at } P[6, -2.5, 3]$

$$\Rightarrow \underline{59.97a_x^1 - 79.09a_y^1 - 20a_z^1} V/m$$

$$D = E_0 \cdot \vec{E}$$

(ii)  $\vec{D} = 0.531 a_x^1 - 0.634 a_y^1 - 0.147 a_z^1 \text{ nC/m}^2$

$$\nabla \vec{D} = \rho_v = (\nabla \cdot \vec{E}) \epsilon_0 = \rho_v$$

$$\nabla \vec{E} = \frac{\partial}{\partial x} [E_x + \frac{\partial}{\partial y} (Ey + \frac{\partial}{\partial z} E_z)], \text{ at } P[6, -2.5, 3]$$

$$\underline{\underline{\nabla \vec{E}}} = 10.00g$$

$$\rho_v = \omega_p = E_0 \cdot (\nabla \vec{E})$$

$$= 88.6193 \text{ pC/m}^2$$

Ans

(iii) W.D

$$W = -Q \int_{A}^{B} \vec{E} \cdot dL = \frac{Q}{4\pi} V_{AB} = Q [V_B - V_A]$$

$$W = 10 \left[ \frac{10}{\alpha^2} \sin \alpha \cos \alpha - \frac{10}{\alpha^2} \cos \alpha \cos \alpha \right]$$

$$W = 10 \left[ \frac{10}{\alpha^2} \sin \alpha \cos 60 - \frac{10}{\alpha^2} \cos 30 \cos 20 \right] \times 10^{-6}$$

$$\underline{\underline{W = 28.1425 \text{ mJ}}}$$

Q. If  $V = \alpha x^4 y + 20z - \frac{4}{x^2 y^2}$ , find  $\vec{E}, \vec{D}$  and  $\rho_v$  at  $(6, -2.5, 3)$ .

$$\text{Ans } \vec{E} = -\nabla V = -\left[ \frac{\partial}{\partial x} V + \frac{\partial}{\partial y} V + \frac{\partial}{\partial z} V \right]$$

$$\begin{aligned} \frac{\partial}{\partial x} V &= \frac{\partial}{\partial x} \left[ \alpha x^4 y + 20z - \frac{4}{x^2 y^2} \right] \\ &= 4xy - 4 \\ &= 4xy + 8 \left[ \frac{4}{(x^4 y^2)^2} \right] \\ &= \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial y} V &= \frac{\partial}{\partial y} \left[ \alpha x^4 y + 20z - \frac{4}{x^2 y^2} \right] \\ &= 2x^4 y + 8 \left[ \frac{y}{(x^4 y^2)^2} \right] \end{aligned}$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z} \left[ \alpha x^4 y + 20z - \frac{4}{x^2 y^2} \right]$$

$$= 20.$$

$$\vec{E} = -4xy - \frac{8x}{(x^4 y^2)^2} + 2x^4 - \frac{8y}{(x^4 y^2)^2} + 20.$$

$$\vec{E} = -\left[ \left( -4xy - \frac{8x}{(x^4 y^2)^2} \right) a_x^1 + \left( 2x^4 - \frac{8y}{(x^4 y^2)^2} + 20 \right) a_z^1 \right]$$



Q. Given potential  $V = \frac{10}{r^2} \sin\theta \cos\phi$  (i) Find  $\vec{D}^{ab}(2, \pi/2, 10)$

(ii) calculate the workdone in moving a  $10\mu C$  from a point  $A(1, 30, 20)$  to a point  $B(4, 90, 160)$

$$\text{Ans} \quad \vec{E} = -\nabla V = -\left[ \frac{\partial V}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{a}_\theta + \frac{1}{r \sin\theta} \frac{\partial V}{\partial \phi} \hat{a}_\phi \right]$$

$$\frac{\partial V}{\partial r} = \frac{\partial}{\partial r} \left( \frac{10}{r^2} \sin\theta \cos\phi \right) = 10 \sin\theta \cos\phi r^{-3}$$

$$= -20 \sin\theta \cos\phi r^{-3}$$

$$\frac{\partial V}{\partial \theta} = \frac{10}{r^2} \cos\phi \cos\theta.$$

$$\frac{\partial V}{\partial \phi} = \frac{10}{r^2} \sin\theta \sin\phi.$$

$$\vec{E} = -\left[ -20 \sin\theta \cos\phi r^{-3} \hat{a}_r + \frac{1}{r} \times \frac{10}{r^2} \cos\phi \cos\theta \hat{a}_\theta + -\frac{1}{r \sin\theta} \frac{10}{r^2} \sin\theta \sin\phi \hat{a}_\phi \right]$$

$$= -\left[ -20 \sin(\pi/2) \cos 0 r^{-3} \hat{a}_r + \frac{10}{r^2} \cos 0 \cos(\pi/2) \hat{a}_\theta - \frac{10}{r^2} \sin 0 \sin(\pi/2) \hat{a}_\phi \right]$$

$$= 2.5 \hat{a}_r // -\frac{10}{r^2} \sin 0 \hat{a}_\theta //.$$

$$\vec{D} = \epsilon_0 \vec{E} = 2.205 \times 10^{-11} \hat{a}_r //$$

$$W_{AB} = Q \times V_{AB} = Q_A (V_B - V_A) = 10 \times 10^{-6} \times \left[ \frac{10}{16} \sin 90 \cos 60 - \frac{10}{1} \sin 0 \cos 120 \right]$$

$$= 28.125 \text{ MJ}$$

If  $V = \frac{Qx^2y^2}{x^2+y^2}$

\* Q. Find the potential b/w A and B at a distance. 0.5m and 0.1m respectively.  $Q = 20 \times 10^{-10} \text{ C}$

$$V = \frac{Q}{4\pi\epsilon_0 s}$$

$$V_{AB} = \frac{Q}{4\pi\epsilon_0 s} \left[ \frac{1}{r_A} - \frac{1}{r_B} \right]$$

$$= \frac{20 \times 10^{-10}}{4\pi\epsilon_0 \times 8.85 \times 10^{-12}} \left[ \frac{1}{0.5} - \frac{1}{0.1} \right] = \frac{20 \times 10^{-10}}{1.1 \times 10^{-11}} \left[ \frac{1}{0.5} - \frac{1}{0.1} \right]$$

$$= 144.0358$$

### Poisson and Laplace equation

To solve boundary value problem, we use Poisson and Laplace equations.

From Gauss law,

$$\nabla \cdot \vec{D} = \rho_v$$

$$\vec{D} = \epsilon_0 \vec{E} \Rightarrow E = -\nabla V.$$

$$\nabla \cdot \epsilon_0 \vec{E} = \rho_v$$

$$\nabla \cdot \epsilon_0 (-\nabla V) = \rho_v$$

$$\cancel{\nabla} \cdot (\epsilon_0 \nabla V) = \rho_v$$

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon_0}}$$

$$\begin{aligned} \nabla^2 V &= \rho_v \\ \epsilon_0 \nabla^2 V &= \rho_v \\ \epsilon_0 \nabla^2 V &= \rho_v \\ \frac{\epsilon_0}{4\pi} \nabla^2 V &= \rho_v \end{aligned}$$

$\Rightarrow$  Poisson's eqn

- when  $\rho v = 0$   
 $\nabla^2 V = 0 \rightarrow$  Laplace eqn

(i) On cartesian co-ordinate system,

$$\nabla^2 V = \frac{\partial^2}{\partial x^2} V + \frac{\partial^2}{\partial y^2} V + \frac{\partial^2}{\partial z^2} V = -\frac{\rho v}{\epsilon}$$

(ii) Laplace equation is

1) Cartesian co-ordinate system [RCG]

$$\nabla^2 V = \frac{\partial^2}{\partial x^2} V + \frac{\partial^2}{\partial y^2} V + \frac{\partial^2}{\partial z^2} V = 0$$

2) Cylindrical co-ordinate system.

$$\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2} \left( \frac{\partial^2 V}{\partial \theta^2} \right) + \frac{\partial^2 V}{\partial z^2}$$

3) Spherical co-ordinate system.

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2}$$

Laplace eqn  $\nabla^2 V = 0$

Poisson's eqn  $\nabla^2 V = -\frac{\rho}{\epsilon}$

- \* 1) Cartesian co-ordinate system.  
 ~~$\nabla^2 V = \frac{\partial^2}{\partial x^2}$~~   
 Q. Determine whether or not the potential field satisfies the  
 i)  $V = \rho \cos \theta + z$  and ii)  $V = \rho \cos \theta + \phi$ .

follows

Laplace eqn

i)

$$V = \rho \cos \theta + z$$

ii)

$$V = \rho \cos \theta + \phi$$

$$\begin{aligned} \nabla^2 V &= \frac{1}{\rho^2} \frac{\partial}{\partial \rho} \left[ \rho^2 \frac{\partial V}{\partial \rho} \right] + \frac{1}{\rho^2 \sin \theta} \frac{\partial}{\partial \theta} \left[ \sin \theta \frac{\partial V}{\partial \theta} \right] \\ &\quad + \frac{1}{\rho^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2} \\ &= \frac{1}{\rho^2} \frac{\partial}{\partial \rho} \left[ \rho^2 (\cos \theta) \right] + \frac{1}{\rho^2 \sin \theta} \frac{\partial}{\partial \theta} \left[ \sin \theta (\cos \theta) \right] \\ &\quad + \frac{1}{\rho^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} (\cos \theta) \\ &= \frac{1}{\rho^2} \frac{\partial}{\partial \rho} [\rho^2 \cos \theta] + \frac{1}{\rho^2 \sin \theta} \frac{\partial}{\partial \theta} [\sin \theta \cos \theta] + 0 \\ &= \frac{1}{\rho^2} [\rho^2 \cos \theta] + \frac{1}{\rho^2 \sin \theta} [-\rho^2 \sin \theta \cos \theta] \\ &= \frac{2}{\rho} \cos \theta - \frac{2}{\rho^2 \sin \theta} \rho \sin \theta \cos \theta \\ &= \frac{2}{\rho} \cos \theta - \frac{1}{\rho \sin \theta} 2 \sin \theta \cos \theta \\ &= \frac{2}{\rho} \cos \theta - \frac{1}{\rho} 2 \cos \theta \\ &= \frac{2}{\rho} \cos \theta - \frac{2}{\rho} \cos \theta = 0 \end{aligned}$$

It satisfies Laplace eqn /

$$(ii) V = p \cos\phi + z$$

$$\nabla^2 V = \frac{1}{p} \frac{\partial}{\partial p} \left[ p \cdot \frac{\partial V}{\partial p} \right] + \frac{1}{p^2} \left[ \frac{\partial^2 V}{\partial \phi^2} \right] + \frac{\partial^2 V}{\partial z^2}$$

$$= \frac{1}{p} \frac{\partial}{\partial p} [p \cos\phi] + \frac{1}{p^2} [-p \cos\phi] + 0$$

$$= \frac{1}{p} \cos\phi + -\frac{1}{p^2} p \cos\phi$$

$$= \frac{\cos\phi}{p} - \frac{\cos\phi}{p} = 0$$

It satisfies laplace eqn

$$\left\{ \begin{array}{l} \frac{\partial V}{\partial p} = \cos\phi \\ \frac{\partial^2 V}{\partial \phi^2} = -p \cos\phi \\ \frac{\partial V}{\partial z} = -p \sin\phi \\ \frac{\partial^2 V}{\partial z^2} = -p \cos\phi \end{array} \right.$$

8 The region b/w 2 concentric right cylinders contains a uniform charge density. Solve the poissoneqns for the potential in the regions.

In conical cable  $E$  in radial direction from inner to outer cylinder. So  $E$  has only the radial component.

consider cylindrical coordinate system

$$\nabla^2 V = \frac{1}{E} \text{ poisson eqn}$$

$\nabla^2 V$  in cylindrical coordinate system.

$$\nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial V}{\partial r} \right] + \frac{1}{r^2} \left[ \frac{\partial^2 V}{\partial \phi^2} \right] + \frac{\partial^2 V}{\partial z^2}$$

one zero eq.

$$\nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \frac{\partial V}{\partial r} \right] = -\frac{p}{E}$$

$$-\frac{p}{E} = \frac{\partial}{\partial r} \left[ r \frac{\partial V}{\partial r} \right]$$

Integrating both sides.

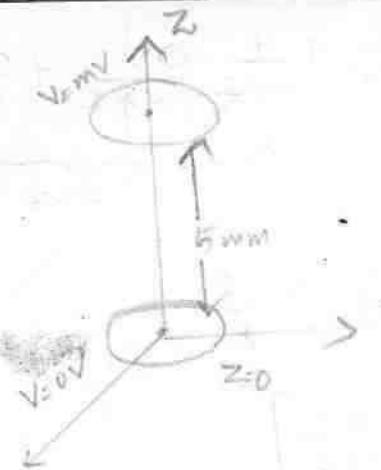
$$\begin{aligned} \frac{\partial V}{\partial r} &= \int -\frac{p}{E} dr = \frac{p}{E} \int r dr \\ &= -\frac{p}{E} \frac{r^2}{2} + C_1 \end{aligned}$$

$C_1, C_2$  constants of integration

$$\begin{aligned} \text{Integrating both sides } V &= \int \left[ -\frac{p}{E} \frac{r^2}{2} + \frac{C_1}{r} \right] dr \\ &= -\frac{p}{2E} \int r dr + \int \frac{C_1}{r} dr + C_2 \\ &= -\frac{p}{2E} \frac{r^2}{2} + C_1 (\ln r) + C_2 \end{aligned}$$

$$V = -\frac{p r^2}{4E} + C_1 (\ln r) + C_2$$

3) 2  $\parallel$  el conducting disc are separated by a distance 5mm at  $z=0$  and  $z=5$  mm. If  $V=0$  at  $z=0$  and  $V=100V$  at  $z=5$  mm. Find the charge density on the disc.



consider cylindrical co-ordinate s/m

potential  $V$  is the position of  $z$  alone and no  $\gamma$  or  $\theta$  coordinate.

$$\nabla^2 V = 0; \frac{\partial V}{\partial z} = 0$$

$$\text{Integrating } \frac{\partial V}{\partial z} = 0 \int dz + c_1 = c_1.$$

$$\text{Integrating } V = \int c_1 dz + c_2 = c_1 z + c_2$$

$$V = c_1 z + c_2 \quad \text{--- (1)}$$

$$\text{At } z=0 \quad V=0V \quad 0=c_2$$

$$\text{At } z=5\text{mm} \quad V=100V \quad 100 = 5c_1 + c_2$$

$$c_1 = \frac{100}{5 \times 10^{-3}} = \frac{20 \times 10^3}{5}$$

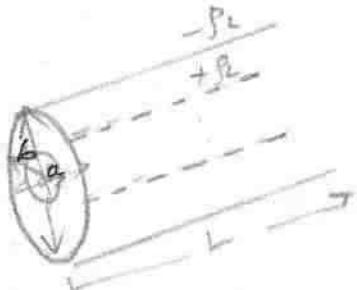
$$V = 20 \times 10^3 z^3 V$$

$$\vec{E} = -\nabla V = -\frac{\partial}{\partial z} [20 \times 10^3 z^3] \hat{a}_z$$

$$\vec{E} = -20 \times 10^3 z^2 \hat{a}_z V/m$$

## Capacitance of a coaxial cable

$$C = \frac{Q}{V} = \frac{\oint_S \vec{E} \cdot \vec{d}s}{-\int_L \vec{E} \cdot dL}$$



$$\Psi = Q = \oint_S \vec{D} \cdot \vec{d}s$$

$$= \oint_S \vec{E} \cdot \vec{E} ds$$

$$V = - \int_L \vec{E} \cdot dL = - \int_L \vec{E} \cdot dL$$

consider a coaxial cable (or) capacitance.

let 'a' be the radius of the inner conductor

let 'b' be the radius of the outer conductor.

The conductors are separated by a dielectric of permittivity  $\epsilon$ . The length of the cable is  $L$  m. The inner conductor carries a charge density  $+ρ_L$  C/m and the outer conductor is  $-ρ_L$  C/m.

Assume cylindrical co-ordinate s/m.  $\vec{E}$  will be radial from inner to outer.

$\vec{E}$  for infinite line charge.

$$\vec{E} = \frac{\rho_L}{2\pi \epsilon_0 z} \hat{a}_z$$

The potential difference in the work done in moving unit charge against  $\vec{E}$  is from  $z=b$

$$to \infty = a$$

$$V = - \int_{-\infty}^a \vec{E} \cdot d\vec{L} = \int_{-\infty}^a \frac{\rho_L}{2\pi\epsilon_0} d\sigma \cdot \hat{d\sigma}$$

$$V = -\frac{\rho_L}{2\pi\epsilon_0} [\ln a - \ln b]$$

$$= \frac{\rho_L}{2\pi\epsilon_0} \ln \left[ \frac{a}{b} \right]$$

$$C = \frac{Q}{V} = \frac{\rho_L L}{V}$$

$$= \frac{\rho_L}{2\pi\epsilon_0} \frac{\epsilon_0 L}{\ln \left[ \frac{a}{b} \right]} = \frac{2\pi\epsilon_0 L}{\ln a/b}$$

$$\text{If, } L = 1m \quad C = \frac{2\pi\epsilon_0}{\ln [a/b]}$$

Capacitance or capacitors is defined as the ratio of magnitude of the total charge on any one of the two conductors and potential difference b/w the conductors is called Capacitance.

$$C = \frac{q}{V}$$

$q$  = charge in coulombs.

$V$  = potential difference in volts.

$$q = \oint \vec{D} \cdot d\vec{s} = \oint \epsilon \vec{E} \cdot d\vec{s}$$

$$\text{potential } V = - \int_L \vec{E} \cdot d\vec{L} \quad \frac{q}{V} = \frac{\oint \epsilon \vec{E} \cdot d\vec{s}}{- \int_L \vec{E} \cdot d\vec{L}}$$

Potential Energy stored in electric field

Workdone is nothing but potential energy stored in potential energy stored in the sum of point charges.

$$W = \frac{1}{2} \sum_{m=1}^M Q_m V_m J$$

$$W = QV \quad V = \frac{W}{Q}$$

Workdone = potential x charge.

Instead of point charges, the region has continuous charge disturbance then,

For line charge  $\rho_L = W_F = \frac{1}{2} \int \rho_L dL \cdot V J$

Surface charge  $\rho_S = W_F = \frac{1}{2} \int \rho_S ds \cdot V J$

Volumic charge  $\rho_V = W_F = \frac{1}{2} \int \rho_V dv \cdot V J$

Energy stored in terms of  $\vec{D}$  and  $\vec{E}$ .

$$W_E = \frac{1}{2} \int_{vd} \vec{D} \cdot \vec{E} dv J = \frac{1}{2} \int_{vd} \frac{D}{\epsilon_0} \cdot \vec{E} dv J$$

$$= \frac{1}{2} \int_{vd} \epsilon_0 |E|^2 dv J = \frac{1}{2} \int_{vd} \epsilon_0 \vec{E} \cdot \vec{E} dv J$$

Energy density  $= \frac{dw_E}{dv} = \frac{1}{2} D \cdot E J/m^2$

Integrating  $W_E = \int_{vd} \left[ \frac{dw_E}{dv} \right] dv$

Potential  $= \frac{w.d}{\text{unit charge}}$

Workdone is nothing but potential stored in a spec-

of the potential field in free space is given by  $V = \frac{50}{r}$ ,  $a \leq r \leq b$  (spherical). shown that  $\oint \vec{v} = 0$  for  $a < r < b$

- Find the energy stored in the region  $a < r < b$

$$(i) W_E = \frac{1}{2} \int_{\text{Vol}} \epsilon_0 |\vec{E}|^2 dV.$$

$$dV = a^2 \sin\theta d\phi d\theta d\phi \quad (\text{sc})$$

$$|\vec{E}|^2 = \left(\frac{50}{82}\right)^2 = \frac{2500}{84}$$

$$W_E = \frac{1}{2} \int_{\text{Vol}} \epsilon_0 \frac{2500}{84} a^2 \sin\theta d\phi d\theta d\phi$$

$$= 1250 \epsilon_0 \int_{x=a}^{b} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \frac{\sin\theta}{a^2} d\phi d\theta dx$$

$$= 1250 \epsilon_0 \left[ -\cos\theta \right]_0^{\pi} (\phi)_0^{2\pi} \left( -\frac{1}{a} \right)_a^b$$

$$N_F = 1.39 \times 10^7 \left[ \frac{1}{a} - \frac{1}{b} \right] J$$

Q. If  $\vec{V} = x\hat{i} + y\hat{j} + z\hat{k}$  v. find  $\vec{E}$  @ (1, 2, 3)  
and energy stored in cube of side 2m centered  
at origin.?

$$\vec{E} = -\nabla V$$

$$\vec{E} = -\left[ \frac{\partial V}{\partial x} \hat{x} + \frac{\partial V}{\partial y} \hat{y} + \frac{\partial V}{\partial z} \hat{z} \right]$$

$$\vec{E} = -[(+y)\hat{x} + (x-1)\hat{y} + z\hat{z}]$$

$$\vec{E} = - \left[ (1+z)\hat{x} + (1-y)\hat{y} + 2\hat{z} \right] \Big|_{(1, 2, 3)}$$

$$\vec{E} = -[3\hat{x} + 2\hat{z}]$$

$$\vec{E} = \underline{-3ax^2 - 2az^2} \text{ V/m.}$$

Energy stored in electric field.

$$W_E = \frac{1}{2} \int_{\text{Vol}} \epsilon_0 |\vec{E}|^2 dV.$$

$$|\vec{E}|^2 = \left( \sqrt{(x+y)^2 + (x-1)^2 + z^2} \right)^2$$

$$= (1+y)^2 + (x-1)^2 + z^2$$

$$= y^2 + 2y + 1 + x^2 - 2x + 1 + z^2$$

$$= y^2 + 2y - 2x + x^2 + 6$$

$$W_E = \frac{1}{2} \int_{x=-1}^1 \int_{y=-1}^1 \int_{z=-1}^1 \epsilon_0 (y^2 + 2y - 2x + x^2 + 6) \left[ dV = \frac{dx dy dz}{a^3} \right]$$

$$= \frac{120}{2} \int_{x=-1}^1 \int_{y=-1}^1 \left[ y^3 + 2y^2 - 2x^2 + x^3 + 6x \right] dx dy$$

$$= \frac{120}{2} \int_{x=-1}^1 \int_{y=-1}^1 \left[ y^2 + 2y - 2x + x^2 + 6 + (y^3 + 2y^2 - 2x^2 + x^3 + 6x) \right] dx dy$$

$$= \frac{120}{2} \int_{x=-1}^1 \int_{y=-1}^1 \left[ 2y^2 + 4y - 4x + 2x^2 + 12 \right] dx dy$$

$$= \frac{120}{2} \int_{x=-1}^1 \left[ 2\frac{y^3}{3} + 4\frac{y^2}{2} - 4xy + 2x^2 y + 12x \right] dx$$

$$\begin{aligned}
 &= \frac{\epsilon_0}{2} \int_{-1}^1 \left[ \frac{2}{3} + \frac{4}{2} - 4x + 2x^2 + 12 \right] dx \\
 &\quad \left[ \frac{2}{3} + \frac{4}{2} - 4x + 2x^2 + 12 \right] \\
 &= \frac{\epsilon_0}{2} \int_{-1}^1 \left[ \frac{4}{3} + 4 - 8x + 4x^2 + 24 \right] dx \\
 &= \frac{\epsilon_0}{2} \left\{ \left[ \frac{8x}{3} - 8x + 4x^2 \right] \right\}_{-1}^1 \\
 &= \frac{\epsilon_0}{2} \left[ \frac{88}{3} - 8 \frac{x^2}{2} + 4 \frac{x^3}{3} \right]_{-1}^1 \\
 &= \frac{\epsilon_0}{2} \left[ \frac{88}{3} - \frac{8}{2} + \frac{4}{3} + \frac{88}{3} - \frac{8}{2} + \frac{4}{3} \right] \\
 &= \frac{\epsilon_0}{2} \left[ \frac{160}{3} \right] = 8.85 \times 10^{-12} \times 26.66 = \underline{\underline{2.36 \times 10^{-9}}} \\
 &\quad = \underline{\underline{2.36 \times 10^{-9} \text{ aF}}}.
 \end{aligned}$$

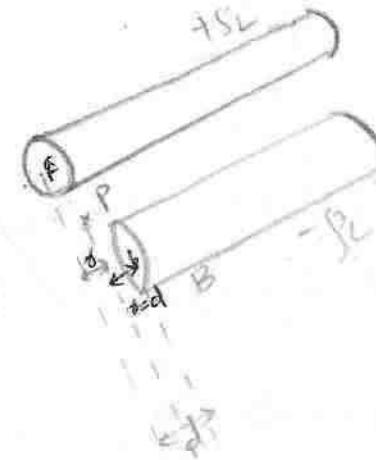
Capacitance of a two wire transmission line.

2 conductors A & B. at a distance  $d'$ .

Radius  $\rightarrow a$ . charge density on A  $\rightarrow \rho_L$

charge density on B  $\rightarrow \rho_L$

Total  $\vec{E}$  at point p is  $\vec{E}_1 + \vec{E}_2$ ,  $\vec{E} = \vec{E}_1 + \vec{E}_2$



The electric field intensity  $E$  of a line charge is

$$\frac{P_L}{2\pi\epsilon_0} a^2 : E = \frac{P_L}{2\pi\epsilon_0} a^2 + \frac{P_L}{2\pi\epsilon_0} a^2 \\
 = \frac{P_L}{2\pi\epsilon_0} \left[ \frac{1}{a} + \frac{1}{(d-a)} \right] a^2.$$

$$V = - \int \epsilon_0^2 dl = \int_{d-a}^a \frac{P_L}{2\pi\epsilon_0} \left[ \frac{1}{a} + \frac{1}{(d-a)} \right] a^2 da$$

$$= \frac{P_L}{2\pi\epsilon_0} \int_{d-a}^a \frac{1}{a} \frac{1}{(d-a)} da = \frac{P_L}{2\pi\epsilon_0} \left[ \ln a - \ln(d-a) \right]_{d-a}^a$$

$$V = - \frac{P_L}{2\pi\epsilon_0} \left[ \ln a - \ln(d-a) \right] - \ln(a-d) - \ln a$$

$$= - \frac{P_L}{2\pi\epsilon_0} \left[ \ln \left[ \frac{a}{d-a} \right] - \ln \left[ \frac{d-a}{a} \right] \right]$$

$$V = - \frac{P_L}{2\pi\epsilon_0} \ln \left[ \frac{a}{d-a} \right] - \frac{P_L}{2\pi\epsilon_0} \ln \left[ \frac{d-a}{a} \right]$$

$$C = \frac{Q}{V} = \frac{\rho_L}{\frac{2\pi\epsilon_0}{\pi\epsilon_0} \ln \left( \frac{d-a}{a} \right)} = \frac{\omega}{\frac{1}{\pi\epsilon_0} \ln \left( \frac{d-a}{a} \right)}$$

$$C = \frac{\pi\epsilon_0 L}{\ln \left( \frac{d-a}{a} \right)}$$

radii of both wires are  $A$ . The charge density on A is  $+g_L$  and that of B is  $-g_L$

### Current and current density

$$I = \frac{dQ}{dt} \quad A.$$

Current density  $\vec{J}$

- denoted by  $\vec{J}$  (vector quantity)
- Unit:  $A/m^2$
- $I = \oint_S \vec{J} \cdot d\vec{s}$  → Relation b/w  $\vec{J}$  &  $I$ .
- $\vec{J} = \int_V \vec{v} dv \rightarrow$  Relation b/w  $\vec{J}$  &  $\vec{v}$

### Continuity equation

Consider a closed surface 'S' with it density  $\vec{J}$

Total current  $I = \oint_S \vec{J} \cdot d\vec{s}$  (outward flow).

- According to Law of conservation of charge Total outward flux is compensated by reducing the the charge inside.

$$\frac{-dQ}{dt} \rightarrow \text{rate of decrease of charge (decreasing so -ve)}$$

$$\oint_S \vec{J} \cdot d\vec{s} = -\frac{dQ}{dt} = I \rightarrow \text{integral form.}$$

$$\oint_S A \cdot d\vec{s} = \int_V \nabla \cdot \vec{A} dv.$$

Applying divergence theorem

$$\oint_S \vec{J} \cdot d\vec{s} = \int_V \nabla \cdot \vec{J} dv.$$

$$\oint_S \vec{J} \cdot d\vec{s} = \frac{-dQ}{dt} \quad Q = \int_V \rho_v dv.$$

$$\int_V \nabla \cdot \vec{J} dv = \frac{d}{dt} \int_V \rho_v dv.$$

$$\boxed{\nabla \cdot \vec{J} = -\frac{d\rho_v}{dt}}$$

In point form

Flux

$$\psi = \int D \cdot d\vec{s}.$$

$$D = \psi/s \quad (C/m^2)$$

$$D = \epsilon_0 E.$$

Flux density

$$E = -\nabla V$$

$$V = \frac{1}{4\pi} \frac{q_e}{\epsilon_0 r}$$

relation b/w fields  
potentials

$$I = C \frac{dv}{dt}.$$

Energy density

$$W_F = \frac{1}{2} D E$$

$$\rho_F = \frac{E^2}{\epsilon_0}$$

Poisson's eqn

$$I = \frac{C dv}{dt}.$$

Field intensity

$$E = V/r \quad (V/m)$$

Electric

## Magnetostatics

- \*  $H = \frac{I}{l} (\text{A/m})$
- \*  $W_m = \frac{1}{2} B \cdot H$
- \*  $\nabla^2 A = -\mu J$
- \*  $B = \mu_0 H$
- \*  $H = -\nabla V_m (J=0)$
- \*  $V = \int \frac{\mu I dI}{4\pi R}$
- \*  $\psi = \int B \cdot ds$
- \*  $\psi = LI$
- \*  $\nabla \psi = \mu \frac{dI}{dt}$

Q. Find the total current in a circular conductor of radius 4 mm. If the current density varies according to  $J = \frac{10^4}{r} \text{ A/m}^2$ .

$$I = \oint J \cdot ds = \iint J \cdot r dr d\phi a_z^2$$

$$= \iint \frac{10^4}{r} dr d\phi a_z^2$$

$$\begin{aligned} &= \iint J \cdot r dr d\phi a_z^2 \\ I &= \int_0^{2\pi} \int_0^{4 \times 10^{-3}} 10^4 r dr d\phi \\ &= \int_0^{2\pi} \int_0^{4 \times 10^{-3}} 10^4 dr d\phi \\ &= \int_0^{2\pi} \left[ 10^4 r \right]_0^{4 \times 10^{-3}} d\phi \\ &= 10^4 \int_0^{2\pi} \left[ 4 \times 10^{-3} \right] d\phi = 10^4 \times 4 \times 10^{-3} [\phi]_0^{2\pi} \\ &= 40^4 \times 4 \times 10^{-3} [2\pi] = \underline{\underline{80\pi}} \quad \int B \cdot ds = \psi \end{aligned}$$

Magnetostatics: Study of steady magnetic field existing in a given field space produced by flow of direct current through a conductor is called Magnetostatics.

### Biot-Savart's law

- States that magnetic field intensity  $dH$  produced at a point  $P$  due to the differential current element  $I \cdot dl$  is proportional to both of  $I$  and differential length  $dl$ . Sign of angle between element of line pointing

Q

→ Inversely proportional to the square of distance between the pt P and element

$$d\vec{H} \propto \frac{Idl}{4\pi R^2} \sin \theta A/m^2$$

$$d\vec{H} = \frac{Idl}{4\pi R^2} \hat{a} \propto A/m^2$$

Magnetic flux  $\phi$

$$\phi = \int_S \vec{B} \cdot d\vec{s}$$

$$\int_S \vec{B} \cdot d\vec{s} = 0 \quad \left. \begin{array}{l} \text{Gauss's Law} \\ \text{no magnetic field} \end{array} \right\} \text{good form in}$$

$$\nabla \cdot \vec{B} = 0 \quad \left. \begin{array}{l} \text{in point form} \\ \text{(d) differential form} \end{array} \right.$$

$$\vec{B} = \mu_0 \vec{H}$$

$$\mu_0 = 4\pi \times 10^{-7}$$

$$\vec{B} = \mu_0 \vec{H}$$

Ampere's law / Ampere's circ law.  $\rightarrow$  Related to  $\vec{B}$ .

$$\oint \vec{H} \cdot d\vec{l} = I_{\text{enclosed}}$$



$$\oint \vec{H} \cdot d\vec{l}$$

$$4\pi R^2$$

By applying Stoke's theorem.

$$\oint_S \vec{H} \cdot d\vec{n} = \int_S \nabla \times \vec{H} \cdot d\vec{s}$$

$$(1) \rightarrow \int_S \nabla \times \vec{H} \cdot d\vec{s} = I_{\text{enc}} \quad \left\{ \begin{array}{l} I = \int_C \vec{J} \cdot d\vec{l} \end{array} \right.$$

$$\int_C \vec{J} \cdot d\vec{l} = \int_S \nabla \times \vec{H} \cdot d\vec{s}$$

$$\boxed{\nabla \times \vec{H} = \vec{J}} \quad \text{Maxwell's eqn} \rightarrow \text{pt form} \quad \left\{ \begin{array}{l} \text{differential form.} \\ \int_S \vec{J} \cdot d\vec{l} \end{array} \right.$$

$$I = \oint_C \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{l} = \int_S \nabla \times \vec{H} \cdot d\vec{s}$$

$$\nabla \times \vec{H} = \vec{J} \Rightarrow \text{pt form.}$$

$$\oint \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{l} \Rightarrow \int l \text{ form.}$$

states that "the line integral of magnetic field intensity  $\vec{H}$  around a closed path is the same as the current  $I$  enclosed by the path."

Applying Stokes

→ It is used to find  $H$  when current distribution is symmet.

→ Law doesn't depend on shape of the path, but it should be closed.

In cylindrical coordinate system  $B = \left(\frac{\partial \phi}{\partial z}\right) \hat{a}_\phi$  tesla.

determine the magnetic flux  $\phi$  choosing the plane surface defined as  $0.5 \leq r \leq 2.5$  m  $0 \leq z \leq 2$  m.

$$\int_S \vec{B} \cdot d\vec{s} \quad \text{In cylindrical}$$

$$\therefore \int \vec{B} \cdot d\vec{s} dz d\phi: \quad ds = r dr d\theta dz$$

$$\Phi = \int_0^2 \int_{-0.5}^{0.5} \left( \frac{2}{\pi} a^2 \right) \cdot dr d\theta dz \quad a\phi \cdot a\theta = 1$$

$$= \int_{-0.5}^{0.5} \left[ \ln \right]_{-0.5}^{0.5} \left[ z \right]^2$$

$$[\ln 2.5 - \ln 0.5] \times 2 = 6.13 \text{ wb}$$

Q. Given that  $\vec{B} = 2.5 \left( \sin \frac{\pi x}{2} \right) e^{-ay} \hat{a}_z \text{ wb/m}^2$   
total magnetic flux  $I = 0; y \geq 0, 0 \leq x \leq 2m$ .

$$\Phi = \int_S \vec{B} \cdot d\vec{s}$$

$$= \int_0^2 \int_{y=0}^y 2.5 \left( \sin \frac{\pi x}{2} \right) e^{-ay} \hat{a}_z \cdot dr dy \quad \hat{a}_z \cdot \hat{a}_z = 1$$

$$= \int_{x=0}^2 \int_{y=0}^y 2.5 \left( \sin \frac{\pi x}{2} \right) e^{-ay} dx dy$$

$$= 2.5 \int_{x=0}^2 \left[ \sin \left( \frac{\pi x}{2} \right) \left[ \frac{e^{-ay}}{-a} \right] \right]_0^y dx$$

$$= 2.5 \int_{x=0}^2 \sin \left( \frac{\pi x}{2} \right) \times \frac{1}{-a} \times [e^{-ay} - 1] dx$$

$$= \frac{2.5}{\pi/2} \int_0^2 \sin \left( \frac{\pi x}{2} \right) dx = \frac{2.5}{\pi/2} \left[ -\cos \left( \frac{\pi x}{2} \right) \right]_0^2$$

$$= \frac{2.5}{\pi/2} \left[ -\cos(\pi) - \left[ -\cos(0) \right] \right] = 1.5915 \text{ wb}$$

$$\Phi = 1.5915 \text{ wb}$$

Consider an long straight conductor carrying current  $I$  placed along  $\vec{x}$ -axis as shown in fig.

Consider a closed circular path of radius  $a$  which encloses a conductor of carrying current  $I$ . Point  $p$  is at a  $r$  distance from conductor. Consider  $d\ell$  at point  $p$  which is in  $\hat{a}\phi$  direction tangential to circular path.

$$dl = r d\phi \hat{a}\phi$$

(P)  $\vec{H}$  at  $p$  from Bio Savart's law due to infinitely long conductor

$$\left. \vec{H} = \frac{I}{2\pi r} \hat{a}\phi \right\}$$

Proof.

$$\vec{H} \cdot d\vec{l} = \frac{I}{2\pi r} d\phi \cdot r d\phi d\vec{l}$$

$$= \frac{I}{2\pi r} \cancel{r} d\phi$$

(ad ∵ ad =)

Integrating over closed path.

$$= \oint \vec{H} \cdot d\vec{l} = \int_{\phi=0}^{2\pi} \frac{I}{2\pi r} d\phi$$

$$= \frac{I}{2\pi} \int_0^{2\pi} d\phi = \frac{I}{2\pi} [\phi]_0^{2\pi} = \frac{I}{2\pi} \times 2\pi$$

$\oint \vec{H} \cdot d\vec{l} = I \rightarrow$  current carrying the conductor

hence law is proved.

Magnetic Scalars and Vector potential

→ Magnetic potential - a types - Scalars & vector may be  
Scalar potential -  $V_m$ .  $[\vec{E} = -\nabla V]$

$$\vec{H} = -\nabla V_m \quad (1) \quad (\text{only for } J=0)$$

Taking curl on both sides.

$$(1) \nabla \times \vec{H} = -\nabla \times \nabla V_m$$

$$\nabla \times \vec{H} = 0$$

According to Ampere's law

$$\nabla \times \vec{H} = J \quad \text{comparing } J=0$$

scalar magnetic potential is defined as source free region [no current and current density].

$$\nabla^2 V_m = 0 \text{ for } \vec{J} = 0$$

Vector magnetic potential (A)

→ Unit is  $\text{wb/m}$  (denoted by  $A$ )

$$\nabla \times \vec{A} = \vec{B}$$

$$\nabla \times \vec{A} = \vec{B}$$

$$\nabla \cdot \vec{B} = 0$$

$$\text{Poisson's eqn } \nabla^2 A = \mu_0 J$$

a. In cylindrical coordinate  $A = 50r^2 a_2 \text{ wb/m}$   
is a vector magnetic potential is certain  
region of freespace and  $H, B, J$  and  
total current  $I$  closing the surface.  
 $0 \leq r \leq 1, 0 \leq \phi \leq 2\pi, z=0$ .

$$\nabla \times \vec{A} = \vec{B} =$$

$$= \left( \frac{\partial A_r}{\partial \phi} + A_\phi - \frac{\partial A_\phi}{\partial r} \right) a_r + \left[ \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right] a_\phi + \left[ \frac{\partial A_\phi}{\partial z} + \frac{\partial A_z}{\partial \phi} \right] a_z$$

$$= \left[ \frac{\partial^2 A_r}{\partial r^2} + \frac{1}{r} \frac{\partial A_r}{\partial r} + \frac{\partial^2 A_r}{\partial z^2} + \frac{\partial^2 A_r}{\partial \phi^2} \right] a_r + \left[ \frac{\partial^2 A_\phi}{\partial r^2} + \frac{1}{r} \frac{\partial A_\phi}{\partial r} + \frac{\partial^2 A_\phi}{\partial z^2} + \frac{\partial^2 A_\phi}{\partial \phi^2} \right] a_\phi + \left[ \frac{\partial^2 A_z}{\partial r^2} + \frac{1}{r} \frac{\partial A_z}{\partial r} + \frac{\partial^2 A_z}{\partial z^2} + \frac{\partial^2 A_z}{\partial \phi^2} \right] a_z$$

$$A_r = 0, A_\phi = 0, A_z = 50r^2$$

$$\boxed{\nabla \times \vec{V} = 0}$$

$$\vec{B} = \frac{1}{\mu_0} \left[ \frac{\partial}{\partial \phi} (50x^2) - 0 \right] \hat{a}_x + \left[ 0 - \frac{\partial}{\partial x} (50x^2) \right] \hat{a}_y \\ + \frac{1}{x} (0-0) \hat{a}_z$$

$$\vec{B} = -100x \hat{a}_y \text{ A/m}^2$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{-100x \hat{a}_y}{\mu_0} \text{ A/m}$$

$$\vec{J} = \nabla \times \vec{H}$$

$$H_x = 0, H_d = \frac{-100x \hat{a}_y}{\mu_0}, H_z = 0$$

$$\nabla \times \vec{H} = \left[ 0 - \frac{\partial}{\partial z} \left( \frac{-100x}{\mu_0} \right) \right] \hat{a}_x + (0-0) + \frac{1}{x} \cdot$$

$$\left[ \frac{\partial}{\partial x} \frac{-100x^2}{\mu_0} - 0 \right] \hat{a}_y$$

$$\vec{J} = -\frac{200}{\mu_0} \hat{a}_y \text{ A/m}^2$$

$$\vec{A} = 50x^2 \hat{a}_z \text{ Wbm.}$$

$$I = \int_S J \, ds$$

$$I = \int_{x=0}^{\infty} \int_{\theta=0}^{2\pi} -\frac{200}{\mu_0} \hat{a}_y \cdot \hat{a}_z \, r \, dr \, d\theta \, \hat{a}_z$$

$$= -\frac{200}{\mu_0} \left[ \frac{x^2}{2} \right]_0^{\infty} [\phi]_0^{2\pi} = -500 \text{ mA}$$

\* A flag perfectly conducting surfaces on  $xy$  plane is placed in a mag. field  $\vec{H} =$   
 $\vec{H} = 3 \cos x \hat{a}_x + 3 \cos x \hat{a}_y \text{ A/m}$  for  $z > 0$ .  
 find current density on the conductor surface?

Ans:

$$\vec{J} = \nabla \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3 \cos x & 3 \cos x & 0 \end{vmatrix}$$

$$= \left( \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) \hat{a}_x - \left( \frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} \right) \hat{a}_y + \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \hat{a}_z$$

$$f = -\cos x \hat{a}_x - 3 \sin x \hat{a}_z$$

$$= \left( \frac{\partial}{\partial y} x_0 - \frac{\partial z}{\partial z} \cos x \right) \hat{a}_x - \left( \frac{\partial x_0}{\partial x} - \frac{\partial z}{\partial z} \right) \hat{a}_y + \left( \frac{\partial z}{\partial x} \cos x - \frac{\partial x}{\partial y} \sin x \right) \hat{a}_z$$

$$= (\alpha - \cos x) \hat{a}_x - (0-0) + (3 \sin x - 0) \hat{a}_z$$

$$= -\cos x \hat{a}_x - \underline{3 \sin x \hat{a}_z}$$

magnetic flux  $\phi = \int_S \vec{B} \cdot d\vec{s}$  wbs.

$\oint_S \vec{B} \cdot d\vec{s} = 0 \rightarrow$  gauss law in integral form

law of conservation of magnetic flux

$\nabla \cdot \vec{B} = 0 \rightarrow$  gauss's law in differential form.

$$\vec{B} = \mu_0 H \quad * \quad \omega_m = \frac{1}{2} \int B H dV$$

$\mu_0 = 4\pi \times 10^{-7}$

$\vec{B} = \mu H$  per unit flux density.  $\vec{B} = \mu H$  intensity

$$\rightarrow \oint \vec{H} \cdot d\vec{l} = I_{enc}$$
 (ampere's law).

$$\rightarrow \nabla \times \vec{H} = \vec{J} \rightarrow$$
 Ampere's law in diff form.

$$\rightarrow V_m = - \int \vec{H} \cdot d\vec{l}$$
 scalar magnetic potential.

$$\rightarrow \vec{H} = -\nabla V_m$$

$$\rightarrow$$
 Energy stored in magnetic field  $U_m = \frac{1}{2} \int M H^2 dV$ .

$$\rightarrow \vec{H}$$
 due to infinite line charge  $\vec{H} = \frac{I}{2\pi r} a^\phi \hat{\phi}$

$$\rightarrow \vec{H}$$
 due to infinite sheet of current  $= \vec{H} = \frac{1}{2} \vec{k} \times a^\phi$

$$\rightarrow \vec{A}$$
 due to the diff. current  $\vec{A} = \frac{\mu_0 I d\sigma}{4\pi R} \hat{\phi}$  wh/m

$\vec{k}$  = infinite sheet of current density

$$\rightarrow \vec{A}$$
 due to the distributed of source  $\vec{A} = \oint_S \frac{\mu_0 \vec{k} \cdot d\vec{s}}{4\pi R} \hat{\phi}$  wh/m

$$\rightarrow \vec{A}$$
 due to the distribution of volume  $\vec{A} = \oint_{vol} \frac{\mu_0 J dV}{4\pi a} \hat{\phi}$  wh/m

$$\rightarrow$$
 Inductance of co-axial cable  $L = \frac{\mu_0}{2\pi} \ln(b/a) \cdot R$

$$\rightarrow$$
 Conductance of a  $\lambda$  wave tuner  $L = \frac{1}{2} \ln(b/a)$   $\Omega$ 

$$G = \frac{M}{\pi} [1/4 + \ln(1/2)] R$$

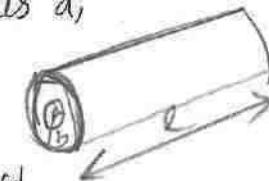
$$\nabla \times \vec{H} = \vec{J} \rightarrow$$
 Amp undiff form.

$$V_m = - \int \vec{H} \cdot d\vec{l}$$
 scalar magnetic potential

$$\vec{H} = -\nabla V_m$$

### Inductance of co-axial cable

Co-axial cable with inner radius 'a', outer conductor radius 'b'.



Let Current through Co-axial cable be 'I'.

The field intensity at any point b/w the inner & outer conductor is

$$H = \frac{I}{2\pi r} a^\phi \text{ across}$$

$$(CCS) \quad \vec{B} = \frac{\mu I}{2\pi r} a^\phi$$

Total flux linkage b/w A & B per unit length is,

$$\Phi = \int_S \vec{B} \cdot d\vec{s}$$
 (not closed surface so not zero).

$$\Phi = \int_S \vec{B} \cdot d\vec{s} = \int_a^b \int_0^l \frac{\mu I}{2\pi r} a^\phi d\sigma da^\phi \cdot \left[ ds = dr da^\phi \right]$$

$$= \int_a^b \frac{\mu I}{2\pi r} \left[ \frac{1}{2} r^2 \right]_0^l da^\phi$$

$$= \int_a^b \frac{\mu I l}{2\pi} \left[ \frac{1}{2} (b^2 - a^2) \right] da^\phi$$

$$= \frac{\mu I l}{2\pi} \left[ \ln(b/a) \right]$$

$$L = \frac{\mu I l}{2\pi} \ln(b/a)$$

$$\text{Inductance } L = \frac{\Phi}{I} = \frac{\mu I l}{2\pi} \ln(b/a)$$

for unit length coaxial cable

$$L = \frac{\mu}{2\pi} \ln\left(\frac{b}{a}\right)$$

Henry

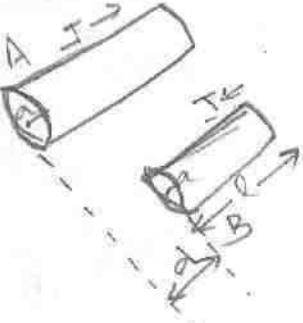
Inductance of a 2 wire transmission line

2 wires A & B separated by a distance 'd'

A carries a current I and B

carries -I

$$\phi_1 = \frac{\mu I d l}{8\pi}$$



flux leakage b/w conductors is given as, (according)

$$\phi_2 = \int_{r=a}^{d-a} \int_{z=0}^l \frac{\mu I}{2\pi r z} dz dr$$

$$= \frac{\mu I}{2\pi} \int_{a=0}^{d-a} \frac{1}{2} \left[ \ln z \right]_0^l dr$$

$$= \frac{\mu I}{2\pi} \int_a^{d-a} (l - 0) dr$$

$$= \frac{\mu I l}{2\pi} \left[ \ln r \right]_a^{d-a} = \frac{\mu I l}{2\pi} \ln\left(\frac{d-a}{a}\right) \quad \textcircled{2}$$

Total flux leakage produced by A  $\phi_1 + \phi_2$

$$\phi_1 + \phi_2 = \frac{\mu I l}{8\pi} + \frac{\mu I l}{2\pi} \cdot l \cdot \left( \ln\left(\frac{d-a}{a}\right) \right)$$

$$= \frac{\mu I l}{2\pi} \left[ \frac{1}{4} + \ln\left(\frac{d-a}{a}\right) \right]$$

By symmetry, some amount of flux will be produced by conductor B, hence total flux leakage produced by 2 conductors S.A & B.

$$\therefore \phi = \alpha [\phi_1 + \phi_2]$$

$$\phi = \frac{\mu \pm l}{\pi} \left[ \frac{1}{4} + \ln\left(\frac{d-a}{a}\right) \right]$$

$\omega$  is given by

$$\omega = \frac{\phi}{I} = \frac{\mu l}{\pi} \left[ \frac{1}{4} + \ln\left(\frac{d-a}{a}\right) \right]$$

usually  $d \gg a \rightarrow$

$$\omega = \frac{\mu l}{\pi} \left[ \frac{1}{4} + \ln\left(\frac{l}{a}\right) \right]$$

$$\omega = \frac{\mu l}{\pi} \left[ \frac{1}{4} + \ln(d/a) \right] \quad \text{[For unit length (l=1)]}$$

Maxwell's equations in phasor form

$$\nabla \times \vec{E} = -j\omega \mu \vec{H}$$

$$\nabla \times \vec{H} = \vec{J} + j\omega \epsilon \vec{E}$$

$$\nabla \cdot \vec{H} = 0$$

$$\nabla \times \vec{E} = \rho/\epsilon$$

Time harmonics.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -j\omega \vec{B} = -j\omega \mu \vec{H}$$

$$\nabla \times \vec{H} = \vec{J} + j\omega \epsilon \vec{E}$$

$$\nabla \cdot \vec{H} = 0$$

$$\nabla \times \vec{E} = \rho/\epsilon$$

Relation  $\propto \vec{E}, \vec{v}, \vec{A}$ .

Maxwell's eqn from faraday's law.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{--- (1)}$$

$$\vec{B} = \nabla \times \vec{A}$$

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t} (\nabla \times \vec{A}) \quad \text{--- (2)}$$

$$\nabla \times \vec{E} + \frac{\partial}{\partial t} (\nabla \times \vec{A}) = 0$$

$$\nabla \times [\vec{E} + \frac{\partial \vec{A}}{\partial t}] = 0$$

If  $\vec{E}$  is curl free, so the expression in field parameters is considered as gradient of scalar.

$$\vec{E} = -\nabla V - \vec{A} \quad (3)$$

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t} \quad \text{V/m.}$$

## Module 2

Maxwell's equation form fundamentals.

proto form Differential form	Integrated form	Significance
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint_C \vec{E} \cdot d\vec{s} = -\frac{\partial \vec{B}}{\partial t}$	Faraday's law
$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{B}}{\partial t}$	$\oint_C \vec{H} \cdot d\vec{l} = I + \int_C \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$	Ampere's law.
$\nabla \cdot \vec{D} = \rho_v$	$\oint_S \vec{D} \cdot d\vec{s} = Q$	Gauss's law for electric field (Law of conservation of charge)
$\nabla \cdot \vec{B} = 0$	$\oint_S \vec{B} \cdot d\vec{s} = 0$	Gauss's law for magnetic field.

Displacement current density & conduction current density.

Maxwell eqn are set of 4 eqn which are derived from faraday's law, ampere, gauss's law is faraday & gauss's law of magnetism.

Conduction current density ( $\vec{J}_C$ ) Displacement current density ( $\vec{J}_D$ )

- Current passes through a conductor or a resistor.
- denoted by  $I_C$  | If  $\rho$  - resistivity
- $I_C = V/R = \frac{\vec{E} \cdot l}{\rho A} = \frac{\vec{E} \cdot A}{\rho}$

$$\frac{I_C}{A} = \frac{\vec{E}}{\rho}$$

$$[\vec{J}_C = \vec{E}/\rho]$$

Conduction current density,

- Current passes through a capacitor when an AC voltage is applied across it.
- denoted by  $I_D$ .
- also called convection current density.

$$I_D = \frac{dQ}{dt} = \frac{d}{dt} (CV) = C \frac{dV}{dt} = C \frac{dE}{dt}$$

$$I_D = \frac{\epsilon_0 A}{d} \cdot \frac{d\vec{E}}{dt} \cdot \frac{d}{A} = \epsilon_0 A \frac{d\vec{E}}{dt}$$

$$\frac{I_D}{A} = \frac{dE}{dt} \cdot \frac{d}{dt} \left[ \frac{\vec{J}_D}{\vec{E}} \right] = \frac{d\vec{J}_D}{dt}$$