

Module 4

Transmission lines are commonly used as power distn at low frequency and communication in higher frequency. also called feedline or feeder.

2 port or 4 port network.

Simplest form of transmission line is 2 wire transmission line.

Transmission line consisting of one or more parallel conductors.

Types of transmission lines

- * Twisted pair - contains 2 wires twisted together.
- * Cables - under ground wire, telephone cables - hundreds of conductors.
- * Insulated individually with paper.
- * Coaxial cable - used to connect TV to antenna, 2 concentric conductors which are coaxially placed.
- * Micro strip lines - used in integrated circuit. - 2 strips placed in either side of a dielectric.
- * wave guide - to transmit electrical wave at microwave frequencies, hollow

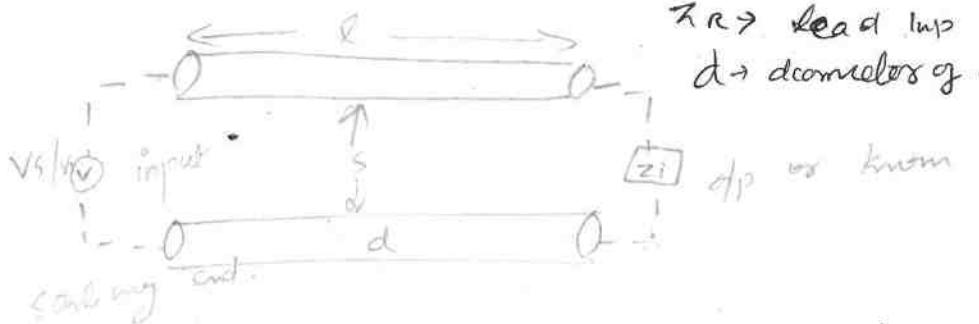
metallic conducting tubes of rectangular hollow circular power transmission.

space b/w 2 conductors

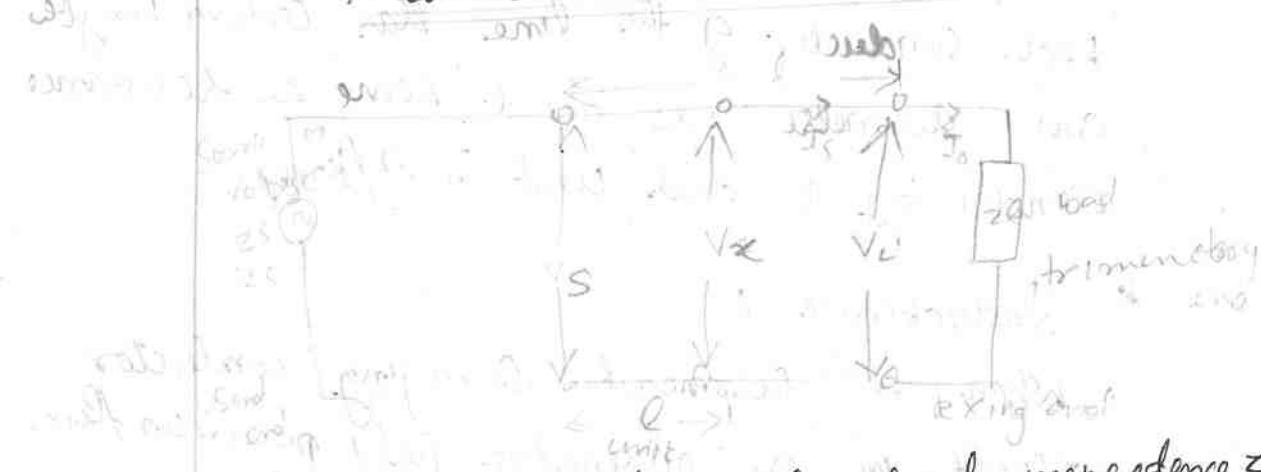
$Z_c \rightarrow$ source in parallel

$Z_R \rightarrow$ load imp

$d \rightarrow$ diameter of conducto



Electrical lines of a transmission line

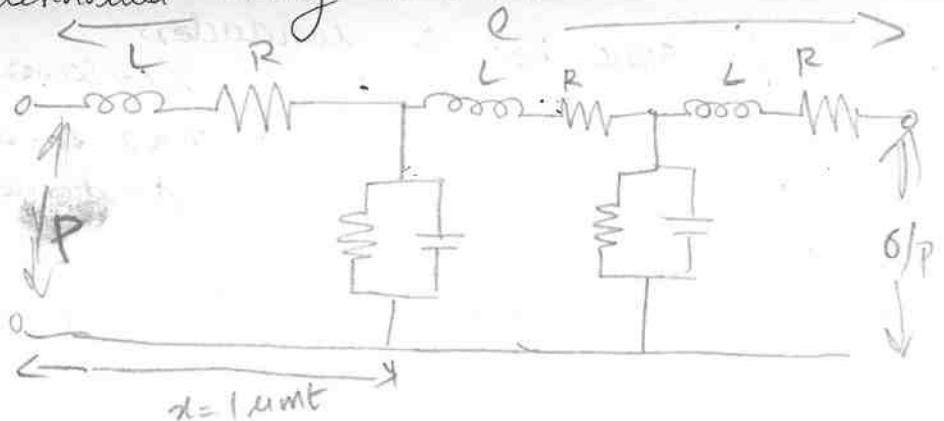


for the operation the load impedance Z_L is connected at load terminals and sinusoidal V_{se} vs is applied at input terminal,

Equivalent circuit representation of transmission line

like a 2 port n/w transmission line has inductance L, resistance R, capacitance C

Capacitance C and conductance G , these are distributed along the line.



* Resistance R .

Each conductor of the line has certain length and diameter, so must have a resistance denoted by R and unit is Ω/km .

* Inductance L .

When a current carrying conductor placed in a magnetic field produces flux. The flux linkage / Φ weber of the current give rise to the effect of inductance and is henry per km.

* Capacitance C

Two parallel conductors separated by air. unit is farad per km.

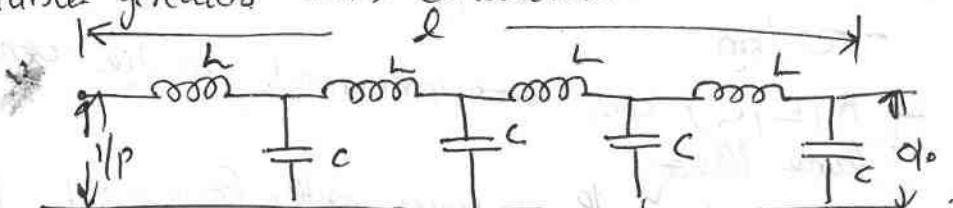
* Conductance G .

The dielectric b/w these 2 lines aren't perfectly insulated, so there's a current leakage through it. This leakage of current gives rise to leakage conductance represented by G .

* Uniform lossless transmission line.

When R, L, C & G are uniformly distributed along the entire length of transmission line it is called uniform transmission line. At RF freq. inductive reactance is much greater than resistive and also capacitance greater than conductance. Hence R & G are neglected ($R \approx 0$).

$R \approx 0$, so it is called lossless and the calculation will be carry at RF. At RF frequency inductive reactance is much greater than resistance and also capacitive suscep-tance greater than conductance. Hence R & G are neglected.



RF equiv. ckt. of tx line l m.

- In order to describe the behaviour of transmission line we consider two ideal cases

- * Infinite length ($l = \infty$)

- * lossless transmission line ($R=0, G=0, C=\infty$)

* Uniform Lossless Transmission Line parameters

Primary constants of transmission line $\xrightarrow{\text{KOO}} R, L, C, G$ are i const;

- & 4 line parameters are called i const.

\rightarrow Resistance (R): loop resistance / unit length.

- sum of resistance of both wires for unit length
- Ω/km .

\rightarrow Inductance (L): loop inductance / unit length.

- sum of inductance of both wires / unit length.
- H/km .

\rightarrow Capacitance (C): shunt capacitance \propto two conductors / unit length of line.

- F/km .

\rightarrow Conductance (G): shunt conductance \propto

- conductors / unit length of line.

- $S\Omega/\text{km}$.

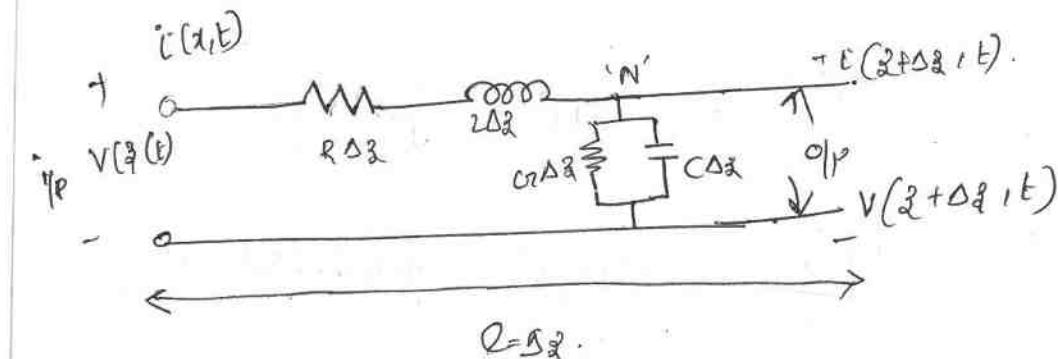
- R, L, C, G are assumed to be const. in transmission line theory.

- Actually they vary with frequency $f = \frac{1}{R} + \frac{1}{Lj\omega}$

- Impedance $Z = R + j\omega L$

- Admittance $Y = G + j\omega C$

X Basic Transmission line equations



By applying KVL. $V(z,t) + V(z+\Delta z,t) \Rightarrow$ instantaneous voltage @ z $\approx (z+\Delta z)$.

$$V(z,t) - i(z,t)R\Delta z - L\Delta z \frac{\partial i(z,t)}{\partial t} - V(z+\Delta z,t) \quad \text{--- (1)}$$

By applying KCL @ node N.

$$i(z,t) - G\Delta z V(z+\Delta z,t) - C\Delta z \frac{\partial V(z+\Delta z,t)}{\partial t} - i(z) = 0 \quad \text{--- (2)}$$

$$\text{--- (1)} \Rightarrow V(z,t) - V(z+\Delta z,t) = R\Delta z i(z,t) + L\Delta z \frac{\partial i(z,t)}{\partial t}.$$

$$\frac{V(z,t) - V(z+\Delta z,t)}{\Delta z} = -\frac{\partial V(z)}{\partial z} = \frac{-\partial V(z)}{\partial z} = \frac{-\partial V(z)}{\partial z}.$$

$$\frac{\Delta z \rightarrow 0}{\Delta z} \frac{-\partial V(z+\Delta z)}{\Delta z} = R i(z,t) + L \frac{\partial i(z,t)}{\partial t} \quad \text{--- (3)}$$

Similarly
 $\text{--- (2)} \rightarrow -\frac{\partial i(z,t)}{\partial z} = G V(z,t) + C \frac{\partial V(z,t)}{\partial t} \quad \text{--- (4)}$

Under steady state logic $C = j\omega C$; $L = j\omega L$

$$\frac{dv(z)}{dz} = -R I(z) - j\omega L I(z) \quad \text{and pos.}$$

$$\frac{dI(z)}{dz} = -G v(z) - j\omega C v(z)$$

$$\frac{dv(z)}{dz} = -T(z) [R + j\omega L] \quad \text{--- (5)}$$

$$\frac{dI(z)}{dz} = -V(z) [G + j\omega C] \quad \text{--- (6)}$$

$$I_2 = \frac{1}{(R+j\omega L)} \frac{d}{dz} - \frac{1}{G+j\omega C} \frac{dI(z)}{dz}$$

$$= \frac{1}{(R+j\omega L)(G+j\omega C)} \frac{d^2 I}{dz^2}$$

$$I_2 = \frac{1}{(R+j\omega L)(G+j\omega C)} \frac{d^2 v(z)}{dz^2}$$

$$\frac{d^2 v(z)}{dz^2} = -(R+j\omega L)(G+j\omega C) v(z) = 0 \quad \text{--- (7)}$$

$$\frac{d^2 v(z)}{dz^2} + \gamma^2 v(z) = 0$$

$$\frac{d^2 T(z)}{dz^2} = \underbrace{-(R+j\omega L)(G+j\omega C)}_{\gamma^2} I(z) = 0 \quad \text{--- (8)}$$

10.

$\overline{j\omega L}$
 \overline{jC}

2nd order diff equation

$$\frac{d^2 z}{dx^2} = \gamma^2 z = 0 \quad (\text{std wave eqn.})$$

$$\text{Sol: } V(z) = V_0 e^{-\gamma z} + V_0 e^{-\gamma z}$$

$$I(z) = V_0 e^{-\gamma z} + V_0 e^{-\gamma z}$$

wave travel in z direction

\Rightarrow propagation const:

$$V = A e^{-\gamma z} + B e^{\gamma z} \quad \text{--- (8)}$$

$$I = C e^{-\gamma z} + D e^{\gamma z} \quad \text{--- (9)}$$

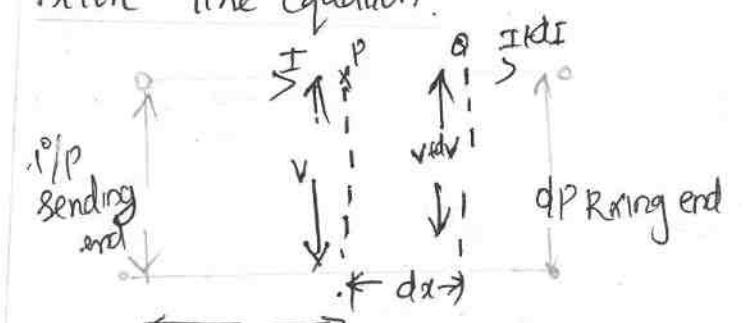
Tx line eqn
or V=I eqn
z =

$$L = \mu, C = \epsilon, G = \sigma$$

$$\frac{\partial^2 V}{\partial z^2} - j\omega L(G + j\omega C)V = 0 \quad \frac{\partial^2 V}{\partial z^2} - \gamma^2 V = 0 \Rightarrow \text{Tx line boundary eqn.}$$

(Uniform lossless Tx line equation)

✓ Tx line equation:



Consider a line of length l

- short section of length dx, at distance z from?

@ p \Rightarrow voltage V, I @ $\alpha \Rightarrow V + dV \propto I + dI$.

for small length dx

$$\text{Impedance } (R + j\omega L)dx = Z$$

$$\text{admittance } (G + j\omega C)dx = Y$$

Voltage drop across pQ $V = IR$.

$$V - (V + dV) = I(R + j\omega L)dx \quad \text{--- (1)}$$

$$\cancel{V} - \frac{-dV}{dx} = I(R + j\omega L)$$

ct' diff before pQ .

$$I - (I + dI) = V(G + j\omega C)dx \quad \text{--- (2)} \quad \frac{I = V}{R} = Y$$

$$\text{--- (3)} \quad \frac{-dV}{dx} = I(R + j\omega L)$$

$$\text{--- (4)} \quad \frac{-dI}{dx} = V(G + j\omega C) \quad \frac{dV}{dx^2} = \frac{dI}{dx} \frac{1}{Z} = \frac{1}{(R + j\omega L)}$$

diff. w.r.t. x & put $\frac{dI}{dx}$ in (4)

$$\frac{d^2V}{dx^2} = \gamma^2 V \quad \text{--- (5)}$$

Trixion line equation

$$\frac{d^2I}{dx^2} = \gamma^2 I \quad \text{--- (6)}$$

$$\gamma^2 = (R + j\omega L)(G + j\omega C) \quad \text{--- (7)}$$

a_{line}

form of 2nd order diff. equation.

$$V = A e^{-\gamma x} + B e^{+\gamma x} \quad \text{--- (8)}$$

$$I = C e^{-\gamma x} + D e^{+\gamma x} \quad \text{--- (9)}$$

Dimension of A, B, C, D ^{of current}

A, B, C, D are arbitrary constants.

$\gamma \rightarrow$ propagation const:

$$\left[\begin{aligned} \gamma^2 &= (R + j\omega L)(G + j\omega C) \\ \cdot \gamma &= \alpha + j\beta \end{aligned} \right]$$

$$\text{--- (10)} \quad V = A e^{-\alpha x - j\beta x} + B e^{+\alpha x + j\beta x}$$

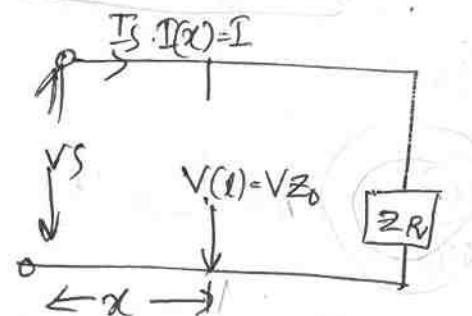
$$I = C e^{-\alpha x - j\beta x} + D e^{+\alpha x + j\beta x} \quad \text{--- (11)}$$

1st term 2nd term.

* 1st term \Rightarrow incident wave

* 2nd term \Rightarrow reflected wave.

General solution of Trixion line terminated with
Any load impedance Z_L .



Trixion line terminating with load impedance $Z_L = Z_0$.

Trixion line eqn
In exponential form

z is replaced by x
distance x , instead
of z .

→ Transmission line equation in hyperbolic form.

When $Z_R \neq Z_0$

i.e., load impedance \neq characteristic impedance.

so the general eqn for vge & ct^t becomes.

$$V = A e^{-\gamma x} + B e^{+\gamma x} \quad \text{--- (1)}$$

$$I = C e^{-\gamma x} + D e^{+\gamma x} \quad \text{--- (2)}$$

From eq. 3 $\frac{-dv}{dx} = I(R+j\omega L) \quad \text{--- (3)}$

diff w.r.t x .

$$(1) \Rightarrow \frac{dv}{dx} = A e^{-\gamma x} (-\gamma) + B e^{\gamma x} (\gamma) \dots$$

Sub in (3).

$$I(R+j\omega L) = \gamma A e^{-\gamma x} - B \gamma e^{\gamma x}$$

$$F = \frac{\gamma}{(R+j\omega L)} (A e^{-\gamma x} - B e^{\gamma x})$$

$$I = \frac{\sqrt{R+j\omega L} (R+j\omega L)}{(R+j\omega L)} A e^{-\gamma x} - B e^{\gamma x}$$

$$= \sqrt{\frac{R+j\omega L}{R+j\omega L}} (A e^{-\gamma x} - B e^{\gamma x}) \quad \text{--- (4)}$$

$$I = \frac{1}{Z_0} (A e^{-\gamma x} - B e^{\gamma x}) \quad \text{--- (5)}$$

$$Z_0 = \frac{R+j\omega L}{j\omega L} \text{ characteristic Imp}$$

$$\cosh \gamma x = \frac{e^{\gamma x} + e^{-\gamma x}}{2}$$

$$\sinh \gamma x = \frac{e^{\gamma x} - e^{-\gamma x}}{2}$$

$$\cosh \gamma x - \sinh \gamma x = \frac{1}{2} [e^{\gamma x} + e^{-\gamma x} - e^{\gamma x} + e^{-\gamma x}]$$

$$\begin{aligned} \cosh \gamma x - \sinh \gamma x &= e^{-\gamma x} \\ \cosh \gamma x + \sinh \gamma x &= e^{\gamma x} \end{aligned} \quad \text{--- (7) } \quad \text{--- (8)}$$

Sub
eqns (7) and (8) in (5) & (6).

$$V = A(\cosh \gamma x - \sinh \gamma x) + B(\cosh \gamma x + \sinh \gamma x)$$

$$I = \frac{1}{Z_0} [\cosh \gamma x - \sinh \gamma x] - B(\cosh \gamma x + \sinh \gamma x)$$

and in terms of hyperbolic function.

$$V = \cosh \gamma x (A+B) - \sinh \gamma x (A-B) \quad \text{--- (9)}$$

$$I = \frac{1}{Z_0} [\cosh \gamma x (A-B) - \sinh \gamma x (A+B)] \quad \text{--- (10)}$$

⇒ At 1/p end, $x=0$, $V=V_S$ & $I=I_S$.

$$\cosh \gamma x = 1; \sinh \gamma x = 0$$

$$\left\{ \begin{array}{l} V = V_S \\ I = I_S \end{array} \right.$$

or If q/b

$$\left[\begin{array}{l} V_S = A+B \\ I_S = \frac{A-B}{Z_0} \end{array} \right] \quad \text{--- (11)}$$

$$\left[\begin{array}{l} V_S = A-B \\ I_S = \frac{A+B}{Z_0} \end{array} \right] \quad \text{--- (12)}$$

$$\left\{ \begin{array}{l} V = V_S \\ I = I_S \end{array} \right.$$

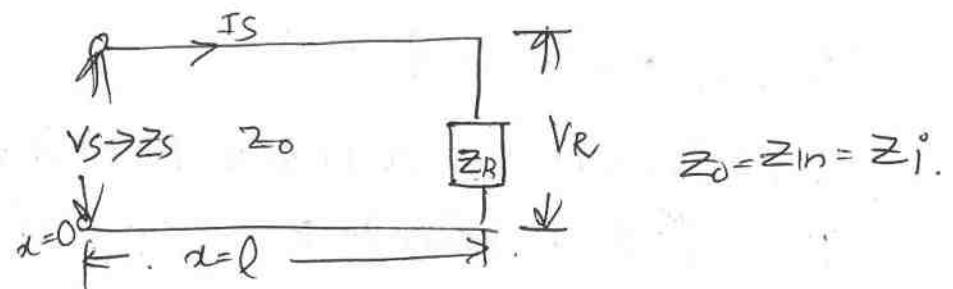
Sub 11 @ 10 in eq 11

$$V = V_s \cosh \gamma x - I_s Z_0 \sinh \gamma x \quad (13)$$

$$I = I_s \cosh \gamma x - \frac{V_s}{Z_0} \sinh \gamma x \quad (14)$$

general
soln.
+
of
Txion
to
V (eq)

Input Impedance of a Txion line terminated with any load impedance (Z_R)



consider a txion line terminated with any load

Impedance Z_R @ $x=l$.

The equation : of voltage & ct' @ a point distance x from the i/p and (or) sending end.

$$V = V_s \cosh \gamma x - I_s Z_0 \sinh \gamma x \quad (13)$$

$$I = I_s \cosh \gamma x - \frac{V_s}{Z_0} \sinh \gamma x \quad (14)$$

→ when the line is terminated with any impedance Z_R @ distance $x=l$.

then voltage across terminated impedance Z_R .

$$V_R = I_R Z_R \quad (15)$$

The general soln of line terminated with Z_R ; is obtained by putting $V=V_R$ and $I=I_R$ in eq (13) and (14)

$$\left[V_R = V_s \cosh \gamma l - I_s Z_0 \sinh \gamma l \right] \quad (16)$$

$$\left[I_R = I_s \cosh \gamma l - \frac{V_s}{Z_0} \sinh \gamma l \right] \quad (17)$$

put (16) & (17) in (10) $V_R = I_R Z_R$.

$$V_s \cosh \gamma l - I_s Z_0 \sinh \gamma l = Z_R \left[I_s \cosh \gamma l - \frac{V_s}{Z_0} \sinh \gamma l \right]$$

→ multiply with Z_0 on both sides.

$$V_s Z_0 \cosh \gamma l - I_s Z_0^2 \sinh \gamma l = Z_R Z_0 I_s \cosh \gamma l - \frac{V_s}{Z_0} Z_0 \sinh \gamma l$$

$$V_s [Z_0 \cosh \gamma l + Z_R \sinh \gamma l] = I_s Z_0 [Z_R \cosh \gamma l + Z_0 \sinh \gamma l]$$

$$\frac{V_s}{I_s} = Z_0 \left[\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l} \right]$$

$$\frac{V_s}{I_s} = Z_0 \left[\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l} \right]$$

$\frac{Z_s - V_s}{I_s}$ \Rightarrow sending Impedance or P/p impedance.

I/p im

$$\frac{Z_s}{\text{OR } Z_0} = Z_0 \left[\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l} \right] \quad (18)$$

up to my
I/p mixed

$$\text{If } Z_R = Z_0 \rightarrow Z_S = Z_0 \left[\frac{Z_0 \cosh \gamma l + Z_R \sinh \gamma l}{Z_0 \cosh \gamma l - Z_R \sinh \gamma l} \right]$$

$$Z_S = Z_0.$$

In equation

In eqn ⑯ ÷ N.R & D.R by $\cosh \gamma l$.

$$Z_S = Z_0 \left[\frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l} \right] \quad \begin{array}{l} \text{eqn of I/p impedance} \\ \text{at alternate form.} \end{array}$$

Voltage & ct¹ at any pt on a transmission line.

$$\text{general equation } V = V_s \cosh \gamma x - I_s Z_0 \sinh \gamma x \quad \text{--- (3)}$$

$$I = I_s \cosh \gamma x - \frac{V_s}{Z_0} \sinh \gamma x \quad \text{--- (4)}$$

Terminated by Z_R .

$$V = V_s \left[\frac{Z_R \cosh \gamma'(l-x) + Z_0 \sinh \gamma'(l-x)}{Z_R \cosh \gamma l + Z_0 \sinh \gamma l} \right]$$

$$I = \frac{V_s}{Z_0} \left[\frac{Z_0 \cosh \gamma'(l-x) + Z_R \sinh \gamma'(l-x)}{Z_R \cosh \gamma l + Z_0 \sinh \gamma l} \right]$$

* Voltage and ct¹ when the line is open circled at far end \Rightarrow

When the line is open circled $Z_R = \infty$ (i)

* I/p impedance Z_S .

$$\text{From ⑯ } Z_S = Z_0 \left[\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l} \right]$$

÷ N.R & D.R by Z_R .

$$Z_S = Z_0 \left[\frac{\cosh \gamma l + \frac{Z_0}{Z_R} \sinh \gamma l}{\frac{Z_0}{Z_R} \cosh \gamma l + \sinh \gamma l} \right] \quad \text{--- (5)}$$

when the line is open circled, $Z_R \rightarrow \text{Infinite}$
No ct¹ flow.

$$Z_R = \infty.$$

$$Z_0/Z_R = \frac{Z_0}{\infty} = 0 ; Z_{oc} = Z_S = Z_0 \left[\frac{\cosh \gamma l + 0}{0 + \sinh \gamma l} \right].$$

$$Z_S = Z_{oc} = Z_0 \coth \gamma l \quad \text{--- (6)}$$

when γ is large OR losses are high $\gamma l \rightarrow l$.

$$\therefore Z_{oc} = Z_0.$$

From eqn (2) — vge and ct¹ @ any pt on the Tx line terminated with Z_R .

$$V = \frac{V_s \cosh \gamma(l-x)}{\sinh \gamma l} \quad \text{--- (21)a}$$

$$I = \frac{V_s \sinh \gamma(l-x)}{Z_0 \cosh \gamma l} \quad \text{--- (21)b}$$

$$Z_0 = \sqrt{Z_0 Z_{sc}}$$

$$\tanh \gamma l = \sqrt{\frac{Z_0}{Z_{sc}}}$$

3 Cases of terminating load.

1. When the terminating end is open ckted i.e. load L is open
2. The terminating end is short ckted. It is shorted by a metallic strip called stub. (Small length of transmission line is called stub).
3. When the load is equal to characteristics impedance.
 $Z_0 = Z_R = \gamma_L$

When two waves are in phase we add it to get the V_{ge} . When 2 waves are opposite phase it will get cancelled.

V_{ge} minima \rightarrow Node

V_{ge} maxima \rightarrow Antinode

node \rightarrow A point of zero V_{ge} / current in standing waves.

Anti \rightarrow V_{ge} max.

Point of maximum V_{ge} / current.

Reflection. f/k

When transmission line is terminated at an impedance = characteristics impedance then there is no reflection. i.e; $Z_0 = Z_R / Z_h \Rightarrow Z_0 = Z_R$ No reflection

2. \rightarrow when $Z_0 \neq Z_R$ reflection will be there. At only some power will be absorb at the load end and rest will be reflected.
 - \rightarrow when $Z_0 \neq Z_R$ constants are not uniformly distributed.
- ### Reflection coefficient (ρ/k)
- It is defined as the ratio of reflected V_{ge} or current to the incident V_{ge} / current denoted by k/ρ , Γ . It is a vector quantity. If its V_{ge} ratio then it's voltage reflection coefficient. If it current then it is current reflection coefficient.

$$k = \frac{V_o}{V_i}$$

$$-\Gamma = \frac{I_o}{I_i}$$

Current reflection coefficient is due to reflection current is -ve.

Mathematical formula for reflection coefficient: $(Z_R \neq Z_0)$
relation b/w Γ

Fundamental txm line eqn.

$$V = A e^{-\gamma x} + B e^{\gamma x}$$

$$I = \frac{1}{Z_0} A e^{-\gamma y} + B e^{\gamma y} \quad \text{--- (2)}$$

Let y be the distance from transmitting load impedance Z_R eqn (1) & (2) in terms of y .
put $\alpha = \gamma$.

$$V = A e^{\gamma y} + B e^{-\gamma y} \quad \text{--- (3)}$$

$$I = \frac{1}{Z_0} (A e^{\gamma y} - B e^{-\gamma y}) \quad \text{--- (4)}$$

When $y=0$

$$V = V_R, \text{ & } I = I_R.$$

$$V_R = A + B.$$

$$I_R = \frac{A - B}{Z_0}$$

$$A + B = V_R \quad \text{--- (5)}$$

$$A - B = Z_0 I_R \quad \text{--- (6)}$$

$$\therefore 2A = V_R + Z_0 I_R.$$

$$A = \frac{V_R + Z_0 I_R}{2}$$

$$(5)-(6) \quad B = \frac{V_R - Z_0 I_R}{2}$$

$$K = \frac{V_R}{V I^0} = \frac{B e^{-\gamma y}}{A e^{\gamma y}} = \frac{B}{A} e^{-2\gamma y}$$

$$@ y=0 \quad K = \frac{B}{A} e^{-2\gamma y} = \frac{B}{A} = \frac{V_R - Z_0 I_R}{V_R + Z_0 I_R} = \frac{V_R - Z_0 I_R}{2}$$

$$K = \frac{V_R - Z_0 I_R}{V_R + Z_0 I_R} = \frac{V_R/I_R - Z_0}{V_R/I_R + Z_0}$$

$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$\boxed{Z_R = 2Z_0} \text{ - both are even.}$$

R.C for mismatched line

$$K = \frac{2 \sqrt{Z_1 Z_2}}{Z_1 f Z_2}$$

* Expression for I/p Impedance in terms of Reflection coeff. IC DD

Input Impedance of transmission line is.

$$Z_S = Z_{in} = Z_0 \left[\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l} \right]$$

Change $\cosh \gamma l$ & $\sinh \gamma l$ is exponential.

$$Z_S = Z_0 \left[\frac{Z_R \cdot \left(\frac{e^{+\gamma l} - e^{-\gamma l}}{2} \right) + Z_0 \left(\frac{e^{+\gamma l} - e^{-\gamma l}}{2} \right)}{Z_0 \left(\frac{e^{+\gamma l} - e^{-\gamma l}}{2} \right) + Z_R \left(\frac{e^{+\gamma l} - e^{-\gamma l}}{2} \right)} \right]$$

$$= Z_0 \left[\frac{\frac{+ \gamma l}{2} \cdot [Z_R + Z_0] + \frac{- \gamma l}{2} [Z_R - Z_0]}{\left(Z_0 + Z_R \right) \frac{\gamma l}{2} + \frac{e^{-\gamma l}}{2} (Z_0 - Z_R)} \right]$$

\therefore NR seen by $\frac{e^{j\omega t}}{2}(Z_R + Z_0)$.

$$Z_S = Z_{in} = Z_0 \left[\frac{1 + e^{-2\gamma l} \left[\frac{Z_R - Z_0}{Z_R + Z_0} \right]}{1 - e^{-2\gamma l} \left(\frac{Z_0 - Z_R}{Z_R + Z_0} \right)} \right]$$

$$Z_S = Z_0 \left[\frac{1 + k e^{-2\gamma l}}{1 - k e^{-2\gamma l}} \right] \quad \text{eqn used is antenna impedance.}$$

Standing wave ratio SWR.

When transmission line is not correctly terminated, the travelling EM wave from generator @ the sending end is reflected completely (OR) partially @ the termination. The combination of incident and reflected waves gives rise to Interference phenomenon occurs and standing wave (cf) and voltage occurs along the line, with minima & maxima of current and voltage.

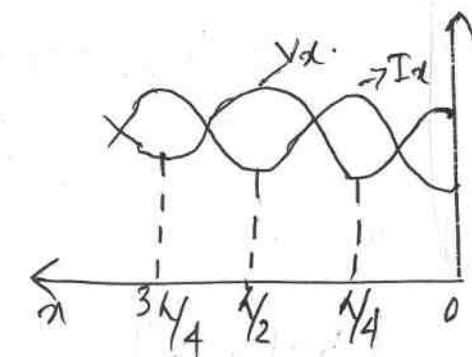
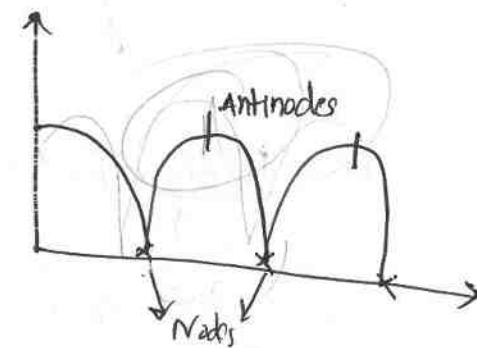
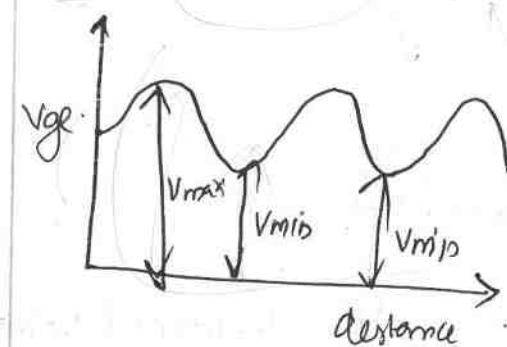
If the line loss is small the vges minima and maxima can be assumed to have equal amplitude then SWR is defined as

"The ratio of maximum to minimum cf or voltage in a line having standing waves!"
Def. by SWR \rightarrow 's
voltage SWR \rightarrow VSWR.

$$\boxed{\text{VSWR} = \left| \frac{V_{max}}{V_{min}} \right|} \quad \text{IC}$$

$$\rho = \left| \frac{V_{max}}{V_{min}} \right| = \left| \frac{I_{max}}{I_{min}} \right|$$

- Value 1 to x.



standing wave on resistor
termination lossless line

$R_L > R_0$

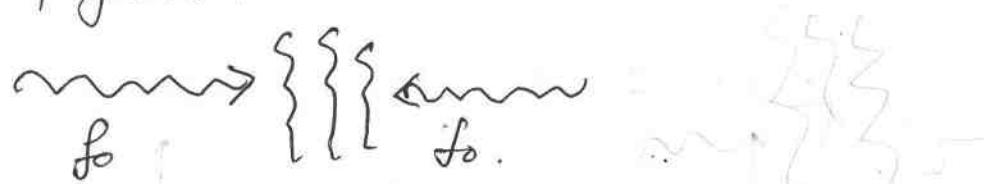
SWR a loss less line

Node - pt of zero vges (or) ct

Antinode - pt of maximum vges (or) ct'

x Standing wave

"When two waves of exactly same mag of freq travelling opposite to each other the result is not a wave - but an oscillation with no propagation".



Relation b/w SWR & Reflection coeffl.

SWR $\propto \Gamma$

V_i^o \Rightarrow Incident voltage (rms)

$$V_{max} = |V_i^o| + |V_r|$$

V_r \Rightarrow Reflected vge (rms value)

$$V_{min} = |V_i^o| - |V_r|$$

$$V_{SWR} = \left| \frac{V_{max}}{V_{min}} \right| = \left| \frac{|V_i^o| + |V_r|}{|V_i^o| - |V_r|} \right|$$

$V \Rightarrow$ inphase $I \Rightarrow$ out of phase

$$\therefore V_i^o \quad V_{SWR} = \frac{1 + \left| \frac{V_r}{V_i^o} \right|}{1 - \left| \frac{V_r}{V_i^o} \right|} \Rightarrow V_{SWR} = \frac{1 + |\kappa|}{1 - |\kappa|}$$

$$\frac{V_{SWR} - 1}{V_{SWR} + 1} = \frac{1 + |\kappa| - 1 + |\kappa|}{1 + |\kappa| + 1 - |\kappa|} = \frac{2|\kappa|}{2}$$

$$|\kappa| = \frac{V_{SWR} - 1}{V_{SWR} + 1}$$

$$|\kappa| = \frac{s-1}{s+1}$$

$$V_{SWR} = \frac{1 + |\kappa|}{1 - |\kappa|}$$

$$\frac{1 + |\kappa|}{1 - |\kappa|} - 1$$

$$\frac{1 + |\kappa| + 1 + |\kappa|}{1 + |\kappa| + 1 - |\kappa|}$$

$V_{max} \Rightarrow V_r \text{ & } V_i^o$ are inphase

$V_{min} \Rightarrow V_r \text{ & } V_i^o$ out of phase

@ V_{max} I_{max} I_{min}

@ V_{min} I_{max}

$$\frac{Z_R - Z_0}{Z_R + Z_0} = \kappa$$

$\left\{ \begin{array}{l} V_{ge} \Rightarrow \text{Incident & reflected wave add} \\ \text{ct} \\ \text{relation} \rightarrow \text{add, subtract} \end{array} \right\}$

$$I_{min} = \frac{|V_i^o| - |V_r|}{Z_0}$$

At this position Impedance is purely resistive and has maximum value.

$$Z_{max} = \frac{V_{max}}{I_{min}} = \frac{|V_i^0| + |V_o|}{\frac{|V_i^0| - |V_o|}{Z_0}}$$

$$Z_{max} = Z_0 \left[\frac{1 + \left(\frac{|V_o|}{|V_i^0|} \right)}{1 - \left(\frac{|V_o|}{|V_i^0|} \right)} \right]$$

$$Z_{max} = Z_0 \left[\frac{1 + |k|}{1 - |k|} \right].$$

$$Z_{max} = Z_0 * V_{SWR}$$

@ V_o maxima pt \Rightarrow C_l minima

$$I_{max} = \frac{|V_i^0| + |V_o|}{Z_0}$$

$$Z_{min} = \frac{V_{min}}{I_{max}} = Z_0 \left[\frac{|V_i^0| - |V_o|}{|V_i^0| + |V_o|} \right] = \frac{Z_0}{\frac{1+|k|}{1-|k|}}$$

$$Z_{min} = \frac{Z_0}{V_{SWR}}$$

continuation

* V_o and C_l when the line is short ckted @ far end

i.e; C_l the $Z_R = 0$; No C_l flow through the load.

$$\text{From (24)} \quad Z_S = Z_0 \left[\frac{\cosh \delta l + \frac{Z_0}{Z_R} \sinh \delta l}{\frac{Z_0}{Z_R} \cosh \delta l + \sinh \delta l} \right]$$

$$Z_C = Z_S = Z_0 \left[\frac{Z_0 \sinh \delta l}{Z_0 \cosh \delta l} \right] \quad (25)$$

$$Z_{SC} = Z_0 \tanh \delta l \quad (26)$$

when losses are higher (or) line is longer $\delta l \rightarrow \infty$.

$$Z_{SC} = Z_0 \quad (27)$$

V_o and C_l are obtained by putting $Z_R = 0$ in eq. 20.

$$V_o = \frac{V_s \sinh \delta l (l-x)}{\sinh \delta l} \quad (28)$$

$$C_l = \frac{V_s \cosh \delta l (l-x)}{Z_0 \sinh \delta l} \quad (29)$$

I/p impedance of lossless RF Transistor Line (OR) Low-loss loss Transistor line.

For a lossless line $R=0$ & $C=0$.

$$\gamma = \alpha + j\beta, \alpha = 0 \text{ as it lossless.}$$

$$\gamma = j\beta$$

Sub this in eq: 24

$$Z_s = Z_0 \left[\frac{Z_R \cosh(j\beta l) + Z_0 \sinh(j\beta l)}{Z_0 \cosh(j\beta l) + Z_R \sinh(j\beta l)} \right]$$

$$\cosh(j\beta l) = \cos \beta l.$$

$$\sinh(j\beta l) = j \sin \beta l.$$

$$Z_s = Z_0 \left[\frac{Z_R \cos \beta l + j Z_0 \sin \beta l}{Z_0 \cos \beta l + j Z_R \sin \beta l} \right] \quad (24)$$

(OR) $\therefore N_t$ and D_t by $\cos \beta l$.

$$Z_s = Z_0 \left[\frac{Z_R + j Z_0 \tan \beta l}{Z_0 + j Z_R \tan \beta l} \right]. \quad (25)$$

$Z_0 \rightarrow$ chara impedance.

$Z_R \rightarrow$ load impedance (terminated)

$$\beta = \frac{2\pi}{\lambda} \rightarrow \text{phase const}$$

$l \rightarrow$ length of the line

$Z_s \rightarrow$ I/p Impedance (or) sendnd end impedance.

* characteristics Impedance (Z_0)

$$\text{here } \frac{V}{I} = Z_0$$

$$\frac{V_s e^{-j\beta l}}{V_s / Z_0 e^{-j\beta l}} = Z_0$$

Ratio of V to $I \rightarrow Z_0$ o/p impedance.

Impedance looking into infinite length

represented by Z_0 .

def:- Impedance measured at the I/p of the line when its length is infinite (or) Impedance looking into an infinite lengths line."

A line having series of shunt impedance \rightarrow must have Z_0 .

Also called surge impedance.

Unit -.

$$-\frac{dv}{dx} = I(R + j\omega L) \quad \textcircled{1}$$

$$V = V_s e^{-\gamma x} \quad ? \quad \text{Val F of infinite line}$$

$$I = I_s e^{-\gamma x} \quad \textcircled{2}$$

$$\frac{V_s}{I_s} = Z_0 = Z_s$$

Sub in eq, \textcircled{1}

$$-\frac{d}{dx} V_s e^{-\gamma x} = I_s e^{-\gamma x} [R + j\omega L]$$

$$-V_s(-\gamma) e^{-\gamma x} = I_s e^{-\gamma x} (R + j\omega L)$$

$$\frac{V_s}{I_s} = \frac{(R + j\omega L)}{\gamma} = \frac{R + j\omega L}{\sqrt{(R + j\omega L)(G + j\omega C)}}$$

$$= \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{Z}{Y}} \quad \textcircled{3}$$

$$Z_0 = \sqrt{Z_0 C Z_{sc}}$$

when freq is very small

$$Z_0 = \sqrt{R/G}$$

- It determines the chara of line

- Equa of chara: imp of line: When freq is very large

- chara of co-axial cable:

$\kappa \Rightarrow$ dielectric constant of insulation.

$D \Rightarrow$ inner dia of ± 1 outer conductor

$d \Rightarrow$ dia of inner conductor

$\delta \Rightarrow$ centre to centre spacing b/w two conductors

$R \Rightarrow$ radius.

chara: Impedance @ RF

$j\omega L \gg R$ & $j\omega C \gg G_L$ (i.e. R & G_L are zero
 $R = 0 = G_L$)

$$Z_0 = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{L/C}$$

$$Z_0 = \sqrt{L/C}$$

Propagation const (γ)

Nature in which wave are propagated.

- le vge and ct¹ vary with distance x.

$$V = V_s \cosh \gamma x - I_s Z_0 \sinh \gamma x$$

$$I = I_s \cosh \gamma x - \frac{V_s}{Z_0} \sinh \gamma x$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$\gamma = \sqrt{Z Y}$$

$$\gamma = \alpha + j\beta$$

when line is lossless

$\gamma \rightarrow$ purely Imaginary

$\gamma \propto$ to freq

* lossy line

$\gamma \rightarrow$ complex

- No unit as its ratio of V to I.

$$\gamma = \log_c \frac{I_s}{I_R} \text{ (OR) } \log_e \frac{V_s}{V_R} \quad \boxed{\text{--- I.}}$$

defined as

* Natural log. of vector ratio of steady state vge (OR) ct¹ entering and leaving the structure.

* Attenuation const : q phase const:

$$\gamma = \alpha + j\beta$$

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{j\omega L \left(1 + \frac{R}{j\omega L}\right) + j\omega C \left(1 + \frac{G}{j\omega C}\right)}$$

$$\gamma = j\omega \sqrt{LC} \sqrt{\left(1 + \frac{R}{j\omega L}\right) \left(1 + \frac{G}{j\omega C}\right)} - j\omega \sqrt{LC} \left(\frac{1+R}{j\omega L}\right)^{1/2} \\ \left(\frac{1+G}{j\omega C}\right)^{1/2}$$

Since $\omega_L \gg R$ & $\omega_C \gg G \Rightarrow$ at RF

$$\frac{R}{\omega L} \ll 1 \quad \frac{G}{\omega C} \ll 1$$

expand using binomial theorem.

$$\gamma = j\omega \sqrt{LC} \left[\left(1 + \frac{1}{2} \frac{R}{j\omega L} + \dots\right) \left(1 + \frac{1}{2} \frac{G}{j\omega C} + \dots\right) \right]$$

$$= j\omega \sqrt{LC} \left[1 + \frac{1}{2} \frac{G}{j\omega C} + \frac{R}{2j\omega L} + \dots \right]$$

$$\gamma = \alpha + j\beta = j\omega \sqrt{LC} + \frac{G}{2} \sqrt{LC} + \frac{R}{2} \sqrt{LC}$$

equating real and imaginary parts

$$\alpha = \frac{R}{2} \sqrt{\frac{C}{L}} + \frac{G}{2} \sqrt{\frac{L}{C}}$$

$$\beta = \omega \sqrt{LC}$$

$$\beta = \omega \sqrt{LC}$$

$$V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

$$V_p = \frac{1}{\sqrt{LC}}$$

$$V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

$$\alpha = \frac{R}{2} \sqrt{\frac{C}{L}} + \frac{G}{2} \sqrt{\frac{L}{C}}$$

$$\alpha = \frac{R}{2} \sqrt{\frac{C}{L}} + \frac{G}{2} \sqrt{\frac{L}{C}}$$

$\alpha \Rightarrow$ attenuation.

- i.e. exp. the reduction of vge of ct¹ along the line
- unit Nepes.

* Expression for α & β in terms of P const

1° constants $\rightarrow R, L, C, G$

2° \propto II $\rightarrow \gamma, \alpha, \beta, Z_0$.

$$\alpha = \pm \sqrt{\frac{1}{2} (R C_T - \omega^2 C) + \sqrt{(R^2 + \omega^2 L^2)(C_T^2 + \omega^2 C^2)}}$$

$$\beta = \pm \sqrt{\frac{1}{2} (\omega^2 L C - R C_T) + \sqrt{(R^2 + \omega^2 L^2)(C_T^2 + \omega^2 C^2)}}$$

telephone
cable. $Z_0 = \sqrt{\frac{R}{\alpha C}} < 45^\circ$

- wavelength (λ). $f = c/\lambda$

$$\lambda = \frac{2\pi}{\beta}$$

definition

→ phase velocity (or) velocity of propagation (v_p).

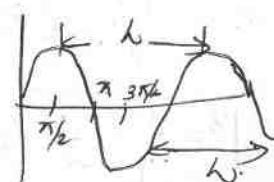
$$v_p = h, \quad v_p = \omega/\beta$$

→ GP velocity v_g

$$v_g = \frac{d\omega}{d\beta} \\ v_g = \frac{\omega_2 - \omega_1}{\beta_2 - \beta_1}$$

h → h is defined as the distance in which the phase change of 2π radians is affected by a travelling wave along the line. (or)
of a cycle.

the distance b/w two successive (or) peaks. $\beta h = 2\pi$
 $h = \frac{2\pi}{\beta}$



$$Z_0 = \sqrt{\frac{Z}{Y}} ; Y = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$Z_0 * Y = R + j\omega L$$

$$\frac{Y}{Z_0} = G + j\omega C$$

GP impedance of open or short circled line.

$$Z_{oc} = Z_0 \tanh h \Gamma l$$

$$Z_{sc} = Z_0 \operatorname{tanh} h \Gamma l$$

$$Z_0 = \sqrt{Z_{oc} Z_{sc}}$$

$$\tanh h \Gamma l = \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

open or short circled lossless transmission line

$$Z_{oc} = j \omega C \beta l$$

$$Z_{sc} = j Z_0 \tan \beta l$$

1. calculate SWR and reflection coefficient of a line having $Z_0 = \frac{300 \Omega}{\text{SWR}}$ and terminated at. $Z_R = 300 + j 400 \Omega$

$$k = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{300 + j 400 - 300}{300 + j 400} = 0.3076 + j 0.46 \\ = 0.555, \angle 56.31^\circ$$

$$S = \frac{1 + |k|}{1 - |k|} = \frac{(1 + 0.3076)}{(1 - 0.3076)} = \frac{1.3076}{0.6924} = 1.875$$

- Q. A line having characteristics impedance of 50Ω is terminated in load impedance $95 + j75\Omega$. Determine the reflection coefficient and V_{SWR}

$$\text{Ans } k = 0.542 < 40.6.$$

$$S = 3.369$$

3. A line with characteristics impedance of $Z_0 = 69\Omega < -12^\circ \text{ rad}$ is terminated in 200Ω resistor. Determine k and S .

$$\text{Ans} = 0.559 < 142.5^\circ$$

$$S = 3.535$$

- 4) A transmission line two miles long operates at 10 kHz and has parameters $R = 50\Omega/\text{mile}$, $C = 80\text{nF/mile}$, $\mu = 2.2\text{mH/mile}$; $G_1 = 20\text{mS/mile}$ propagation constant α , find the characteristic impedance Z_0 , γ , phase constant β .

$$Z_p = \sqrt{\gamma}$$

$$Z = R + j\omega L$$

$$Y = G_1 + j\omega C$$

$$\kappa = 30 + 2\pi \times 10 \times 10^3 \times 2 - 2 \times 10^{-6}$$

$$\gamma = 20 \times 10^{-9} + j 2\pi \times 10 \times 10^3 \times 80 \times 10^{-9}$$

$$Z = 30 + 44\pi j = 30 + 138 j$$

$$Y = 20 \times 10^{-9} + 502 \times 10^{-9} j$$

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{30 + j \cdot 138}{20 \times 10^{-9} + j \cdot 5.02 \times 10^{-3}}} = \sqrt{\frac{30 + j \cdot 138}{(20 \times 10^{-9}) + j \cdot 5.02 \times 10^{-3}}} = \sqrt{\frac{30.00031 \angle 0.263}{5.02 \times 10^{-3} \angle 89.99}} = \underline{5.444 \angle 0.512}$$

$$0.0 + j0.85 \angle 9.46^\circ$$

$$= 4 \angle 30^\circ \angle -8.97^\circ$$

$$\gamma = \sqrt{ZY} = \sqrt{(30 + j \cdot 138) \cdot (20 \times 10^{-9} + j \cdot 5.02 \times 10^{-3})} = \sqrt{(30.00031 \angle 0.263) \times (5.02 \times 10^{-3} \angle 89.99)} = 0.3880 \angle 9.998$$

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{22.11 \angle 62.68^\circ}{4.85 \times 10^5 \angle 89.65^\circ}} = \underline{4.357284 \angle -1.55^\circ}$$

- Q. transmission line has $f = 100 \text{ MHz}$
 $R = 0.15 \Omega$ $L = 3.93 \text{ mH}$ $C = 0.0097 \mu\text{F}$
 $G = 0.29 \mu\text{S}$. determine Z_0 at γ .

$$Z = R + j\omega L$$

$$= 0.15 + j \frac{2\pi \times 10^8 \times 3.93 \times 10^{-3}}{10^6} = 0.15 + j \frac{2\pi \times 10^8 \times 0.0097 \times 10^{-3}}{10^6} = \underline{10.15 + j246.92j}$$

$$= 10.15 + j19.65$$

$$Z = 22.11 \angle 62.68^\circ$$

$$Y = G + j\omega C$$

$$= 0.29 \times 10^{-6} + j \frac{2\pi \times 10^8 \times 500 \times 10^{-9}}{10^6} = 0.29 \times 10^{-6} + j4.85 \times 10^{-5}$$

$$Y = 4.85 \times 10^{-5} \angle 89.65^\circ$$

- Q. A transmission line has $f = 500 \text{ MHz}$ $R = 5 \Omega$ $C = 300 \text{ pF}$ $L = 0.1 \mu\text{H}$ $G = 0.01 \text{ S}$, Z_0 , γ obtains same parameter for a lossless line?

$$R = 5 \Omega$$

$$Z_0 = \sqrt{\frac{Z}{Y}} \quad Y = \sqrt{ZY}$$

$$Z = R + j\omega L = 5 + j \frac{2\pi \times 10^8 \times 0.1 \times 10^{-6}}{10^6} = 5 + j314.159 = 314.198 \angle 89.088^\circ$$

$$Y = G + j\omega C = 0.01 + j \frac{2\pi \times 10^8 \times 300 \times 10^{-12}}{10^6} = 0.9425 \angle 89.39^\circ$$

$$Y = \sqrt{\frac{Z}{Y}} = 18.26 \angle 0.15^\circ$$

$$Y = \sqrt{ZY} = 17.20 \angle 18.89^\circ$$

$$\gamma = R + j\beta = 16.27 + j5.56$$

$$\alpha = 16.27 \quad \beta = 5.56$$

for lossless $R = G = 0 \therefore \alpha = 0$

$$\begin{aligned} \beta &= \omega\sqrt{LC} = 2\pi \times 500 \times 10^6 \sqrt{0.1 \times 10^{-6} \times 300 \times 10^{-12}} \\ &= 17.20 \text{ rad/m.} \quad Z_0 = \sqrt{L/C} \\ &= 18.25 \Omega \end{aligned}$$

3 special cases

- 1* The line is terminated at its characteristic impedance ie $Z_L = Z_0$ $(R \rightarrow Z_L)$

$$Z_{in} = Z_0$$

- 2* If the line is short circuited at the receiver end
ie $Z_L = 0$

$$Z_{in} = Z_0 \tanh h\gamma L$$

$$Z_{in} = Z_{sc}$$

- 3* If the line opens circuited at the receiver end
ie $Z_L = \infty$ $Z_{in} = Z_0 \tanh h\gamma L$

$$Z_{in} = Z_0$$

$$Z_0 = \sqrt{Z_0 C Z_{sc}}$$

lossless TXION since $R = 0, G = 0$.

$$\begin{aligned} Z_0 &= \sqrt{\frac{L}{C}} \\ \beta &= j\omega\sqrt{LC} \end{aligned}$$

$$\begin{aligned} \alpha &= 0 \\ \beta &= j\omega\sqrt{LC} \end{aligned}$$

Distortion less line

$$\frac{R}{G} = \frac{L}{C}$$

A line is said to be distortion less if it has an attenuation constant α independent of frequency and β which is linearly depending up on frequency and characteristic impedance Z_0 .
Independent of frequency.

distortion less line means no frequency or phase distortion

$$\begin{aligned} Z_0 &= \sqrt{\frac{Z}{Y}} \\ &= \sqrt{R+j\omega L} \\ &= \sqrt{G+j\omega C} \end{aligned}$$

$$\text{We have } \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{Zy}$$

$$= \sqrt{j\omega L \left(1 + \frac{R}{j\omega L}\right) j\omega C \left(1 + \frac{G}{j\omega C}\right)}$$

$$= j\omega \sqrt{LC} \sqrt{\left(1 + \frac{G}{j\omega C}\right) \left(1 + \frac{R}{j\omega L}\right)} \quad \left\{ \begin{array}{l} \frac{R}{G} = \frac{L}{C} \\ \frac{R}{L} = \frac{G}{C} \end{array} \right.$$

$$= j\omega \sqrt{LC} \left(1 + \frac{G}{j\omega C}\right)$$

$$Y' = j\omega \sqrt{LC} + j\omega G \cdot \frac{G}{j\omega C}$$

$$= j\omega \sqrt{LC} + \sqrt{LG} \cdot \frac{G}{C}$$

$$Y' = j\omega \sqrt{LC} + \sqrt{\frac{L}{C}} G$$

$$Y' = G\sqrt{\frac{L}{C}} + j\omega \sqrt{LC}$$

$$\alpha = G\sqrt{\frac{L}{C}}$$

$$\beta = \omega \sqrt{LC}$$

If we take R of G_2 out side.

$$\gamma = \sqrt{RG_1} + j\omega \sqrt{2C}$$

$$\alpha = \sqrt{RG_2} \quad \beta = \omega \sqrt{2C}$$

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{R(1+j\omega L)}{G(1+j\omega C)}}$$

$$\frac{R}{G} = \frac{L}{C}; \quad \frac{L}{R} = \frac{C}{G}$$

$$Z_0 = \sqrt{\frac{R(1+j\omega C)}{G(1+j\omega L)}}$$

$$Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$$

$$Y_{Z_0} = R + j\omega L$$

$$\frac{\gamma}{Z_0} = \alpha + j\omega C$$

$$\tanh \gamma l = \sqrt{Z_0 Z_{SC}}$$

Input Impedance of a half wavelength ~~at~~ $\lambda/2$ line
which is loss less

$$Z_0 = \sqrt{Z_0 \cdot Z_{SC}}$$

$$\text{i.e. } Z_0 = Z_{SC}$$

$$\text{Velocity } v = \frac{\omega}{\beta} = f\lambda$$

	γ	Z_0
General	$\sqrt{Rfj\omega L} + j\omega C$	$\sqrt{Rfj\omega L}$
lossless	$j\omega C$	$\sqrt{\gamma_C + j\omega}$
Distortion less.	$\sqrt{R_0} + j\omega C$	$\sqrt{\gamma_C + j\omega}$

- Q. A generator of 1V, 1kHz supplies power to a ~~length~~ 100km longline terminated in Z_0 having the following constants $R = 10.4 \Omega/km$.

$$L = 0.0036 H/km \quad G_L = 0.8 \times 10^{-6} \Omega/km$$

$$C = 0.0084 \times 10^{-6} F/km$$

Calculate Z_0 , γ , α , β and V , λ .

$$Z_0 = \sqrt{\frac{Z}{\gamma}}$$

$$Z = R + j\omega L = 10.4 + j2\pi \times 10^3 \times 0.0036 = 10.4 + 22.6j$$

$$Y = G_L + j\omega C = -8 \times 10^{-6} + j2\pi \times 10^3 \times 0.0084 \times 10^{-6} = -8 \times 10^{-6} + 5.29 \times 10^{-6}$$

$$Z_0 = \sqrt{\frac{24.8 \times 65.2}{5.4 \times 10^{-6} \times 88.15}} = 62 \angle 647.6^\circ \angle -1.86^\circ$$

$$\gamma = \sqrt{24}$$

$$= \sqrt{24.8 \times 5.4 \times 10^{-5}} < 65.2 + 88.15$$

$$= 0.0365 < 17.81$$

$$\gamma = \alpha + \beta j = 0.034 + 0.010j$$

$$\alpha = 0.034$$

$$\beta = 0.010$$

$$V = \frac{\omega}{\beta} = \frac{2\pi \times 10^3}{0.010} = 628 \times 10^3 \text{ km/h} = \frac{3 \text{ h}}{\frac{2\pi}{\beta}} = \frac{3 \text{ h}}{0.010} = 628.31 \text{ km}$$

- Q. A transmission line has following parameters per km. $R = 15.2$, $C = 15 \mu F$, $L = 1 mH$ and $G_L = 1 \mu \Omega$. Find the additional inductance to give distortionless transmission?

$$\frac{R}{G} = \frac{L'}{C}$$

$$L' = \frac{RC}{G} = \frac{15 \times 10^{-6} \times 15}{1 \times 10^{-6}} = 2.25 \times 10^{-6} \text{ H}$$

$$\text{Additional Inductance } 2.25 - 1 \times 10^{-6} = 224.919$$

Q. A distortion line has $Z_0 = 60\Omega$ $\alpha = 20 \text{ mNp/m}$

$u = 0.6c$, where c is the speed of light in
(c , parameter) vacuum. Find R, L, C, G at $\lambda @ 100 \text{ MHz}$?

Ans

$$\frac{R}{G} = \frac{k}{\alpha}$$

$$\frac{R}{G} = \frac{L}{C}$$

$$G = \frac{RC}{L}$$

$$G = \frac{RC}{L}$$

$$Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$$

$$(60) \left(\sqrt{\frac{L}{C}} \right)^2$$

$$\alpha = \sqrt{RG}$$

$$= \sqrt{R \cdot \frac{RC}{L}} = R \sqrt{\frac{C}{L}}$$

$$\alpha = \frac{R}{Z_0} \quad \text{--- (3)}$$

$$R = \alpha Z_0 \quad \text{--- (3)} = 20 \times 10^3 \times 60 = 1.2 \Omega \text{ per m}$$

$$u = \frac{\alpha}{B} = \frac{1}{VLC} \quad \text{--- (4)}$$

$$\text{--- (1)} \div \text{--- (4)} \quad L = \frac{Z_0}{u} = \frac{60}{0.6 \times 10^3 \times 10^8} = 333 \text{ nH}$$

$$\therefore G = \frac{\alpha^2}{R} = \frac{400 \times 10^6}{1.2} \\ = 333 \text{ Msi/m}$$

$$\alpha = \sqrt{RG}$$

$$\sqrt{RG} = \frac{R}{u}$$

$$(\sqrt{RG})^2 = \frac{(R)^2}{(u)^2}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$u = \frac{1}{\sqrt{LC}}$$

$$\frac{Z_0}{u} = \frac{\sqrt{L}}{\sqrt{C}} \Rightarrow L = \frac{Z_0}{u}$$

$$RG = \frac{R^2}{u^2} \times \frac{L}{K}$$

$$Z_0^2 = RG$$

$$C = \frac{\alpha^2}{R}$$

$$u Z_0 = \frac{1}{\sqrt{LC}} \sqrt{L} = \frac{1}{C}$$

$$C = \frac{1}{u Z_0} = \frac{1}{0.6 \times 3 \times 10^8 \times 60}$$

$$C = 92.59 \text{ pF/m}$$

$$L = \frac{Z_0}{u} = 1.8 \text{ H}$$

airline - loss less line

Q. A loss less transmission line has characteristic impedance 40Ω and $B = 3 \text{ rad/m}$ at 100 MHz . Calculate the inductance / m of the capacitance / m?

$$G=0$$

$$R=\alpha=0$$

$$\omega=0$$

$$Z_0 = \sqrt{L/C} \quad \text{--- (1)}$$

$$\beta = \omega\sqrt{LC} \quad \text{--- (2)}$$

$$\textcircled{1} : \textcircled{2} \quad \left(\frac{Z_0}{\beta}\right)^2 = \left(\frac{\sqrt{L}}{\sqrt{C}}\right)^2$$

$$= \frac{L}{C} \times \frac{1}{\omega^2 LC}$$

$$= \frac{1}{C^2 \omega^2}$$

$$\frac{Z_0}{\beta} = \frac{1}{C\omega}$$

$$C = \frac{\beta}{Z_0 \omega} = \frac{3}{70 \times 2\pi \times 100 \times 10^6}$$

$$= \underline{68.2 \text{ pF/m}}$$

$$Z_0 = \sqrt{L/C}$$

$$Z_0 \times V_C = VL$$

$$L = Z_0^2 C = 834.18 \times 10^{-9}$$

$$= \underline{334.18 \text{ nH/m}}$$

→ A lossless line is distortion less line but distortion less line ^{is} need not be lossless.

Q. A transmission line operating @ 500 MHz has

$$Z_0 = \cancel{80 \Omega} \quad \alpha = 0.04 \text{ Np/m} \quad \beta = 1.5 \text{ rad/m}$$

find line parameters R, L, C &

$$\gamma Z_0$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(C + j\omega C)} \quad \text{--- (1)}$$

$$\gamma = \cancel{0.04 + j1.5}$$

$$Z_0 = \frac{(R + j\omega L)}{(C + j\omega C)} \quad \text{--- (2)}$$

$$(\sqrt{R_f j\omega L})^2$$

$$\omega L = 120$$

$$L = \frac{120}{2\pi \times 500 \times 10} = \underline{38.19 \text{ nH}}$$

$$\gamma Z_0 = \cancel{(R + j\omega L)(C + j\omega C)} \times \frac{\cancel{(R + j\omega L)}}{\cancel{(C + j\omega C)}}$$

$$\gamma Z_0 = R + j\omega L = (0.04 + j1.5) 80 = 3.2 + j120$$

$$R = \underline{3.2}$$

$$\frac{\gamma}{Z_0} = \frac{\cancel{(R + j\omega L)(C + j\omega C)}}{\sqrt{R_f L}} \times \frac{\cancel{(C + j\omega C)}}{\cancel{VR + j\omega C}} = G + j\omega C$$

$$\frac{0.04 + j1.5}{80} = 5 \times 10^{-4} + 0.018 + j$$

$$C = \cancel{5 \times 10^{-4} \Omega} = \underline{5.16 \text{ pF}}$$

$$C = \frac{0.018 + j}{2\pi \times 500 \times 10} = \underline{5.16 \text{ pF}}$$

$$C = \frac{0.018 + j}{2\pi \times 500 \times 10} = \underline{5.16 \text{ pF}}$$

- Q. A transmission line with $Z_0 = 300\Omega$ terminated in a resistive load, the minimum of Max voltage in the line is $V_{min} = 5mV$ & $V_{max} = 7.5mV$. What is the value of load impedance?

$$K = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$S = \frac{1+|K|}{1-|K|}$$

$$S = \left| \frac{V_{max}}{V_{min}} \right| = \left| \frac{7.5mV}{5mV} \right| = \frac{1.5}{0.5} = 1.5$$

$$K = \frac{S-1}{S+1} = \frac{1.5-1}{1.5+1} = \frac{1}{2.5} = 0.4$$

$$0.2 = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (e) \quad 0.2Z_L + 0.2Z_0 = Z_L Z_0$$

$$0.2Z_L - Z_L = -Z_0 - 0.2Z_0$$

$$-0.8Z_L = -1.2Z_0$$

$$-0.8Z_L = -1.2 \times 300 = -360$$

$$Z_L = \frac{-360}{-0.8} = 450\Omega$$

- Q. A lossless transmission line 30 m long operates @ 2 MHz, the line is terminated with a load $60+j40\Omega$. If $a = 0.6c$ on the line find k , s and Z_{in} . $Z_0 = 50\Omega$

$$k = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$= \frac{60+j40 - 50+0j}{60+j40 + 50+0j}$$

$$= \frac{10+j40}{110+j40} = 0.090 \quad 0.19+0.80+0.29j \\ = 0.35 \angle 55.89^\circ$$

~~RE~~

$$|k| = 0.35$$

$$S = \frac{1+|k|}{1-|k|} = \frac{1+0.35}{1-0.35} = 2.07$$

$$Z_{in} = \frac{Z_0 \sin \beta l}{\cos \beta l}, \quad \beta = \frac{\omega}{a}$$

$$\beta l = \frac{\omega l}{a} = \frac{\pi \times 2 \times 10^6 \times 30}{0.6 \times 3 \times 10^8} = 120^\circ$$

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right] = 50 \left[\frac{60+j40 + j50 \tan 120^\circ}{50 + j(60 \tan 120^\circ)} \right] \\ = 24.0 \angle 3.22^\circ$$

Q A transmission line working @ RF has following constants

$\lambda = 9 \text{ m}$ $C = 60 \text{ pF/m}$ The line is terminated with a resistance load of 1000Ω find the reflection coefficient and SWR.

$$Z_L = 1000 \Omega$$

$$Z_0 = \sqrt{4C}$$

Rp

RF \rightarrow hyper

$$K = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$Z_0 = 750 \Omega$$

$$k = 0.142$$

Q. Find the 1° and 2° coefficients of reflection line when

Z_{sc} measured by a bridge @ 400 Hz is $286 \angle 40^\circ \Omega$ and Z_{sc} is $1550 \angle 16^\circ$

$$\tanh h \gamma l = \sqrt{Z_0 \epsilon / Z_{sc}}$$

$$= \sqrt{\frac{286 \angle 40^\circ}{1550 \angle 16^\circ}} = 0.429 \angle 2.32^\circ$$

$$= \frac{219.08 + 18.1 \cdot .83j}{1489.95 + 927.237} = \frac{219.08 + 15.1 \cdot .83j}{1489.95 + 927.237}$$

$$= 0.3834 + j$$

$$= 0.428 + 0.01736j$$

$$\tanh h \gamma l = \frac{e^{2\gamma l} - 1}{e^{2\gamma l} + 1} = 0.428 + 0.01736j$$

$$e^{2\gamma l} - 1 = (e^{2\gamma l})(0.428 + 0.01736j)$$

$$= 0.428 e^{2\gamma l} + 0.428 + 0.01736j e^{2\gamma l} + 0.01736j$$

$$e^{2\gamma l} - 1 = e^{2\gamma l} (0.428 + 0.01736j) + (0.428 + 0.01736j)$$

$$e^{2\gamma l} = e^{2\gamma l} (0.428 + 0.01736j) + (0.428 + 0.01736j)$$

$$e^{2\gamma l} - e^{2\gamma l} (0.428 + 0.01736j) = 1.428 + 0.01736j$$

$$e^{2\gamma l} (1 - 0.428 - 0.01736j) = 1.428 + 0.01736j$$

$$e^{2\gamma l} (0.572 + 0.01736j) = 1.428 + 0.01736j$$

$$e^{2\gamma l} = \frac{1.428 + 0.01736j}{0.572 + 0.01736j} = 2.492 \angle 0.163^\circ$$

$$\ln(a) = x \angle 0.163^\circ \quad 9.31 \angle 81.09^\circ$$

$$\ln(a+b) = \ln(a+jb) \quad a, b$$

$$2\gamma l = \ln [a <]$$

$$\gamma = \frac{1}{2l} [\ln a + \ln b]$$

$$= \alpha + j\beta$$

$$\alpha = ? \quad \beta = ?$$

$$Z_0 = \sqrt{Z_0 C Z_{SC}}$$

$$Z_0 l = R + j\omega L$$

$$R = ?$$

$$j\omega L = ?$$

$$L = ?$$

$$\frac{\gamma}{Z_0} = \alpha + j\omega C$$

$$G = ? \quad C = ?$$

$$[> 2] \Rightarrow \gamma = 3 \text{ rad}$$

Q. A lossless transmission line $l = 0.3 \text{ m}$.

$$Z_R = Z_L = 30 - j20 \Omega, Z_0 = 15 \Omega. \text{ find } K, S, Z_S.$$

$$Z_{in} = Z_S$$

$$K = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{30 - j20 - 15}{30 - j20 + 15}$$

$$= \frac{-15 - j20}{105 - j20} = \underline{-0.378 - 0.262j}$$

$$S = \frac{1 + |K|}{1 - |K|} = \frac{1 + 0.459}{1 - 0.459} = \frac{1.4599}{1 - 0.459} = \underline{2.07}$$

$$Z_S = Z_{in} = Z_0 \left[\frac{Z_L \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_L \sinh \gamma l} \right]$$

$\gamma = ?$ since the line is lossless

$$\alpha = 0 \quad \gamma = j\beta$$

$$\cosh \gamma l = \cosh j\beta l = \cos \beta l$$

$$\sinh \gamma l = \sinh j\beta l = j \sin \beta l$$

$$\beta l = \frac{2\pi}{\lambda} \times 0.3 \text{ m} = 0.67 \text{ rad}$$

$$Z_{in} = \frac{1}{j} \left[\frac{(30-j20) \cos(0.6\pi) + j5 \sin(0.6\pi)}{45 \cos(0.6\pi) + j5 \sin(0.6\pi)} \right] \\ = (202 - j5.19) \Omega$$

$$\text{Impedance} = \frac{j16.18}{3.6804}$$

$$\underline{Z_s} = (202 - j5.19) \Omega$$

- Q. Calculate the characteristics impedance, attenuation α , B , of following line. If following measurements made
 $Z_{DC} = 550 \angle 60^\circ \Omega$ $Z_{SC} = 500 \angle 14^\circ \Omega$.

$$Z_0 = \sqrt{Z_{DC} \cdot Z_{SC}} = \sqrt{550 \times 500 \angle 60 + 14^\circ} \\ = 524.404 \angle 11.486^\circ = 513.908 + j104.3 \Omega$$

$$\tan h \gamma l = \sqrt{\frac{Z_{SC}}{Z_0}} = 1.095 \angle 4.004^\circ \\ = 1.095 + j0.0731 \Omega$$

$$= \frac{e^{2\gamma l} - 1}{e^{2\gamma l} + 1} = 1.095 + j0.0731$$

$$(e^{2\gamma l} - 1) = (e^{2\gamma l} + 1) (1.095 + j0.0731)$$

$$e^{2\gamma l} - 1.095 e^{2\gamma l} - 0.0731 e^{2\gamma l} = 1.095 + j0.0731 + 1$$

$$e^{2\gamma l} (1 - 1.095 - j0.0731) = 2.095 + j0.0731$$

$$e^{2\gamma l} = -0.0232 - j0.0341 \\ = 0.041 \angle -124.04^\circ$$

$$2\gamma l = \ln(0.041) - j124.04^\circ$$

$$2\gamma l = -3.1941 - j124.04^\circ$$

$$\gamma = \frac{1}{2\lambda} [-3.1941 - j124.04^\circ]$$

$$= -1.597 - j62.03^\circ$$

$$Z_0 = \sqrt{Z_{DC} Z_{SC}} = \sqrt{(550 \angle 60^\circ) \cdot (500 \angle 14^\circ)} \\ = 524.404$$

$$\alpha = \frac{1}{2} \ln(-1.597) = 0.234 \text{ NP/km.}$$

$$\beta = \sqrt{\frac{Z_{DC}}{Z_{SC}}} = \sqrt{\frac{550 \angle 60^\circ}{500 \angle 14^\circ}} = 1.098 \text{ rad/km.}$$

- Q. A generator of 1V, 1kHz supplies power to a hundred km open wire line. It terminated in 200 ohms resistance. Line parameters are $R = 10 \Omega/\text{km}$, $L = 3.8 \text{ mH/km}$, $G = 1 \times 10^{-6} \text{ S/km}$, $C = 0.0085 \text{ F/km}$. Calculate the P/p impedance reflection coefficient?

$$R = 10 \Omega/\text{km}, L = 3.8 \text{ mH/km}, G = 1 \times 10^{-6} \text{ S/km}, C = 0.0085 \text{ F/km}$$

$$Z_{in} = Z_0 \left[\frac{Z_L \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_L \sinh \gamma l} \right]$$

$$Z_0 = \sqrt{\frac{Z}{Y}}$$

$$Y = \sqrt{ZY}$$

$$Z_0 = \sqrt{\frac{R + j\omega B}{G + j\omega C}} = \sqrt{\frac{10 + j2\pi \times 10^3 \times 3.8 \times 10^{-3}}{1 \times 10^{-6} + j2\pi \times 10^3 \times 0.0085 \times 10^{-6}}}$$

$$= \sqrt{\frac{10 + 23.8j}{1 \times 10^{-6} + 44 \times 10^{-6} \times 5.34 \times 10^{-5}}} = \sqrt{\frac{25.88(67.26)}{5.3409 \times 10^{-5}}} < 88.92.$$

$$= 696.16 < -1.23 = 695.19 + 14.7j$$

$$\text{i.e. } Y = \sqrt{(10 + 23.8j)(1 + 10^{-6} + 5.34 \times 10^{-5}j)}$$

$$= \sqrt{28.88 \times 10^{-5} - 3409 \times 10^{-10}} (691.20 + 14.7j)$$

$$= 14.75 < 17.63$$

$$\alpha =$$

$$\beta =$$

$$k = \frac{Z_R - Z_0}{Z_R + Z_0}$$

$$k =$$

$$e^{rl} = e^{\alpha l + j\beta l} = e^{\alpha l} e^{j\beta l}$$

$$=$$

$$e^{rl} = e^{\alpha l} e^{j\beta l}$$

$$e^{rl} = e^{-\alpha l} e^{-j\beta l}$$

$$Z_S = Z_0 \left[\frac{e^{rl} + ke^{-rl}}{e^{rl} - ke^{-rl}} \right]$$

A generator at 1 kHz supplies power to a
100 km open-wire line terminated in z_0

$$l = 100 \text{ km}$$

$$Z_s = Z_0 \left[\frac{z_R \cosh \gamma l + z_0 \sinh \gamma l}{z_0 \cosh \gamma l + z_R \sinh \gamma l} \right]$$

$$\cosh \gamma l = \cosh h(x + j\beta)$$

Brewster's angle

Also called polarizing angle denoted by θ_B .

definition

Angle of incidence for which there is no reflection then this angle θ_B is called brewster angle

$$\theta_B = \tan^{-1} \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \tan^{-1} \frac{n_2}{n_1}$$

$$\text{Reflection coefficient } \Gamma_{11} = \frac{E_2}{E_1} = \frac{\eta_2 \cos \theta - \eta_1 \cos \theta}{\eta_2 \cos \theta + \eta_1 \cos \theta}$$

By defn., $\Gamma_{11} = 0$

$$\textcircled{1} \text{ becomes } 0 = \frac{\eta_2 \cos \theta - \eta_1 \cos \theta}{\eta_2 \cos \theta + \eta_1 \cos \theta}$$

$$\eta_2 \cos \theta = \eta_1 \cos \theta \quad \text{--- } \textcircled{2}$$

For non-magnetic medium $\mu_1 = \mu_2 = \mu_0$

$$V_1 = \frac{1}{\sqrt{\mu_0 \epsilon_1}} \quad V_2 = \frac{1}{\sqrt{\mu_0 \epsilon_2}}$$

$$\frac{\sin \theta}{\sin \theta_1} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} = \frac{\eta_1}{\eta_2}$$

~~$V_1 \propto \sqrt{\mu_0 \epsilon_1}$~~ \checkmark ~~$V_2 \propto \sqrt{\mu_0 \epsilon_2}$~~

$$\text{--- } \textcircled{1} \text{ becomes } \sqrt{\frac{\epsilon_1}{\epsilon_2}} \cos \theta = \sqrt{\frac{\epsilon_0}{\epsilon_1}} \cos \theta$$

$$\frac{1}{\sqrt{\epsilon_2}} \cos \theta = \frac{1}{\sqrt{\epsilon_1}} \cos \theta$$

$$\sqrt{\epsilon_2} \cos \theta = \sqrt{\epsilon_1} \cos \theta \quad \text{--- } \textcircled{3}$$

$$\sqrt{\epsilon_2} \cos \theta = \sqrt{\epsilon_1} (1 - \sin^2 \theta)$$

$$\frac{\sin \theta}{\sin \theta_1} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \quad \therefore \sin \theta = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \sin \theta_1 \quad \text{--- } \textcircled{4}$$

Sub in $\textcircled{3}$

$$\sqrt{\epsilon_2} \cos \theta = \sqrt{\epsilon_1} \sqrt{1 - (\sqrt{\frac{\epsilon_1}{\epsilon_2}} \sin \theta)^2}$$

Squaring both sides.

$$\epsilon_2 \cos^2 \theta = \epsilon_1 (1 - \frac{\epsilon_1}{\epsilon_2} \sin^2 \theta)$$

$$\epsilon_2 \cos^2 \theta = \epsilon_1 - \frac{\epsilon_1^2}{\epsilon_2} \sin^2 \theta$$

$$\frac{\epsilon_1^2}{\epsilon_2} \sin^2 \theta = \epsilon_1 - \epsilon_2 \cos^2 \theta \\ = \epsilon_1 - \epsilon_2 (1 - \sin^2 \theta)$$

$$\frac{\epsilon_1^2}{\epsilon_2} \sin^2 \theta = \epsilon_1 - \epsilon_2 \cos^2 \theta = \epsilon_1 - \epsilon_2 (1 - \sin^2 \theta)$$

$$\frac{\epsilon_1^2}{\epsilon_2} \sin^2 \theta = \epsilon_1 - \epsilon_2 + \epsilon_2 \sin^2 \theta$$

$$\frac{\epsilon_1^2 \sin^2 \theta}{\epsilon_2} - \epsilon_2 \sin^2 \theta = \epsilon_1 - \epsilon_2$$

$$\frac{\sin^2 \theta}{\epsilon_2} (\epsilon_1^2 - \epsilon_2^2) = \epsilon_1 - \epsilon_2$$

$$\sin^2 \theta \left[\frac{(E_1 + E_2)(E_1 - E_2)}{E_2} \right] = (E_1 - E_2)$$

$$\sin^2 \phi_i (\epsilon_1 + \epsilon_2) = \epsilon_2$$

$$\sin^2 \theta_1 = \frac{E_2}{E_1 + E_2}$$

$$\begin{aligned}
 F_2 \cos^2 \theta i &= E_1 \left[1 - \frac{E_1}{E_2} \sin^2 \theta i \right] \\
 &= E_1 \left[1 - \frac{E_1}{E_2} (1 - \cos^2 \theta i) \right] \\
 &= E_1 - \frac{E_1^2}{E_2} + \frac{E_1^2 \cos^2 \theta i}{E_2}
 \end{aligned}$$

$$E_2 \cos^2 \theta_i - \frac{E_1}{E_2} \cos \theta_i = E_1 - \frac{E_1^2}{E_2}$$

$$\cos^2 \phi_1^o (E_2 - \frac{E_1}{E_2}) = \frac{E_1 E_2 - E_1^2}{E_2}$$

$$\cos^2 \theta_1 \left[\frac{E_2^2 - E_1^2}{E_2} \right] = \frac{E_1 E_2 - E_1^2}{E_2}$$

$$ws^2\theta_1(E_2+E_1)(E_2-E_1) = E_1(E_2-E_1)$$

$$\frac{\cos^2 \theta_j}{\epsilon_2 + \epsilon_1} = \frac{E_1}{E_2 + E_1} \quad \text{--- (6)}$$

$$\tan^2 \theta = \frac{\frac{E_2}{E_1 + E_2}}{\frac{E_1}{E_2 + E_1}} = \frac{E_2}{E_1}$$

$$\tan \theta = \sqrt{\frac{e_2}{e_1}}$$

According to definition of Brewster angle

$$\theta_1 = \theta_2$$

$$O_{B,11} = \tan^{-1} \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \tan^{-1} \frac{m \omega}{n}$$

- a) The dielectric constant of pure water is given
 find the Brewster angle for \perp polarisation
 σ_B^{\perp} and corresponding angle of transmission
 b) A plane wave with \parallel polarisation is incident
 from air on water surface $\sigma_1 = \sigma_B^{\parallel}$
 find the reflection and transmission coefficient.

$$\text{Ans: } \theta_B = \tan^{-1} \sqrt{\frac{E_2}{E_1}} = 83.62^\circ$$

$$\begin{aligned}
 \sin\theta t &= \frac{\sin\theta}{\sqrt{\ell_2}} \\
 \theta t &= \sin^{-1} \left(\sin\theta \sqrt{\frac{\ell_2}{\ell_1}} \right) \\
 &= 81^\circ \left(\sin 83.62^\circ \sqrt{\frac{1}{80}} \right) \\
 &= 6.38
 \end{aligned}$$