

Module 2

Maxwell's equations in phasor form

$$\nabla \times \vec{E} = -j\omega \mu \vec{H}$$

$$\nabla \times \vec{H} = \vec{J} + j\omega \epsilon \vec{E}$$

$$\nabla \cdot \vec{H} = 0$$

$$\nabla \times \vec{E} = \rho/\epsilon$$

Time harmonics.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -j\omega \vec{B} = -j\omega \mu \vec{H}$$

$$\nabla \times \vec{H} = \vec{J} + j\omega \epsilon \vec{E}$$

$$\nabla \cdot \vec{H} = 0$$

$$\nabla \times \vec{E} = \rho/\epsilon$$

Relation $\propto \vec{E}, \nu, \vec{A}$.

Maxwell's eqn from faraday's law.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{--- (1)}$$

$$\vec{B} = \nabla \times \vec{A}$$

$$\nabla \times \vec{A} = -\frac{\partial}{\partial t} (\nabla \times \vec{A}) \quad \text{--- (2)}$$

$$\nabla \times \vec{E} + \frac{\partial}{\partial t} (\nabla \times \vec{A}) = 0$$

$$\nabla \times \left[\vec{E} + \frac{\partial \vec{A}}{\partial t} \right] = 0$$

If \vec{E} is curl free, so the expression in field parameters is considered as gradient of scalar.

$$\vec{E} = -\nabla V - (\text{3})$$

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t} \quad \text{v/m.}$$

Maxwell's equation form fundamentals.

proto form Differential form	Integral form	Significance.
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint_C \vec{E} \cdot d\vec{s} = -\frac{\partial \vec{B}}{\partial t}$	Faraday's law
$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{B}}{\partial t}$	$\oint_C \vec{H} \cdot d\vec{l} = I + \int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$	Ampere's law. (Law of force of charge).
$\nabla \cdot \vec{D} = \rho_v$	$\oint_S \vec{D} \cdot d\vec{s} = Q$	Gauss's law for electric field.
$\nabla \cdot \vec{B} = 0$	$\oint_S \vec{B} \cdot d\vec{s} = 0$	Gauss's law for magnetic field.

Displacement current density & Conduction current density.

Maxwell eqn are set of 4 eqn which are derived from faraday's law, amperes, gauss's law is faraday & gauss's law of magnetic.

Conduction current density (\vec{J}_c) Displacement current density (\vec{J}_D)

- Current passes through a conductor or a resistor.
- denoted by I_c | μ_0 -resistivity
- $I_c = V/R = \frac{\vec{E} \cdot l}{\rho l} = \frac{\vec{E} \cdot A}{\rho}$

$$\frac{I_c}{A} = \frac{\vec{E}}{\rho}$$

$$[\vec{J}_c = \vec{E}/\rho]$$

Conduction current density,

- Current passes through a capacitor when an AC voltage is applied across it.
- denoted by I_D .
- also called convection current density.

$$I_D = \frac{d\phi}{dt} = \frac{d}{dt} \left(\epsilon_0 \frac{A}{2\pi r} \int_{r_1}^{r_2} E \cdot dr \right) = \epsilon_0 A \frac{dE}{dr}$$

$$\frac{I_D}{A} = \frac{dE}{dr} \quad \boxed{[\vec{J}_D = \frac{d\vec{E}}{dr}]}$$

Total current density (modification of amperes law).

According to amperes law.

$$\nabla \times \vec{H} = \vec{J} \quad \text{--- ①}$$

taking divergence on both side

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J} \quad \text{--- ②}$$

(divergence of curl of a vector is zero)

$$\nabla \cdot \vec{J} = 0 \quad \text{--- ③}$$

$$\text{from continuity equation} \rightarrow \nabla \cdot \vec{J} = - \frac{\partial \rho_v}{\partial t} \quad \text{--- ④}$$

displacement current density $\rightarrow \vec{J}_D$
from eq ③.

$$\nabla \times \vec{H} = \vec{J} + \vec{J}_D$$

taking divergence.

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot (\vec{J} + \vec{J}_D) \quad \text{from eq ③.}$$

$$\nabla \cdot \vec{J} + \nabla \cdot \vec{J}_D = 0.$$

$$\nabla \cdot \vec{J}_D = \frac{\partial \rho_v}{\partial t} \quad \text{--- ⑤.}$$

According to gauss's law.

$$\nabla \cdot \vec{D} = \rho_v$$

$$④ \quad \nabla \cdot \vec{J}_D = \frac{\partial \nabla \cdot \vec{D}}{\partial t} \quad \text{--- ⑥.}$$

comparing ④ & ⑥

$$\vec{J}_D = \frac{\partial \vec{D}}{\partial t}$$

then eq ① b/w.

$$\nabla \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J}_c + \vec{J}_D$$

$$\vec{J} = \vec{J}_c + \vec{J}_D$$

\vec{J} = total current density.

\vec{J}_c = conduction current density.

\vec{J}_D = displacement ".

Maxwell's equation from gauss's law of E (law of C.E.)

from gauss's law.

$$\nabla \cdot \vec{D} = \rho_v$$

divergence = $\oint_S \vec{D} \cdot d\vec{s}$

$$\text{gauss's law } \oint_S \vec{D} \cdot d\vec{s} = Q.$$

Apply divergence theorem.

$$\oint_S \vec{D} \cdot d\vec{s} = \int_V \nabla \cdot \vec{D} dv.$$

$Q = \int_V \rho_v dv$

$$\int_V \nabla \cdot \vec{D} dv = Q. \quad \text{to compare left & right } Q \text{ is changes to volume term}$$

$$\int_V \nabla \cdot \vec{D} dv = \int_V \rho_v dv$$

$$\nabla \cdot \vec{D} = \rho_v \quad \text{--- differential or pt form}$$

$$\oint_S \vec{D} \cdot d\vec{s} = Q \quad \text{--- integral form}$$

ii) Maxwell's equation from gauss's law of magnetic field.

$$\nabla \cdot \vec{B} = 0$$

$$\text{gauss's law } \oint_C \vec{B} \cdot d\vec{s} = 0$$

apply divergence theorem.

$$\oint_S \vec{B} \cdot d\vec{s} = \int_V \nabla \cdot \vec{B} \cdot dv = 0$$

$dV \neq 0$

$$\nabla \cdot \vec{B} = 0 \quad \text{pt form}$$

$$\oint_S \vec{B} \cdot d\vec{s} = 0 \quad \text{integral form}$$

1) Maxwell's equation from faraday's law.

faraday's law states that induced emf is only closed surface is proportional to the rate of change of magnetic flux enclosed by the closed path with the path.

$$\text{Faraday's law emf} = -N \frac{\partial \phi}{\partial t}$$

N = no. of turns in
d = induced emf.

$$\text{If } N=1 \text{ Induced emf} = -\frac{\partial \phi}{\partial t}$$

Proof

$$\text{As per faraday's law, induced emf} = -N \frac{\partial \phi}{\partial t} \quad \text{--- (1)}$$

We knew that potential = $\int_L \vec{E} \cdot d\vec{l}$.

$$-\frac{N \partial \phi}{\partial t} = \int_L \vec{E} \cdot d\vec{l} \quad \text{--- (2)}$$

N=1

$$-\frac{\partial \phi}{\partial t} = \int_L \vec{E} \cdot d\vec{l}$$

$$\text{But } \phi = \oint_S \vec{B} \cdot d\vec{s}$$

$$-\frac{d}{dt} \oint_S \vec{B} \cdot d\vec{s} = \int_L \vec{E} \cdot d\vec{l}$$

$$\boxed{\int_L \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \oint_S \vec{B} \cdot d\vec{s}} \quad \rightarrow \text{ integral form}$$

$$\phi = \vec{B} \cdot d\vec{s}$$

applying stokes theorem

$$\int_L \vec{E} \cdot d\vec{l} = \int_S \nabla \times \vec{E} \cdot d\vec{s}$$

$$\textcircled{3} \quad \int_S \nabla \times \vec{E} \cdot d\vec{s} = \frac{d}{dt} \int_S \vec{B} \cdot d\vec{s} = \int_S \frac{d}{dt} \vec{B} \cdot d\vec{s}$$

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \quad \rightarrow \text{ pot form.}$$

2) Maxwell's equations from amperes law.

According to amperes law.

$$\oint \vec{H} \cdot d\vec{l} = I_{\text{enc}} \quad \text{--- (1)}$$

we know.

$$I = \int_S \vec{J} \cdot d\vec{s} \quad \text{--- (2)}$$

$$\oint \vec{H} \cdot d\vec{l} = \int_S \vec{J} \cdot d\vec{s} \quad \text{--- (3)}$$

taking displacement current density into account.

$$\textcircled{3} \rightarrow \oint \vec{H} \cdot d\vec{l} = \int_S (\vec{J}_F + \vec{J}_D) \cdot d\vec{s}$$

$$\boxed{\oint \vec{H} \cdot d\vec{l} = \int_S \vec{T} + \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}} \rightarrow \text{ integral form.}$$

apply stokes theorem.

$$\oint_L \vec{H} \cdot d\vec{l} = \int_S \nabla \times \vec{H} \cdot d\vec{s}$$

$$\int_S \nabla \times \vec{H} \cdot d\vec{s} = \int_S \vec{T} + \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}$$

$$\boxed{\nabla \times \vec{H} = \vec{T} + \frac{\partial \vec{D}}{\partial t}}$$

On free space.

charge density $\rho = 0$ & $\epsilon = 0$

Maxwell's equation for free space ($\sigma = 0$) dielectrics.

for free space is a conducting medium.

so $\sigma = 0$, also no charge can exist ($\rho = 0$).

$\rho \neq 0 \rightarrow$ for free space.

Pnt form

$$(i) \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$(ii) \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \quad \left\{ \begin{array}{l} T = \sigma \vec{E} \text{ here } \sigma = 0 \\ \therefore \vec{T} = 0 \end{array} \right.$$

$$(iii) \nabla \cdot \vec{D} = 0 \quad \left\{ \nabla \cdot \vec{B} = \rho_v \text{ here } \rho_v = 0 \right.$$

$$(iv) \nabla \cdot \vec{B} = 0$$

Integral form.

$$(i) \oint \vec{E} \cdot d\vec{l} = - \int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}.$$

$$(ii) \oint \vec{H} \cdot d\vec{l} = \int_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}$$

$$(iii) \oint \nabla \cdot \vec{D} \cdot d\vec{s} = 0$$

$$(iv) \oint \vec{B} \cdot d\vec{s} = 0$$

Maxwell's eqn for good conductor.

for conducting medium $\sigma \gg \omega$ (current is negligible)

so $\vec{T} \gg \vec{J}_0$ & $\rho_v = 0$ i.e., charge inside the conductor is zero.

Point form

$$i) \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$ii) \nabla \times \vec{H} = \vec{J}$$

$$iii) \nabla \cdot \vec{D} = 0$$

$$iv) \nabla \cdot \vec{B} = 0$$

Integral form

$$i) \oint \vec{E} \cdot d\vec{l} = - \int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

$$ii) \oint \vec{H} \cdot d\vec{l} = I = \int_S \vec{J} \cdot d\vec{s}$$

$$iii) \oint \vec{D} \cdot d\vec{s} = 0$$

$$iv) \oint \vec{B} \cdot d\vec{s} = 0$$

- a) In the magnetic field $\vec{H} = 3x \cos \theta + 6y \sin \theta \hat{a}^z$.
 find current density \vec{J} if fields are invariant
 with time.

$$\frac{\partial \vec{D}}{\partial t} = 0 \quad (\text{field is time invariant})$$

Ans.

$$6 \sin \theta \hat{a}^x + 6 \cos \theta \hat{a}^y$$

$$\vec{H} = 3x \cos \theta + 6y \sin \theta \hat{a}^z$$

Maxwell's eqn from amperes law.

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

fields are invariant with time ; $\frac{\partial \vec{D}}{\partial t} = 0$, $\nabla \times \vec{H} = \vec{J}$.

$$\vec{J} = \begin{bmatrix} a_x^1 & a_y^1 & a_z^1 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{bmatrix} = a_x^1 \left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right] - a_y^1 \left[\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} \right] + a_z^1 \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right]$$

$$J = a_x [6 \sin \theta]$$

$$H = 3x \cos \theta + 6y \sin \theta \hat{a}^z$$

$$a_x (6 \sin \theta \hat{a}^z - 0) - a_y (6 \sin \theta \hat{a}^z - 0) - a_z (3x \cos \theta)$$

$$= 6 \sin \theta \hat{a}^z - 3 \cos \theta \hat{a}^y$$

$$Q_1 \quad \vec{D} = 10x\hat{a}_x - 4y\hat{a}_y + k_2\hat{a}_z \text{ nC/m}^2$$

$$\vec{B} = 2y\hat{a}_z \text{ MT}$$

$$k=0; \sigma=0, \vec{f}_V=0.$$

$$\nabla \cdot \vec{D} = P_V.$$

$$\nabla \cdot \vec{D} = 0.$$

$$\frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z = 0.$$

$$\frac{\partial}{\partial x} (10x) + \frac{\partial}{\partial y} (-4y) + \frac{\partial}{\partial z} (k_2) = 0.$$

$$(10t+4) + K = 0$$

$$K = \underline{-6}$$

Q Show that the displacement current through a capacitor is same as the conduction current I_c where $B \propto V = V_m \sin \omega t$.

$$I_D = I_C ?$$

$$V = V_m \sin \omega t$$

$$I_D = \frac{dQ}{dt} = \frac{d}{dt} CV = C \frac{d}{dt} V_m \sin \omega t$$

$$I_D = \underline{C V_m \omega \cos \omega t}$$

$$\frac{I_d}{A} = \frac{\partial}{\partial t} \epsilon \vec{E} = A \frac{\partial}{\partial t} \epsilon \vec{E} = A E \frac{d}{dt} (\nu/d)$$

$$= \frac{A E V_m \omega}{A} \sin \omega t = \frac{A E V_m \omega \cos \omega t}{d}$$

$$I_D = \underline{C \omega V_m \cos \omega t}$$

$$I_c = I_D$$

$$Q. \text{ Given } \vec{E} = E_m \sin(\omega t - \beta_2) \hat{a}_y.$$

$$\text{Find } \vec{D}, \vec{B} \text{ & } \vec{H} ?$$

$$\vec{E} = E_m \sin(\omega t - \beta_2) \hat{a}_y$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -\frac{dB}{dt}$$

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_m \sin(\omega t - \beta_2) & 0 \end{vmatrix} = -\frac{dB}{dt}$$

$$\hat{a}_x \left(-\frac{\partial}{\partial z} (E_m \sin(\omega t - \beta_2)) \right) \hat{y} [0] + \hat{a}_z \left[\frac{\partial}{\partial x} (E_m \sin(\omega t - \beta_2)) \right]$$

$$\hat{a}_x \left(-\frac{\partial}{\partial z} (E_m \sin(\omega t - \beta_2)) \right) \hat{y} [0] + \hat{a}_z \left[\frac{\partial}{\partial x} (E_m \sin(\omega t - \beta_2)) \right] = -\frac{dB}{dt}$$

$$\beta E_m \cos(\omega t - \beta_2) \hat{a}_x = \frac{dB}{dt}$$

$$\int \beta E_m \cos(\omega t - \beta_2) \hat{a}_x dt = \int dB$$

$$B = -\frac{E_m B}{\omega} \left[\sin(\omega t - B_0) \right] \hat{a}_z$$

$$H = -\frac{E_m B}{\omega \mu_0} \left[\sin(\omega t - B_0) \right] \hat{a}_x$$

Boundary condition from fundamental laws

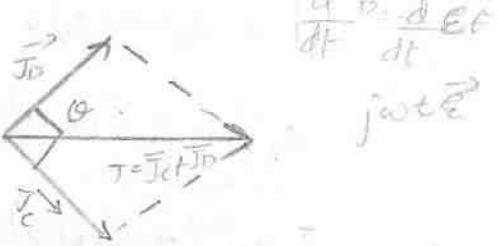
In electromagnetic materials are classified into.

1. Conductor
2. Semiconductor
3. Insulator (dielectric)

→ Dividing line is called \Rightarrow Tangent loss tangent.

$$\left| \frac{\vec{J}_c}{\vec{J}_d} \right| = \left| \frac{\sigma \vec{E}}{j \omega \epsilon \vec{E}} \right| = \left| \frac{\sigma}{j \omega \epsilon} \right| = \frac{\sigma}{\omega \epsilon}$$

$$\text{Tang} \theta = \frac{\sigma}{\omega \epsilon}$$



the conduction
current density and displacement current density are equal

- Q. Calculate the frequency at which the conduction current density and displacement current density are equal
in medium $\sigma = 2 \times 10^5 \text{ S/m}$ $\epsilon_r = 80$

$$\vec{J}_c = \vec{J}_d$$

$$\therefore \text{Tang} \theta = \frac{\vec{J}_c}{\vec{J}_d} = 1$$

$$\therefore \text{Tang} \theta = \frac{\sigma}{\omega \epsilon}$$

$$1 = \frac{\sigma}{2\pi f \epsilon}$$

$$\epsilon = \epsilon_0 \epsilon_r$$

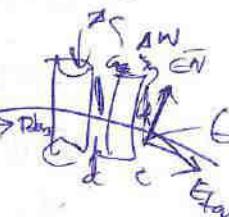
$$f = \frac{\sigma}{2\pi \epsilon} = \frac{2 \times 10^5}{2\pi \times 80 \times 8.85 \times 10^{-12}} = 4.49 \times 10^{12} \text{ Hz}$$

If $\sigma \neq 0$ there will be three cases.

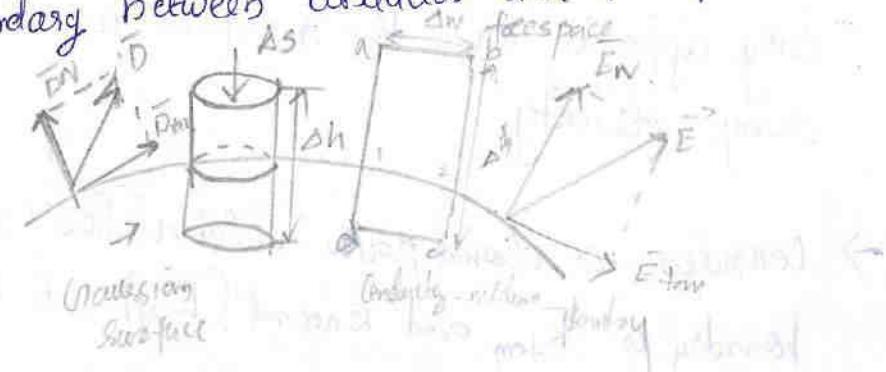
* $\frac{\sigma}{\omega \epsilon} \ll 1$ ie $\sigma \gg \omega \epsilon \Rightarrow$ Dielectric

* $\frac{\sigma}{\omega \epsilon} \approx 1$ ie $\sigma \sim \omega \epsilon \Rightarrow$ Semiconductor

* $\frac{\sigma}{\omega \epsilon} \gg 1$ ie $\sigma \ll \omega \epsilon \Rightarrow$ Conductor



Boundary between conductor and free space.



E_N, D_N

E_{tan}, D_N

Inside the conduct.

\vec{E} electric field
 D charge density.
 f charge density.

$f_s \rightarrow$ surface charge density.

The conditions exist at the boundary of a medium when field passes from one medium to another.
 Two spaces b/w conductor and free space and b/w 2 dielectrics with different properties.

i. boundary condition b/w cond^t and f.s.

→ Consider a boundary b/w cond and D.E. if free space is considered as dielectric.

Conductor is idle hence having infinite conductivity.
 Field intensity \vec{E} , D^{app} inside the conductor is 0.
 But the charge No charge exist within conductor,
 only appears on the surface as surface charge density.

fig - 1

→ Consider a closed path electric field intensity at boundary is E_{tan} and $E_{normal} (E_N)$. E makes some angle with the boundary.

we know that $\oint \vec{E} \cdot d\vec{l} = 0$. \square

-closed cont consider a \square or closed path a-b-c-d-a as shown in figure. expression can be divided into 4 parts -

$$\int_a^b \vec{E} \cdot d\vec{l} + \int_b^c \vec{E} \cdot d\vec{l} + \int_c^d \vec{E} \cdot d\vec{l} + \int_d^a \vec{E} \cdot d\vec{l} = 0 \quad \square$$

The closed contours is placed in such way that its 2 sides AB and CD are \perp to boundary. b-c and a-d are \parallel to boundary. Also half of the surface b-d are \perp to boundary. Also half of the surface is in free space up other half is a conductor. height of the rectangle is Δh and width Δw .

so $\int_c^d \vec{E} \cdot d\vec{l} = 0$. [Because because \vec{E} inside the conductor = 0]

$$\therefore \int_a^b \vec{E} \cdot d\vec{l} + \int_b^c \vec{E} \cdot d\vec{l} + \int_d^a \vec{E} \cdot d\vec{l} = 0 \quad \square$$

Δw is very small over E so E is considered as constant.

$$\int_a^b \vec{E} \cdot d\vec{l} = E \int_a^b d\vec{l} = E \Delta w \quad | \Delta h = \Delta w$$

$$\vec{E} = \vec{E}_{tan}$$

$$\int_a^b \vec{E} \cdot d\vec{l} = E_{tan} \Delta w \quad \square$$

$b-c$ is \perp to the normal component so $E = E_N$
 E_N over Δh is constant as $\Delta h \rightarrow 0$ (small)

$$b-c \text{ is divided into } \int_b^x \vec{E} \cdot d\vec{l} + \int_x^c \vec{E} \cdot d\vec{l} \quad \left. \begin{array}{l} \vec{E} \text{ also} \\ \text{inside} \\ \text{cont} \end{array} \right\}$$

$$\int_b^x \vec{E} \cdot d\vec{l} = \int_b^x \vec{E}^N \cdot d\vec{l} = E_N (\Delta h/2) \quad \textcircled{5}$$

ie, d-a is equal to b-c w.r.t. direction

$$\int_d^a \vec{E} \cdot d\vec{l} = - E_N (\Delta h/2) \quad \textcircled{6}$$

Sub 4, 5, 6 in ③

$$E_{tan} \Delta h + E_N (\Delta h/2) + - E_N (\Delta h/2)$$

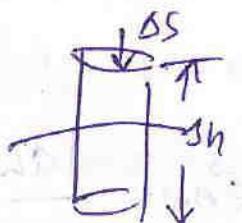
$$E_{tan} \Delta h + E_N \Delta h/2 + 0 - E_N \Delta h/2 = 0$$

$$\Delta h E_{tan} = 0$$

$$\boxed{E_{tan} = 0 \cdot}$$

$$\vec{D} = \epsilon_0 \vec{E}$$

$$\boxed{D_{tan} = 0}$$



To find normal component of D_N , select a closed glass surface in the form of circular cylinder, its in Δh , it's placed in such a way that $\Delta h/2$ is in cylinder and remaining is free space axis is normal direction. circular cylinder height Δh . gauss law,

$$\oint \vec{D} \cdot d\vec{s} = 0 \quad \textcircled{7}$$

take $\Delta h \gg 0$

$$\int_{\text{top}} \vec{D} \cdot d\vec{s} + \int_{\text{bottom}} \vec{D} \cdot d\vec{s} + \int_{\text{lateral}} \vec{D} \cdot d\vec{s} = 0$$

$$ds = 2\pi r \Delta h = 0$$

$$\int_{\text{top}} \vec{D} \cdot d\vec{s} + \int_{\text{bottom}} \vec{D} \cdot d\vec{s} = 0 \quad \textcircled{8}$$

$$\int_{\text{bottom}} \vec{D} \cdot d\vec{s} = 0$$

$$\textcircled{3} \Rightarrow \int_{\text{top}} \vec{D} \cdot d\vec{s} = 0 \quad \textcircled{9}$$

$$D_N \Delta S = 0 \quad \textcircled{10}$$

$$D_N \Delta S = P_S \quad \textcircled{11}$$

$$\boxed{D_N = P_S}$$

$$E_N = \frac{D_N}{\epsilon_0}$$

$$\boxed{E_N = \frac{P_S}{\epsilon_0}}$$

$$Q = \int_S ds = P_S \Delta S$$

$$D = \epsilon_0 E$$

$$\star E_{tan} = 0$$

$$D_{tan} = 0$$

$$D_N = P_S$$

$$E_N = P_S / \epsilon_0$$

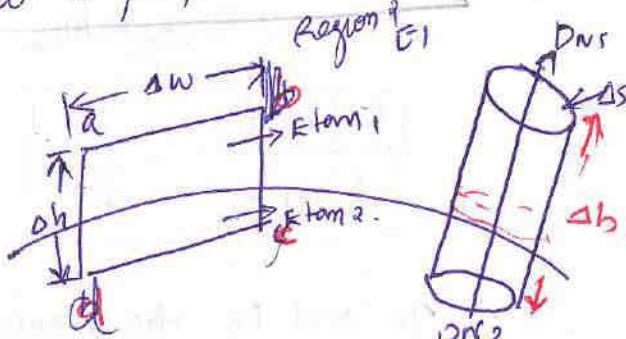
② Boundary conditions b/w 2 perfect dielectrics.

Consider boundary b/w 2 dielectrics,

one dielectric has permittivity ϵ_1 and 2nd has ϵ_2

consider a closed path abc of ϵ_2 region

rectangular shape Δh in width or way that $1/2 \ln 0$ at $1/2 \Delta h$



$$\oint \vec{E} \cdot d\vec{L} = 0 \quad \text{--- (1)}$$

a-b-c-d-a

$$\int_a^b \vec{E} \cdot d\vec{L} + \int_b^c \vec{E} \cdot d\vec{L} + \int_c^d \vec{E} \cdot d\vec{L} + \int_d^a \vec{E} \cdot d\vec{L} = 0 \quad \text{--- (2)}$$

to solve $\Delta h = 0$

$$\int_a^b \vec{E} \cdot d\vec{L} = E \tan_1 \Delta w.$$

$$\int_b^c \vec{E} \cdot d\vec{L} = E N_1 (\Delta h)$$

$$\int_c^d \vec{E} \cdot d\vec{L} = E \tan_2 \Delta w$$

$$\int_d^a \vec{E} \cdot d\vec{L} = -E N_2 \Delta h.$$

$$E \tan_1 \Delta w + E N_1 \Delta h - E \tan_2 \Delta w - E N_2 \Delta h = 0$$

$$E \tan_1 \Delta w - E \tan_2 \Delta w = 0$$

$$E \tan_1 = E \tan_2$$

$$D \tan_1 = \epsilon_1 E \tan_1$$

$$D \tan_2 = \epsilon_2 E \tan_2$$

$$\left| \frac{E \tan_1 \epsilon_1}{E \tan_2 \epsilon_2} = \frac{\epsilon_1}{\epsilon_2} \right|$$

→ To find the normal component consider a closed surface in the form of circular cylinder placed in such a way that half is dielectric 1 & other dielectric 2.

$$\oint \vec{D} \cdot d\vec{s} = Q \quad \text{--- (1)}$$

$$\int_{\text{top}} \vec{D} \cdot d\vec{s} + \int_{\text{bottom}} \vec{D} \cdot d\vec{s} + \int_{\text{lateral}} \vec{D} \cdot d\vec{s} = Q \quad \text{--- (2)}$$

assume $\Delta h = 0$

$$\int_{\text{lateral}} \vec{D} \cdot d\vec{s} = 0$$

$$(2) \Rightarrow \int_{\text{top}} \vec{D} \cdot d\vec{s} + \int_{\text{bottom}} \vec{D} \cdot d\vec{s} = Q$$

$$D N_1 \Delta S - D N_2 \Delta S = Q$$

$$D N_1 \Delta S - D N_2 \Delta S = P_S \Delta S$$

$$D N_1 - D N_2 = P_S \quad \text{--- (4)}$$

for ideal dielectric

$$P_S = 0$$

$$\boxed{D N_1 = D N_2}$$

$$\frac{D N_1}{D N_2} = \frac{\epsilon_1 E N_1}{\epsilon_2 E N_2}$$

$$\text{Hence, } D N_1 = D N_2.$$

$$1 = \frac{\epsilon_1 E N_1}{\epsilon_2 E N_2}$$

$$\boxed{\frac{E N_1}{E N_2} = \frac{\epsilon_1}{\epsilon_2}}$$

Magnetic-Boundary condition

Boundary b/w two magnetic material of two different permeability.

Boundary conditions b/w conductor & free space

$$E \tan_1 = 0 \quad E_N = P_S / \epsilon_0$$

$$D \tan_1 = 0 \quad D_N = P_S$$

Boundary condition b/w 2 dielectric

$$E \tan_1 = E \tan_2 \quad D_N = D N_2$$

$$\frac{D \tan_1}{D \tan_2} = \frac{E \tan_1}{E \tan_2} \cdot \frac{E_N}{E_N} = \frac{\epsilon_1 \sigma_1}{\epsilon_2 \sigma_2}.$$

Boundary condition b/w 2 dielectric

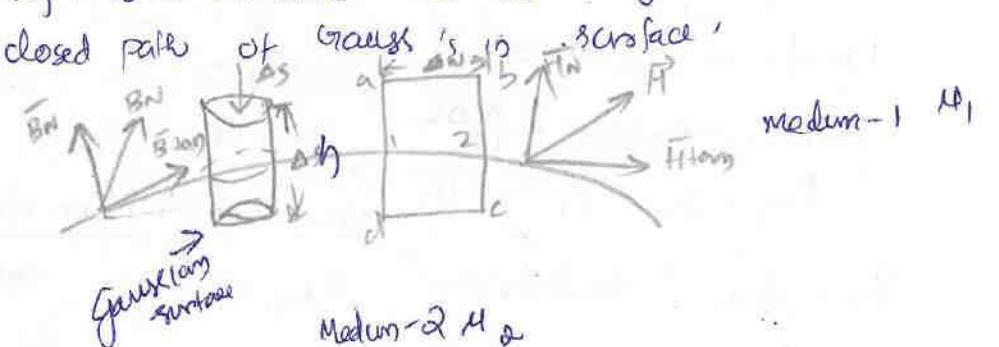
$$B_N_1 = B_N_2$$

$$\frac{H_N_1}{H_N_2} = \frac{\mu_1 \sigma_2}{\mu_2 \sigma_1}$$

$$H \tan_1 = H \tan_2$$

$$\frac{B \tan_1}{B \tan_2} = \frac{\mu_1}{\mu_2},$$

Condition of magnetic field exist at the boundary of a material when the magnetic field passes from one medium to other are called magnetic boundary conditions. To study the magnetic conditions we need solve $B \times H$ in 2 components. tangential & normal component (T₁₂ to the boundary). Consider a boundary b/w 2 material with μ_1 & μ_2 . Shown in fig. to determine the boundary conditions consider a closed path of Gauss's law.



→ Boundary condition for normal component.
o consider a closed path in the form of circular cylinder
 Δh o area of top & bottom is Δh .

According to gauss's law for magnetic field.

$$\oint \vec{B} \cdot d\vec{s} = 0 \quad \text{--- (1)}$$

$$\int_{\text{top}} \vec{B} \cdot d\vec{s} + \int_{\text{bot}} \vec{B} \cdot d\vec{s} + \int_{\text{lat}} \vec{B} \cdot d\vec{s} = 0$$

$$\text{assume } \Delta h \rightarrow 0; d\vec{s}_{\text{lat}} = dT \times dh = 0.$$

$$\int_{\text{top}} B ds = 0 \quad , \quad \int_{\text{top}} \vec{B} \cdot d\vec{s} + \int_{\text{bot}} \vec{B} \cdot d\vec{s} = 0 \quad \text{--- (2)}$$

$$\int_{\text{top}} B_{N1} ds + \int_{\text{bot}} B_{N2} ds = 0 \quad \text{--- (3)}$$

$$B_{N1} \Delta h - B_{N2} \Delta h = 0 \quad (\text{opp. each other})$$

$$B_{N1} = B_{N2}$$

$$B = \mu H; B_{N1} = \mu_0 \mu_{r1} H_{N1}$$

$$B_{N2} = \mu_0 \mu_{r2} H_{N2}$$

$$\frac{B_{N1}}{B_{N2}} = \frac{\mu_0 \mu_{r1} H_{N1}}{\mu_0 \mu_{r2} H_{N2}}$$

$$\frac{H_{N1}}{H_{N2}} = \frac{\mu_{r1}}{\mu_{r2}}$$

$$B_{N1} = B_{N2} \quad I = \frac{\mu_0 \mu_{r1} H_{N1}}{\mu_0 \mu_{r2} H_{N2}}$$

⇒ Boundary condition for tangential component.

Consider closed path abda in form of \square with width Δh . $1/2$ - 1 medium & $1/2$ - medium 2.

According to Ampere's law,

$$\oint \vec{H} \cdot d\vec{l} = I_{\text{enc}} \quad \text{--- (1)}$$

$$\int_a^b \vec{H} \cdot d\vec{l} + \int_b^c \vec{H} \cdot d\vec{l} + \int_c^d \vec{H} \cdot d\vec{l} + \int_d^a \vec{H} \cdot d\vec{l} = I_{\text{enc}}$$

$$\int_a^b H_{tang} dl + \int_b^c H_{tang} dl + \int_c^d H_{tang} dl + \int_d^a H_{tang} dl = I$$

$$H_{tang1} \Delta h + H_{tang1} (\Delta h/2) + H_{tang2} (\Delta h/2) + H_{tang2} (\Delta h/2)$$

$$\Delta h \rightarrow 0$$

$$H_{tang1} \Delta h = H_{tang2} \Delta h = I_{\text{enc}}$$

$$H_{tang1} \Delta h - H_{tang2} \Delta h = \bar{K} \Delta h$$

$$H_{tang1} - H_{tang2} = \bar{K} \quad \text{--- (3)} \quad \text{if } K=0$$

$$B_{tang1} = \mu_0 \mu_{r1} H_{tang1} \quad B_{tang2} = \mu_0 \mu_{r2} H_{tang2}$$

$$\frac{B_{tang1}}{B_{tang2}} = \frac{\mu_{r1}}{\mu_{r2}}$$

1. Given that $\bar{K} = 2ax^2 - 3ay^2 + 5az^2 \text{ V/m}$ at an charge free dielectric interface are show that find E_z

$$D_2 = D_{N2} + D_{tan2}$$

$$E_{N1} = 5 a_0^2 \text{ V/m}$$

$$\vec{E}_1 = E_{N1} + E_{tan1}$$

$$E_{tan1} = 2a_0^2 - 3a_0^2 \text{ V/m}$$

$$E_{tan1} = E_{tan2} = 2a_0^2 - 3a_0^2$$

$$D_2 = D_{N2} + D_{tan2}$$

$$= \epsilon_0 \epsilon_{r1} E_{N1} + \epsilon_0 \epsilon_{r2} E_{tan2} \text{ by plane } z$$

$$= \epsilon_0 [2(5a_0^2) + 5(2a_0^2 - 3a_0^2)] \quad \text{so } \epsilon_r = \epsilon_{r2}$$

$$= 8.85 \times 10^{-2} (10a_0^2 + 10a_0^2 - 15a_0^2)$$

$$D_2 = 8.85 \times 10^{-2} (dx - 13a_0^2 + 8a_0^2)$$

$$D_{tan2} = \epsilon_0 \epsilon_{r2} E_{tan2}$$

$$D_{N1} = \epsilon_0 \epsilon_{r1} E_{N1} = D_{N2}$$

Phase velocity (v_p)

- denoted as v_p (or) v .
- It is defined as the velocity with which the phase of the wave travels.

$$v_p = \frac{\omega}{\beta} \quad \omega$$

$\beta \rightarrow \text{phase constant.}$

$$v_p = \frac{1}{\sqrt{\mu\epsilon}} \quad \beta = \omega\sqrt{\mu\epsilon}$$

For a uniform plain wave. $v = 3 \times 10^8 \text{ m/s.}$

- for free space $\sigma = 0, \mu = \mu_0, \epsilon = \epsilon_0$.

travel with the velocity of light.

parameter Intrinsic Impedance (η):

\rightarrow denoted by η .

$$\rightarrow \sqrt{\mu/\epsilon} \approx$$

$$\rightarrow \text{for free space } \eta = 377 \Omega = 120\pi$$

$$\therefore \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}} =$$

$$\rightarrow \eta \text{ in terms of } \eta_0 \rightarrow \eta = \eta_0 \sqrt{\frac{\mu_r}{\epsilon_r}}$$

$$\eta \Rightarrow \frac{E}{H}$$

Propagation constant (γ)

$$\gamma = [j\omega\mu(\sigma + j\omega\epsilon)]$$

is a complex quantity so $\gamma = \alpha + j\beta$.

$\alpha \rightarrow \text{attenuation constant } \beta \rightarrow \text{phase constant.}$

\rightarrow for free space.

$$\alpha = 0.$$

$$\beta = \omega\sqrt{\mu_0\epsilon_0}.$$

$$\alpha = \omega \sqrt{\frac{\mu\epsilon}{2} \left(\sqrt{1 + \left(\frac{\omega}{\omega_E} \right)^2} - 1 \right)}$$

$$\beta = \omega \sqrt{\frac{\mu\epsilon}{2} \left(\sqrt{1 + \left(\frac{\omega}{\omega_E} \right)^2} + 1 \right)}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

Wavelength (λ)

The distance that must travel by the wave to change the phase by 2π radian is called wavelength.

$$\rightarrow \lambda = \frac{2\pi}{\beta}$$

$$\rightarrow \lambda = \frac{C}{f} = \frac{v_p}{f} = \frac{\omega}{Bf} = \frac{2\pi f}{B\lambda} = \frac{2\pi}{B} //$$

for free space.

Skin depth

- Depth of penetration (δ) delta.
- The distance through which the amplitude of travelling wave decreases to $36\% \left(\frac{1}{e}\right)$ of the original amplitude is called skin depth.

$$\delta = \frac{1}{\omega} = \frac{1}{\beta} = \frac{1}{\sqrt{\mu_0 \epsilon_0 f}}$$

$$E_{at d} = E_0 e^{-\alpha d}$$

$$\rightarrow \delta \propto \frac{1}{f}$$

A homogeneous medium σ, μ, ϵ throughout the medium uniform plan wave E, H have same value at any fixed instant.

Skin effect

In microwave large skin depth is very small for all good conductors all fields ^{and current} be confined to a very thin layer near the surface of conductor. This thin layer but the skin of the conductor hence it is called skin effect.

Solution of wave equation

General wave equation

- Linear homogeneous medium with parameters σ, μ, ϵ .
- Medium is source free & charge free.
- And it obeys ohm's law $\vec{J} = \sigma \vec{E}$
- As per Maxwell's equation.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (1)$$

$$\vec{B} = \mu \vec{H}$$

$$\nabla \times \vec{E} = -\frac{\partial \mu \vec{H}}{\partial t}$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (2)$$

Taking curl on both sides

$$\nabla \times \nabla \times \vec{E} = -\mu \nabla \times \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \nabla \times \vec{E} = \mu \frac{\partial}{\partial t} \nabla \times \vec{H} \quad (3)$$

As per maxwell's equation.

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (4)$$

Sub in (3)

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t} \left[\vec{J} + \frac{\partial \vec{D}}{\partial t} \right] \quad (5)$$

$$\vec{J} = \sigma \vec{E} \quad \& \quad \vec{D} = \epsilon \vec{E}$$

$$\Rightarrow \nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t} \left[\sigma \vec{E} + \frac{\partial \epsilon \vec{E}}{\partial t} \right]$$

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial \sigma \vec{E}}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad (6)$$

According to vector identity.

$$\nabla \times \nabla \times \vec{E} = \nabla (\nabla \cdot \vec{E}) - \nabla^2 \vec{E} \quad (1)$$

$$\nabla \cdot \vec{D} = 0 \text{ so } \nabla \cdot \vec{E} = 0$$

$$(1) \rightarrow \nabla \times \nabla \times \vec{E} = -\nabla^2 \vec{E}$$

Substitute in (1)

$$-\nabla^2 \vec{E} = \mu_0 \frac{\partial \vec{E}}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\boxed{-\nabla^2 \vec{E} = \mu_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$

wave equation for electric field.

Multipplied with ϵ_0 .

$$\boxed{\nabla^2 \vec{D} = \mu_0 \frac{\partial \vec{D}}{\partial t} + \mu_0 \epsilon_0 \frac{\partial^2 \vec{D}}{\partial t^2}} \quad \text{D eqn}$$

→ Wave equation of homogeneous, isotropic, conducting medium.

→ general wave equation is obtained from Maxwell's eqn.

Let us consider electric and mag. field exist in linear isotropic parameter σ, μ, ϵ .

Wave equation for Magnetic field.

Ans.

→ Maxwell's equation from Ampere's law

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (2)$$

Taking curl on both side.

$$\nabla \times \nabla \times \vec{H} = \nabla \times \vec{J} + \frac{\partial}{\partial t} \nabla \times \vec{D} \quad (2)$$

$$\text{Sub } \vec{J} = \sigma \vec{E} \text{ & } \vec{D} = \epsilon \vec{E}$$

$$\nabla \times \nabla \times \vec{H} = \sigma \nabla \times \vec{E} + \epsilon \frac{\partial}{\partial t} \nabla \times \vec{E} \quad (3)$$

Maxwell's equation.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (4)$$

$$\nabla \times \nabla \times \vec{H} = \sigma \left(-\frac{\partial \vec{H}}{\partial t} + \epsilon \frac{\partial}{\partial t} \left(-\frac{\partial \vec{H}}{\partial t} \right) \right).$$

$$\nabla \times \nabla \times \vec{H} = -\sigma \epsilon \left(\frac{\partial^2 \vec{H}}{\partial t^2} \right) - \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2} \quad (5)$$

According to vector identity.

$$\nabla \times \nabla \times \vec{H} = \nabla (\nabla \cdot \vec{H}) - \nabla^2 \vec{H}$$

$$\nabla \cdot \vec{H} = 0$$

$$\nabla \times \nabla \times \vec{H} = -\nabla^2 \vec{H}$$

$$(6) \rightarrow -\nabla^2 \vec{H} = -\sigma \mu_0 \left(\frac{\partial \vec{H}}{\partial t} \right) - \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}$$

$$\boxed{\nabla^2 \vec{H} = \sigma \mu_0 \frac{\partial \vec{H}}{\partial t} + \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}}$$

Wave eqn for magnetic field.

$$\boxed{\nabla^2 \vec{B} = \sigma \mu_0 \frac{\partial \vec{B}}{\partial t} + \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}}$$

for \vec{B} in uniform plane wave.

Propagation of EM waves in perfect dielectric.

Consider a uniform plane wave propagating through a perfect dielectric.

For perfect dielectric $\sigma = 0, H = H_0 \sin kx, t = t_{\text{tot}}$
as $\sigma = 0$, it's called lossless medium.

$$\text{velocity of propagation } V = \frac{1}{\sqrt{\mu\epsilon}}$$

$$V = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0}} = \frac{1/\sqrt{\mu_0 \epsilon_0}}{\sqrt{\mu_r}} = C / \sqrt{\epsilon_r} \text{ m/s}$$

$$V = \frac{1}{\sqrt{\mu\epsilon}} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{\omega}{\beta} \quad \left| \begin{array}{l} \beta = \omega\sqrt{\mu\epsilon} \\ V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}} \end{array} \right.$$

Propagation constant.

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} \text{ m}^{-1}$$

Here $\sigma = 0, \mu = \mu_0 \mu_r, E = E_0 \epsilon_r$

$$\gamma = \pm j\omega\sqrt{\mu\epsilon} \cdot \text{m}^{-1}$$

$$\therefore \gamma = \alpha + j\beta \quad \alpha = 0$$

Attenuation constant = 0 for perfect dielectric

phase const. $\beta = \sqrt{\mu\epsilon}$ rad/m

Intrinsic impedance, $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$

$$\sigma = 0, \eta = \sqrt{\frac{j\omega\mu}{0 + j\omega\epsilon}}$$

$$\sqrt{\frac{j\omega\mu}{j\omega\epsilon}} = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \times \sqrt{\frac{\mu_r}{\epsilon_r}}$$

$$= \eta_0 \sqrt{\frac{\mu_r}{\epsilon_r}} = 377 \sqrt{\frac{\mu_r}{\epsilon_r}} \Omega$$

$$\boxed{\eta = 377 \sqrt{\frac{\mu_r}{\epsilon_r}}}$$

$$\eta = \frac{E}{H}$$

General wave eqn:

$$\nabla^2 E = \mu_0 \sigma E - \mu E \frac{\partial^2 E}{\partial t^2} \quad \text{--- (1)}$$

$$\nabla^2 H = \mu_0 \sigma H + \mu E \frac{\partial^2 H}{\partial t^2} \quad \text{--- (2)}$$

Here $\sigma = 0$, sub (1) & (2).

$$\nabla^2 E = \mu \epsilon \frac{\partial^2 E}{\partial t^2} \quad \text{eq } \nabla^2 H = \mu E \frac{\partial^2 H}{\partial t^2}$$

$$\nabla^2 E = \mu E j^2 \omega^2 E \quad \text{eq } \nabla^2 H = \mu E j^2 \omega^2 H$$

$$\nabla^2 E = \gamma_E^2 E \quad \text{eq } \nabla^2 H = \gamma_H^2 H$$

$$\gamma_E = j\omega\sqrt{\mu\epsilon}$$

$$\nabla^2 E = \sqrt{\mu\epsilon} E$$

$$\nabla^2 H = \sqrt{\mu\epsilon} H$$

EM wave propagation in lossy dielectric

due to certain conductivity ($\sigma \neq 0$) certain loss in the medium takes place & hence the wave travelling in the medium get attenuated ($\alpha \neq 0$) such dielectric is called lossy dielectric $\sigma \neq 0$.

$$\text{general wave eqn, } \nabla^2 E = \mu \sigma \frac{\partial E}{\partial t} + \mu \epsilon \frac{\partial^2 E}{\partial t^2} \quad \text{---(1)}$$

$$\nabla^2 H = \mu \sigma \frac{\partial H}{\partial t} + \mu \epsilon \frac{\partial^2 H}{\partial t^2} \quad \text{---(2)}$$

$$\begin{aligned} \nabla^2 E &= \mu \sigma j \omega E + \mu \epsilon j^2 \omega^2 E \\ &= [j \omega \mu (\sigma + j \omega \epsilon)] E \quad \text{---(3)} \end{aligned}$$

$$\text{Similarly, } \nabla^2 H = [j \omega \mu (\sigma + j \omega \epsilon)] H \quad \text{---(4)}$$

$$\text{As we } j \omega (\sigma + j \omega \epsilon) = j^2$$

$$\nabla^2 E = j^2 E = 0 \quad \text{and} \quad \nabla^2 H = j^2 H = 0.$$

$$\text{Propagation constant } \gamma = \pm \sqrt{j \omega \mu (\sigma + j \omega \epsilon)}.$$

$$\text{Rearranging, } \gamma = \pm \sqrt{j \omega \mu \left[1 + \frac{\sigma}{j \omega \epsilon} \right]} \sqrt{j \omega \mu}.$$

$$\gamma = \pm \sqrt{j \omega \epsilon (\sigma + j \omega \epsilon)} \sqrt{j \omega \mu}.$$

$$\gamma = \omega \beta = j \omega \sqrt{\mu \epsilon} \sqrt{1 - j \frac{\sigma}{\omega \epsilon}}.$$

$$\text{expanding } (1 - j \frac{\sigma}{\omega \epsilon})^{1/2} = 1 - \frac{1}{2} j \frac{\sigma}{\omega \epsilon}.$$

$$\gamma = j \omega \sqrt{\mu \epsilon} \left(1 - \frac{\sigma}{2 \omega \epsilon} \right) = j \omega \sqrt{\mu \epsilon} + \frac{\omega \sqrt{\mu \epsilon}}{2 \epsilon}.$$

$$\gamma = \frac{\omega}{2} \sqrt{\mu \epsilon} + j \omega \sqrt{\mu \epsilon}.$$

$$\alpha = \frac{\omega}{2} \sqrt{\mu \epsilon} \quad \text{and} \quad \beta = \omega \sqrt{\mu \epsilon}$$

Attenuation const & phase const.

Propagation of EM wave in good conductor

$$\alpha = \sqrt{\pi \mu \sigma f} = \sqrt{\frac{\omega \mu \sigma}{2}} \quad \left[\frac{\sigma}{\omega \epsilon} > > 1 \right]$$

$$\beta = \sqrt{\pi \mu f} = \sqrt{\frac{\omega \mu \sigma}{2}} \quad \left[2 \pi f = \alpha \right. \\ \left. \pi f = \omega \frac{1}{2} \right].$$

$$\eta = \sqrt{\frac{\mu \sigma}{2 \epsilon}} (1 + j) \quad \text{or} \quad \sqrt{\frac{\mu \sigma}{\epsilon}} < 45 = \sqrt{\frac{\omega \mu}{\sigma}}$$

$$V_p = \omega / \beta = \frac{\omega}{\sqrt{\omega \mu \frac{1}{2}}} = \sqrt{\frac{2 \omega}{\mu \sigma}}.$$

$$\alpha = \beta = \sqrt{\frac{\mu \sigma}{2}} \quad \gamma = \sqrt{j \omega \mu (\sigma + j \omega \epsilon)} = \sqrt{j \omega \mu \sigma (1 + \frac{\sigma}{j \omega \epsilon})}.$$

$$\begin{aligned} 1 + j &= 1 + j \omega \epsilon \\ &= 1 + j \frac{\sigma}{\omega \epsilon} \\ &= 1 + j \frac{1}{2} \frac{\sigma}{\omega} \\ &= 1 + j \frac{\sigma}{\omega \epsilon} \end{aligned}$$

$$\gamma = \sqrt{\mu_0 \sigma}.$$

Q. For a material for which $\sigma = 5 \text{ Siemens/m}$.

$E_s = 1$ and $\bar{E} = 250 \sin 10t \text{ V/m}$. Find the conduction current density and displacement current density?

$$J_c = \sigma \bar{E} = 5 \times 250 \sin 10t$$

$$= \underline{1250 \sin 10t}$$

$$\bar{J}_d = \frac{\partial \bar{D}}{\partial t} = \frac{\partial}{\partial t} \epsilon_0 \epsilon_r E = 8.85 \times 10^{-12} \times 1 \times \frac{\partial E}{\partial t}$$

$$= 8.85 \times 10^{-12} [250 \cos 10t \times 10]$$

$$= \underline{22.125 \cos 10t \text{ A/m}^2}$$

Q. A uniform plane wave is travelling at a velocity of $2.5 \times 10^8 \text{ m/s}$ having a wavelength $\lambda = 0.25 \text{ mm}$ in a non magnetic good conductor calculate the force of wave of the conductivity of the medium?

$$f = \frac{V}{2} = \frac{2.5 \times 10^8}{0.25 \times 10^3} = 10 \text{ Hz}$$

$$V_p = \frac{\omega}{\beta} = \frac{\alpha \pi f}{\beta} = \frac{2\pi f}{\alpha \pi h} = \frac{1}{h} = 4 \times 10^4$$

$$\beta = 2\pi/h$$

$$= 2\pi/\underline{0.25 \times 10^{-3}} = \underline{25.13 \times 10^3} \text{ rad/m.}$$

$$\approx \underline{1.57 \times 10^5}$$

$$\sigma = \beta = \sqrt{\mu_0 \sigma} =$$

Q. Calculate the intrinsic impedance η ? γ ? V_p for a conducting medium in which $\sigma = 58 \text{ Ms/m}$ $\mu_r = 1$ $\epsilon_{rr} = 1$ at a frequency of 100 MHz

$$\eta = 377 \sqrt{\frac{\mu_0}{\epsilon_0}} = \sqrt{\frac{\mu_0}{\sigma}} = 368 \times 10^3 \angle 45^\circ$$

$$\gamma = \sqrt{j\omega \mu_0} = 2.319 \times 10^5 \angle 45^\circ$$

$$\gamma = \alpha_f j\beta = 1.51 \times 10^3 + j 1.81 \times 10^5$$

$$V = \omega/\beta = \alpha \pi f / \beta = 4.15 \times 10^3 \text{ m/s.}$$

$$\begin{aligned} e^{j\theta} &= J/J \\ e^{j\theta/2} &= \gamma \\ \sqrt{j} &= \frac{j\pi}{2} \\ \theta &= 45^\circ \end{aligned}$$

$$V_j = 45^\circ$$

Lorentz equation / Relation b/w \vec{V} & \vec{A}

Considering Maxwell's eqn.

$$\nabla \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t} \quad \text{--- (1)}$$

$$B = \mu \vec{H}, \vec{D} = \epsilon \vec{E} \text{ & } \vec{B} = \nabla \times \vec{A}$$

$$\frac{\nabla \times \vec{B}}{\mu} = \vec{j} + \frac{\partial \epsilon \vec{E}}{\partial t}$$

$$\frac{1}{\mu} \nabla \times (\nabla \times \vec{A}) = \vec{j} + \frac{\partial \epsilon \vec{E}}{\partial t}$$

$$\nabla \times \nabla \times \vec{A} = \mu \vec{j} + \mu \epsilon \nabla \left(\frac{\partial^2 \vec{A}}{\partial t^2} \right) \quad \text{--- (2)}$$

But from vector identity

$$\nabla \times \nabla \times \vec{A} = \nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$

$$\nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = \mu \vec{j} - \mu \epsilon \nabla \frac{\partial V}{\partial t} - \mu \epsilon \frac{\partial^2 \vec{A}}{\partial t^2}$$

$$\nabla \cdot \vec{A} - \mu \epsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu \vec{j} + \left(\mu \epsilon \frac{\partial V}{\partial t} + \nabla \cdot \vec{A} \right)$$

$$\boxed{\nabla \cdot \vec{A} = \mu \epsilon \frac{\partial V}{\partial t} = 0} \rightarrow \text{Lorentz equation}$$

$$\boxed{\nabla \cdot \vec{A} = -\mu \epsilon \frac{\partial V}{\partial t}} \text{ Relation b/w } \vec{A} \text{ & } V$$

- Q. Find the displacement current density and electric field. In free space $\vec{H} = 0.5 \omega E_0 \cos(\omega t - 50x) \hat{a}_x \text{ A/m}$

Maxwell's equations

$$\nabla \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t} = \vec{0} + \frac{\partial \epsilon \vec{E}}{\partial t}$$

for free space $\sigma = 0, \epsilon = \epsilon_0, \mu = \mu_0$.

$$\text{--- (1)} \quad \nabla \times \vec{H} = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_z \end{vmatrix}$$

$$= \hat{a}_x \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) - \hat{a}_y \left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} \right) + \hat{a}_z (0-0)$$

$$= 0 - \hat{a}_y \left[\frac{\partial H_x}{\partial z} \right] + 0$$

$$= -\hat{a}_y \left[\frac{\partial (0.5 \omega E_0 \cos(\omega t - 50x))}{\partial z} \right]$$

$$= -\hat{a}_y [0.5 \omega E_0 \vec{B}_{15} \cos(\omega t - 50x) \times -50].$$

$$= [-2.5 \omega E_0 \sin(\omega t - 50x)] \hat{a}_y = \nabla \times \vec{H}$$

$$-2.5 \omega E_0 \sin(\omega t - 50x) \hat{a}_y = \frac{\partial \vec{D}}{\partial t}$$

$$-2.5 \omega \sin(\omega t - 50x) \hat{a}_y = \frac{\partial \vec{E}}{\partial t}$$

f w.r.t t.

$$\vec{E} = -2.5 \frac{\partial E}{\partial t} \cos(\omega t - 50x) \hat{a}_y$$

$$\vec{E} = 2.5 \cos(\omega t - 50x) \hat{a}_y$$

$$T_D = \frac{\partial D}{\partial t} = \frac{\partial \epsilon_0 \vec{E}}{\partial t}$$

$$T_D = -2.5 \omega \epsilon_0 \sin(\omega t - 50x) \hat{a}_y$$

Q. In a region, homogeneous region where $\mu_r = 1$ and $\epsilon_{r0} = 50$. The fields are given. $\vec{E} = 20\pi r e^{j(\omega t - \beta z)} \hat{a}_y$ v/m.

$$\vec{B} = \mu_0 H_m e^{j(\omega t - \beta z)} \hat{a}_y \text{ T} \quad \text{find } \omega \text{ if } H_m = ?$$

$$\lambda = 1.75 \text{ m.}$$

Assume lossless medium $\sigma = 0$.

$$\lambda = \frac{2\pi}{\beta} \quad \beta = \frac{2\pi}{\lambda} = 3.59 \text{ rad/m}$$

for lossless medium.

$$\beta = \omega \sqrt{\mu \epsilon}$$

$$\omega = \frac{\beta}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}} = 1.522 \times 10^8 \text{ rad/sec.}$$

Magnetic field is given by

$$\vec{B} = \mu_0 H_m e^{j(\omega t - \beta z)} \hat{a}_z$$

$$\vec{B} = \mu_0 H$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{\mu_0 H_m e^{j(\omega t - \beta z)}}{\mu_0}$$

$$\vec{H} = H_m e^{j(\omega t - \beta z)}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = 53.278 \Omega. \quad \begin{array}{l} \text{depending upon} \\ \text{the medium} \end{array}$$

$$\text{But } \eta = \frac{R_m}{H_m}$$

$$H_m = \frac{R_m}{\eta}$$

$$R_m = 20\pi$$

$$H_m = \frac{20\pi}{53.278 \Omega} = 1.179 \text{ A/m}$$

Q. A uniform plane wave propagating in a medium has $\vec{E} = 2e^{-\alpha z} \sin(10t - \beta z) \hat{a}_y$ v/m. If the medium is characterised by $\epsilon_r = 1$, $\mu_r = 20$, $\sigma = 3.2 \text{ S/m}$. α & $\beta = ?$

$$\alpha = \omega \sqrt{\frac{\mu}{\epsilon}} \left(\sqrt{1 + \left(\frac{R}{\omega \epsilon} \right)^2} - 1 \right)^{1/2}$$

$$\beta = \omega \sqrt{\frac{4\epsilon}{\alpha} \left(\sqrt{1 + \frac{\alpha}{4\epsilon}} \right)^2 + 1}$$

Q. In free space it is given that $\epsilon = 80\pi$

$$E = 30\pi e^{j(\omega t + \beta z)} a_y \text{ V/m.}$$

$$H = H_0 e^{j(\omega t + \beta z)} a_y \text{ A/m.}$$

Find β and H_0 ?

For space $\sigma = 0 \quad v = c = 3 \times 10^8 \text{ m/s.}$

$$V_p = \frac{c}{\beta}$$

$$\beta = \frac{\omega}{V_p} = \frac{70}{3 \times 10^8} = 0.33 \text{ rad/m.}$$

$$\eta = \left| \frac{E}{H} \right| = \eta_0 = 120\pi.$$

$$\eta_0 = \frac{E_0}{H_0} = \frac{30\pi}{H_0} = 120\pi$$

$$H_0 = \frac{30\pi}{120\pi} = \underline{0.25}$$

Module-III

Poynting Theorem

Poynting theorem states that vector product of electric field intensity E and magnetic field intensity H at any point is measure of energy flow / unit area at that point i.e. $\vec{P} = \vec{E} \times \vec{H}$ where \vec{P} is Poynting vector.

vector quantity
direction \perp to both \vec{E} and \vec{H} .

Proof

Consider Maxwell's equations.

$$-\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = \mu_0 \frac{\partial \vec{H}}{\partial t} \quad \text{--- (1)}$$

$$\nabla \times \vec{H} = \vec{T} + \frac{\partial \vec{D}}{\partial t} = \sigma \vec{E} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{--- (2)}$$

Taking dot product on both sides of eqn.

(1) with \vec{E}

$$\vec{E} \cdot (\nabla \times \vec{H}) = \vec{E}^2 (\sigma \vec{E}) + \epsilon_0 \vec{E} \left(\frac{\partial \vec{E}}{\partial t} \right)$$

$$\nabla \cdot (\vec{E} \times \vec{H}) = \vec{E} \cdot \vec{E} \sigma + \vec{E} \cdot \left(\epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \quad \text{--- (3)}$$

Using identity.

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$

$$\text{put } \vec{A} = \vec{E} \text{ & } \vec{B} = \vec{H}$$

$$\nabla \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{H}).$$

(3) sub in (3)

$$(3) \rightarrow \vec{H} \cdot (\nabla \times \vec{E}) - \nabla \cdot (\vec{E} \times \vec{H}) = \sigma \vec{E}^2 + \vec{E} \cdot \left(\epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) \quad \text{--- (4)}$$

consider 1st term on LHS of eqn (4) & put it value of $\nabla \times \vec{E}$ in (1).

$$\nabla \cdot (\nabla \times \vec{E}) = \vec{H} \cdot \left(\mu_0 \frac{\partial \vec{H}}{\partial t} \right) = \mu_0 \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} \quad \text{--- (5)}$$

Consider the term.

$$\begin{aligned} \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} &= \vec{H} \cdot \frac{\partial}{\partial t} \vec{H} + \vec{H} \cdot \frac{\partial}{\partial t} \vec{H} \\ &= \frac{\partial}{\partial t} \vec{H} [\vec{Q} \vec{H}] \end{aligned}$$

$$\frac{\partial}{\partial t} \vec{H} \cdot \vec{H} = \frac{\partial}{\partial t} \vec{H}^2$$

$$\frac{\partial}{\partial t} \vec{H} = \frac{\partial}{\partial t} \vec{H} (\vec{Q} \vec{H})$$

$$\frac{1}{2} \frac{\partial}{\partial t} \vec{H}^2 = \vec{H} \cdot \frac{\partial}{\partial t} \vec{H} \quad \text{--- (6)}$$

$$\text{Hence } \frac{1}{2} \frac{\partial}{\partial t} \vec{H}^2 = \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} \quad \text{--- (7)}$$

Sub - 5, 6, 7 in (4)

$$-\mu_0 \frac{1}{2} \frac{\partial}{\partial t} \vec{H}^2 - \nabla \cdot (\vec{E} \times \vec{H}) = \sigma \vec{E}^2 + \frac{1}{2} \epsilon_0 \frac{\partial}{\partial t} \vec{E}^2.$$

$$-\nabla \cdot (\vec{E} \times \vec{H}) = \sigma \vec{E}^2 + \frac{1}{2} \frac{\partial}{\partial t} [4\vec{H} \cdot \vec{E} + \vec{E} \cdot \vec{E}] \quad \text{pnt form}$$

$$-\nabla \cdot (\vec{P}) = \sigma \vec{E}^2 + \frac{1}{2} \frac{\partial}{\partial t} [4\vec{H} \cdot \vec{E} + \vec{E} \cdot \vec{E}] \quad \text{--- (8)}$$