MODULE - V

$\lambda/2$ + $\lambda/4$

[redacted]			
V	Transmission line as circuit elements (L and C).	2	20
	Half wave and quarter wave transmission lines.	1	
	Development of Smith chart - calculation of line impedance and VSWR using smith chart.	2	
	Single stub matching (Smith chart and analytical method).	2	

Normal to Smith chart →

x, r c/c.

Txion Line-CIRCUIT ELEMENTS: (L&C) 4

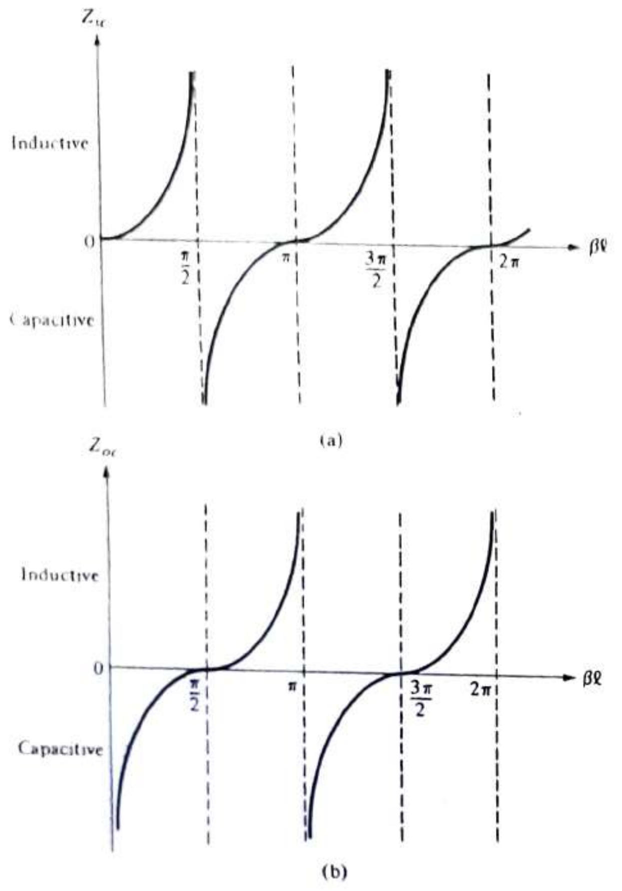


Figure 11.8 Input
a lossless line: **(a)**
(b) when open.

Transmission Line as ckt Elements. (L&C)

under ~~to~~ normal frequency TL are used as txion channels.
At very high frequency they can be used as circuit elements
Assume that the line is lossless.

$\gamma = j\beta \quad (\alpha = 0)$.

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh\gamma l}{Z_0 + Z_L \tanh\gamma l} \right]$$

$\tanh\gamma l = \tan h(\alpha + j\beta l) = j\tan\beta l = j\tan\beta l$.

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{\quad} \right] \quad \text{—— (1)}$$

Open circuited line: open circuited Line

Let the line is open circuited at the load

$Z_L \to \infty$.

$$(i) \Rightarrow \quad Z_{in} = Z_0 \left\{ \frac{\left(1 + \frac{jZ_0}{Z_L}\tan\beta l\right)}{\frac{Z_0}{Z_L} + j\tan\beta l} \right\}.$$

$$Z_{in} = Z_0 \frac{(1+0)}{0 + j\tan\beta l} = Z_0 \frac{1}{j\tan\beta l} \quad \text{—— (2)}$$

For a very short length $\tan(\beta l) \approx \beta l$.

$$(2) \Rightarrow \quad Z_{in} = \frac{Z_0}{j\beta l} = \frac{\sqrt{L/C}}{j\omega\sqrt{LC} \times l} = \frac{1}{j\omega C l} \quad \text{—— (3)}$$

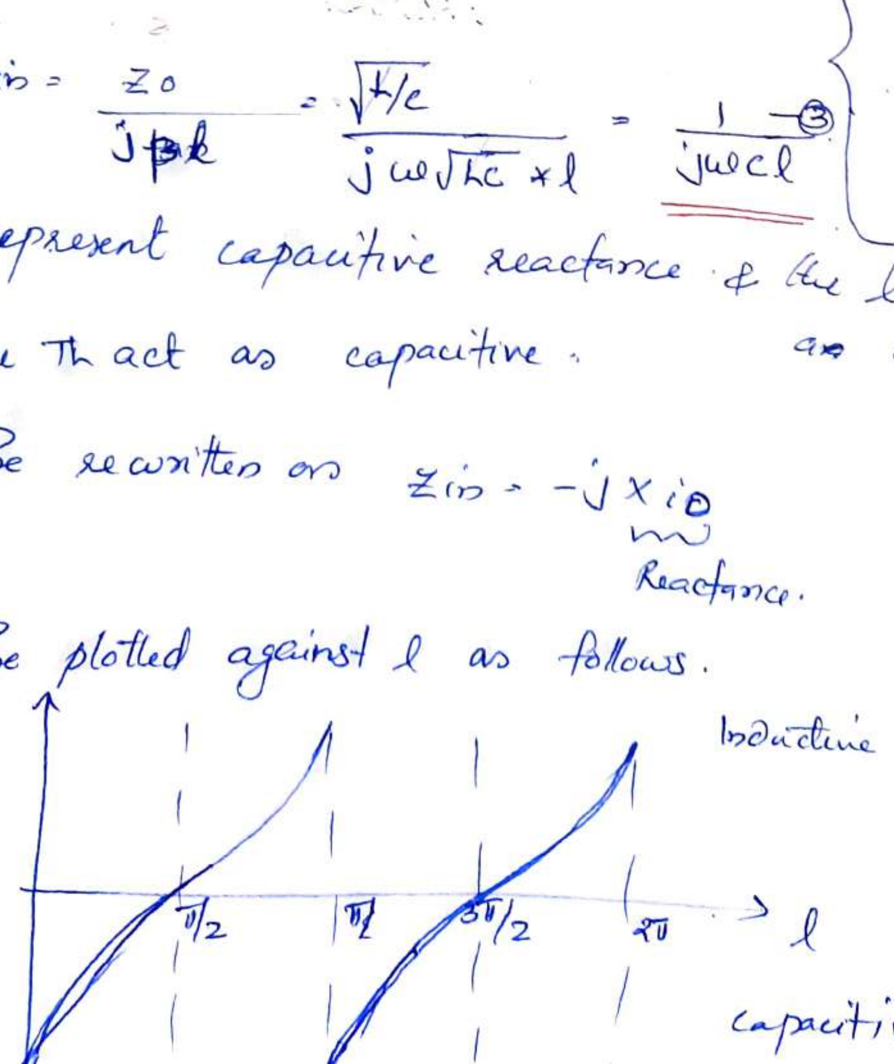
For lossless line
$R = G = 0$.
$Z_0 = \sqrt{L/C}$.
$\beta = \omega\sqrt{LC}$.

This represent capacitive reactance & the line become as capacitive

So the Tl act as capacitive.

(2) can be rewritten as $Z_{in} = -j\underbrace{X_{io}}_{\text{Reactance}}$. $X_{io} = Z_0 \cot\beta l$

X_{io} can be plotted against l as follows.



short ckted, Line: Short circuited Line

- Here the load terminals are short circuited

$Z_L = 0$

$$Z_{in} = Z_0 \frac{\left[Z_L + Z_0 \tanh \gamma l \right]}{\left[Z_0 + Z_L \tanh \gamma l \right]}$$

$\gamma = \alpha + j\beta$ as $\alpha = 0$

$\gamma = j\beta$

$$Z_{in} = Z_0 \left[\frac{0 + Z_0 \tanh \gamma l}{Z_0 + 0} \right] = Z_0 \left[\frac{\cancel{Z_0} \tanh \gamma l}{\cancel{Z_0}} \right]$$

$$Z_{in} = Z_0 \tanh \gamma l \quad \text{—④}$$

$\gamma = j\beta$

(i) $Z_{in} = j Z_0 \tanh \beta l$

$\tanh \beta l \approx \tan \beta l$

$\left.\begin{array}{l} \beta = \omega\sqrt{LC} \\ Z_0 = \sqrt{L/C} \end{array}\right\}$ For lossless line

$\therefore Z_{in} = j Z_0 \tan \beta l \quad \text{—⑤}$

For short wave length

$\tan \beta l \approx \beta l$

$Z_{in} = j Z_0 \beta l = j \sqrt{L/C} * \omega \sqrt{LC} * l = j\omega L * l$

$Z_{in} = j\omega L \cdot l \quad \text{—⑥}$

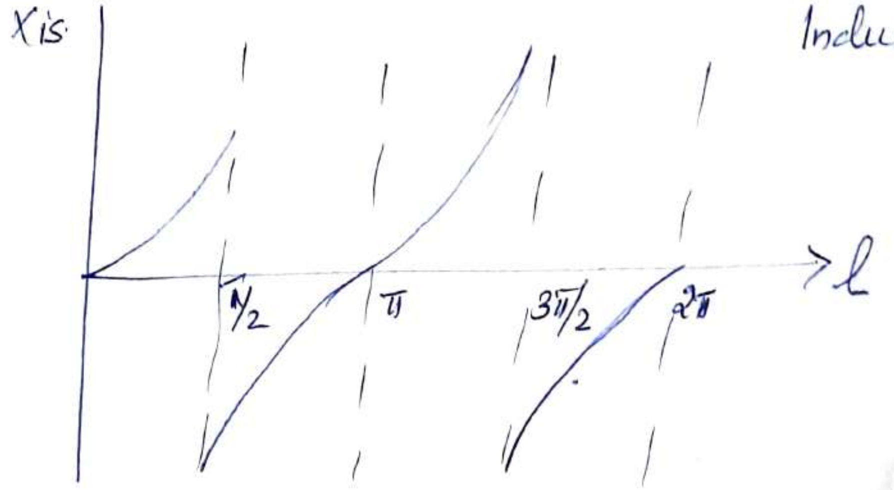
This represents inductive reactance and the line become an INDUCTOR.

Equ ⑥ can be rewritten as

$Z_{in} = j X_{is}$

$X_{is} = Z_0 \tan\beta l$.

The variation of X_{is} w.r.to l is shown as b



Quater wave length line $(\lambda/4)$ - Quarter wave transform

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \beta l}{Z_0 + Z_L \tanh \beta l} \right] \quad \text{——— (1)}$$

(Refer page 3

Here $l = \lambda/4$ i.e. length of TL is $\lambda/4$. Assume that its a lossless line. so $\gamma = j\beta l$. & $\tanh j\beta l = \tan h\beta l \approx \tan\beta l$.

(1) →

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh j\beta l}{Z_0 + Z_L \tanh j\beta l} \right] = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right]$$

put

$l = \lambda/4 \quad \therefore \beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$.

$$\Rightarrow Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} \right]$$

$\tan\beta l = \infty$.

$$= Z_0 \left[\frac{\left(\frac{Z_L}{\tan\beta l} + jZ_0\right)\tan\beta l}{\left(\frac{Z_0}{\tan\beta l} + jZ_L\right)\tan\beta l} \right] = Z_0 \left[\frac{\frac{Z_L}{\infty} + jZ_0}{0 + jZ_L} \right]$$

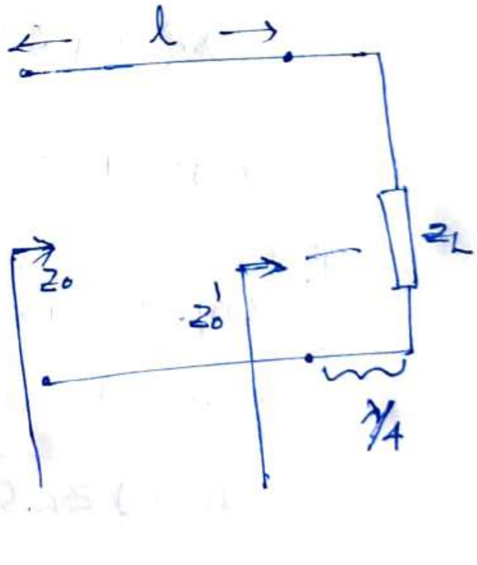
$$= Z_0 \left[jZ_0 / jZ_L \right]$$

IMPEDANCE MATCHING:

- When a line is terminated with characteristics impedance Z_0, then the line is said to be ~~termina~~ matched.
- The process of making $Z_L = Z_0$, is known as impedance matching.
- When $Z_L \neq Z_0$, reflections occur (mismatch).
- Impedance matching can be done in two ways.
  - * Using quarterwave transformer
  - * using stubs - stub matching.

Impedance matching Using Quarter wave transformer

The mismatch between TL & Load impedance can be nullified using a quarter wave length TL.



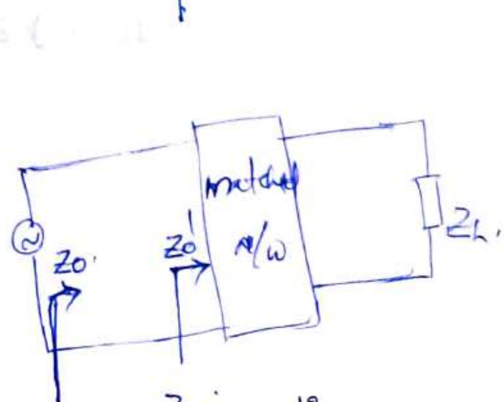
Let Z_0' be the chara. impedance of $\lambda/4$ line

For quarter wave section i/p impedance is $Z_{in} = \frac{Z_0^2}{Z_L}$.

Select Z_0' in such a way that

$\therefore Z_0' = \sqrt{Z_0 Z_L}$.

Here $Z_0 = Z_0'$



$$\therefore Z_{in\,\lambda/4} = \frac{Z_0'^2}{Z_L} = \frac{\left(\sqrt{Z_0 Z_L}\right)^2}{Z_L} = Z_0$$

$$Z_{in} = \frac{Z_0'^2}{Z_L} = Z_0.$$

$$Z_0' = \sqrt{Z_0 Z_L}.$$

$$\therefore \boxed{Z_{in} = Z_0}$$

- The i/p impedance of $\lambda/4$ line is Z_0. This impedance act as load impedance of the actual TL having length 'l'. So TL will feel it is be properly terminated. Thus we have achieved impedance matching

Advantages:-

* Simple method

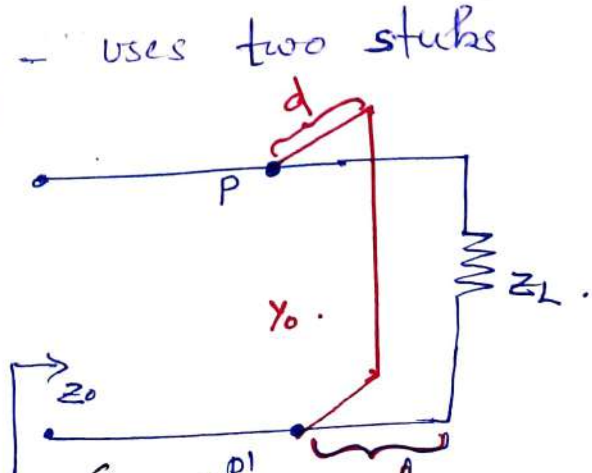
Disadvantages:

* Narrow band operation
* Frequency sensitivity
* Length of line is fixed.
* used in load which is purely resistive..

2* STUB MATCHING:

- Here matching is obtained by connecting a short section of txion line called STUB across the actual TL.
- Two types of stub matching.
  * Single stub matching - using single stub.
  * Double stub matching - uses two stubs

SINGLE STUB MATCHING:



Here a single short ckted stub is connected across the TL at point PP' as shown in figure.

Let 'l' be the location of stub i.e. (length) distance $\times$ load & stub

d be the length of the stub Z_0 & Y_0 are characteristic impedance and admittance of the T.L.

- Let line admittance at position pp' is $Y_0 + jB$

where B is susceptance ($1/X$) ie $B = \frac{1}{\text{Reactance}}$

- Let the admittance of the short ckt stub be $-jB$.

- Total admittance at point pp' is $Y_0 + jB + (-jB) = Y_0 = \frac{1}{Z_0}$

ie the line become properly terminated, since the loca of stub is very much close to the load end.

$$\boxed{\begin{aligned} l &= \frac{\lambda}{2\pi}\tan^{-1}\sqrt{\frac{Z_L}{Z_0}} \\ d &= \frac{\lambda}{2\pi}\tan^{-1}\left[\frac{\sqrt{Z_L Z_0}}{Z_L - Z_0}\right] \end{aligned}}$$

Draw backs :-

* Narrow BW operation
* if load impedance varies stub length need to vary.

Single stub matching - Derivation of l & d (Analytical method)

Let the i/p impedance of TL be

$$Z_{in} = Z_0\left[\frac{Z_L + Z_0\tanh\gamma l}{Z_0 + Z_L\tanh\gamma l}\right] \quad \text{---} (i)$$

Assume the line is loss less $\gamma = \alpha + j\beta$

$= j\beta$

$\tanh j\beta l = j\tanh\beta l$ - $j\tan\beta l$

$(1) \Rightarrow Z_{in} = Z_0 \left[\frac{Z_L + j Z_0 \tan\beta l}{Z_0 + j Z_L \tan\beta l} \right]$ ——— (2)

~~Let~~ equ: (2) can be written in terms of admittance.

$(2)\ Y_{in} = Y_0 \left[\frac{Y_L + j Y_0 \tan\beta l}{Y_0 + j Y_L \tan\beta l} \right]$ ——— (3)

Let us normalized the equ:(3) by deviding it with Y_0.

Normalized impedance $\bar{Z}_{in} = \frac{Z_{in}}{Z_0}$ ($\div$ by chara: impedance)

$(3)\ \frac{\bar{Y}_{in}}{Y_0} = Y_0 \left[\frac{Y_L + j Y_0 \tan\beta l}{Y_0 + j Y_L \tan\beta l} \right] \div Y_0$.

$\bar{Y}_{in} = \frac{\left[\frac{Y_L}{Y_0} + j \frac{Y_0 \tan\beta l}{Y_0}\right]}{\left[Y_0/Y_0 + j \frac{Y_L}{Y_0} \tan\beta l\right]} = \left[\frac{\bar{Y}_L + j \tan\beta l}{1 + j \bar{Y}_L \tan\beta l} \right]$ ——— (4)

multiplied and devided equ: (4) by the complex conjugate of denominator.

$(4) \Rightarrow \bar{Y}_{in} = \frac{(\bar{Y}_L + j \tan\beta l)(1 - j \bar{Y}_L \tan\beta l)}{(1 + j \bar{Y}_L \tan\beta l)(1 - j \bar{Y}_L \tan\beta l)}$

$= \frac{\bar{Y}_L - j \bar{Y}_L^2 \tan\beta l + j \tan\beta l - j^2 \bar{Y}_L \tan^2\beta l}{1 - (j^2 \bar{Y}_L \tan\beta l)^2}$

$= \frac{\bar{Y}_L - j \bar{Y}_L^2 \tan\beta l + j \tan\beta l + \bar{Y}_L \tan^2\beta l}{1 + \bar{Y}_L^2 \tan^2\beta l}$

$$= \frac{\overline{Y_L}\left(1+\tan^2\beta l\right) + j\tan\beta l\left(1-\overline{Y_L}^2\right)}{1+\overline{Y_L}^2\tan^2\beta l}.$$

$$\overline{Y_{in}} = g+jb = \frac{\overline{Y_L}\left(1+\tan^2\beta l\right) + j\tan\beta l\left(1-\overline{Y_L}^2\right)}{1+\overline{Y_L}^2\tan^2\beta l}.$$

equating real & imaginary parts.

$$g = \frac{\overline{y_L}\left(1+\tan^2\beta l\right)}{1+\overline{Y_L}^2\tan^2\beta l} \quad \text{———} \quad (5)$$

$$b = \frac{\tan\beta l\left(1-\overline{y_L}^2\right)}{1+\overline{Y_L}^2\tan^2\beta l}. \quad \text{———} \quad (6)$$

For no reflection $\overline{Y_{in}}$ should be equal to $1+j0$.

$$\overline{Y_{in}} = 1+j0 = g+jb.$$

$$g=1, \quad b=0.$$

The stub has to be connected where there is no

ie $g=1$

Apply this in (5)

$$(5) \quad g = \frac{\overline{Y_L}\left(1+\tan^2\beta l\right)}{1+\overline{y_L}^2\tan^2\beta l} = 1..$$

$$1+\overline{Y_L}^2.\tan^2\beta l = \left(1+\tan^2\beta l\right)\overline{Y_L}$$

$$1+\overline{Y_L}^2\tan^2\beta l = \overline{Y_L} + \overline{Y_L}\tan^2\beta l.$$

$$\overline{Y_L}^2\tan^2\beta l - \overline{Y_L}\tan^2\beta l = \overline{Y_L}-1$$

$$\overline{Y_L}\tan^2\beta l\left(\overline{Y_L}-1\right) = \overline{Y_L}-1$$

$$\overline{Y_L}\tan^2\beta l = \frac{\cancel{\overline{Y_L}-1}}{\cancel{\overline{Y_L}-1}} = 1.$$

$$\tan^2\beta l = 1/\overline{Y_L} \qquad \therefore \tan\beta l = \frac{1}{\sqrt{\overline{Y_L}}}$$

$$\tan\beta l = \frac{1}{\sqrt{\bar{Y}_L}} = \frac{1}{\sqrt{\frac{Y_L}{Y_0}}} ;$$

$$\tan\beta l = \sqrt{\frac{Y_0}{Y_L}} \quad \text{——— (7)}$$

$$\beta l = \tan^{-1}\sqrt{\frac{Y_0}{Y_L}}$$

$$l = \frac{1}{\beta}\tan^{-1}\sqrt{Y_0/Y_L} \qquad \left\{\beta = \frac{2\pi}{\lambda}\right.$$

$$\boxed{l = \frac{\lambda}{2\pi}\tan^{-1}\sqrt{\frac{Z_L}{Z_0}}} \quad \text{——— (8)}$$

The susceptance @ the stub location is given by (6).

$$b = \frac{\tan\beta l\,(1 - \bar{Y}_L^{2})}{1 + \bar{Y}_L^{2}\tan^{2}\beta l}.$$

where 'b' is the normalized susceptance. $\left\{\tan\beta l = \sqrt{Y_0/Y_L}\right.$

$$b = \frac{B}{Y_0} = \frac{\tan\beta l\left(1 - \frac{Y_L^{2}}{Y_0^{2}}\right)}{1 + \frac{Y_L^{2}}{Y_0^{2}}\tan^{2}\beta l}. = \frac{\sqrt{\frac{Y_0}{Y_L}}\left(Y_0^2 - Y_L^2\right)/\cancel{Y_0^2}}{Y_0^2 + Y_L^2\cdot\frac{Y_0}{Y_L}\,/\cancel{Y_0^2}}.$$

$$= \sqrt{\frac{Y_0}{Y_L}}\left[\frac{\cancel{(Y_0 + Y_L)}(Y_0 - Y_L)}{Y_0\cancel{(Y_0 + Y_L)}}\right] = \sqrt{\frac{Y_0}{Y_L}}\left[\frac{Y_0 - Y_L}{Y_0}\right]$$

$$\frac{B}{\cancel{Y_0}} = \sqrt{\frac{Y_0}{Y_L}}\left[\frac{Y_0 - Y_L}{\cancel{Y_0}}\right]$$

$$\boxed{B = \sqrt{\frac{Y_0}{Y_L}}\left[Y_0 - Y_L\right]} \quad \text{——— (9)}$$

Equ(9) is the susceptance of the line, the position were the stub is located. This susceptance was cancelled by the susceptance of the stub

The i/p impedance of the short ckted. stub is given

$Z_{sc} = j Z_0 \tanh \gamma l \quad Z_{st} = j Z_0 \tan \beta l$

$Y_{st} = \frac{1}{j Z_0 \tan \beta l} = -j \cancel{Z_0} \cot \beta l$

$Y_{st} = G_{st} + j B_{st} = -j \cancel{Y_0} \cot \beta l.$

$G_{st} = 0$ &

$B_{st} = -Y_0 \cot \beta l. \quad \text{—— (10)}$

Equ:(10) is the stub succeptance at stub location. The sum of the line succeptance & stub succeptance should be zero.

$B + B_{st} = 0. \quad \text{—— (11)}$

Substitude Equ: (9) & (10) in Equ: (11)

$\sqrt{\frac{Y_0}{Y_L}}(Y_0 - Y_L) + (-Y_0 \cot \beta l) = 0$

$\sqrt{\frac{Y_0}{Y_L}}(Y_0 - Y_L) - Y_0 \cot \beta l = 0$

$\sqrt{\frac{Y_0}{Y_L}}(Y_0 - Y_L) = Y_0 \cot \beta l$

$\cot \beta l = \frac{\sqrt{\frac{Y_0}{Y_L}}(Y_0 - Y_L)}{Y_0} = \frac{Y_0 - Y_L}{\sqrt{Y_0 Y_L}}$

$\tan \beta l = \frac{\sqrt{Y_0 Y_L}}{Y_0 - Y_L}$

$\beta l = \tan^{-1} \frac{\sqrt{Y_0 Y_L}}{Y_0 - Y_L} \qquad \beta = 2\pi/\lambda$

$l = \frac{1}{\beta} \tan^{-1} \frac{\sqrt{Y_0 Y_L}}{Y_0 - Y_L} = \frac{1}{2\pi/\lambda} \tan^{-1} \frac{\sqrt{Y_0 Y_L}}{Y_0 - Y_L}$

$\{ \tanh \beta$

$l = \frac{\lambda}{2\pi}$

He

(12)

l

$$l = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{Y_0 Y_L}}{Y_0 - Y_L} \quad \text{——— (12)}$$

Here $l = d$, the length of the stub (we usually denoted 'd')

(12) becomes

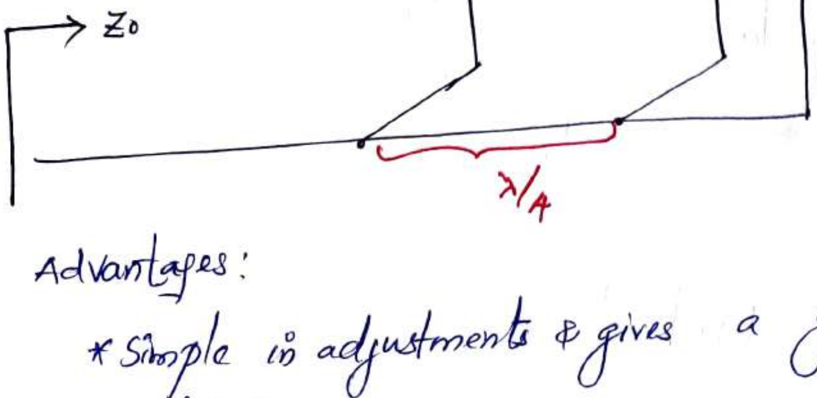
$$d = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{Y_0 Y_L}}{Y_0 - Y_L} = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{\frac{1}{Z_0} \frac{1}{Z_L}}}{\frac{1}{Z_0} - \frac{1}{Z_L}}.$$

$$= \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{\frac{1}{Z_0 Z_L}}}{Z_L - Z_0 / Z_0 Z_L} = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{\frac{1}{Z_0 Z_L}} * Z_0 Z_L}{Z_L - Z_0}.$$

$$\boxed{d = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{Z_0 Z_L}}{Z_L - Z_0}} \quad \text{——— (13)}$$

Double stub matching

- It over comes the limitations of single stub matching. Here two stubs are connected at arbitary points separated by a distance $\lambda/4$, here d_1, d_2 are length of stub 1 & stub 2.



Advantages:

* Simple in adjustments & gives a greater range of impedance matching.
* matching can be done easily by adjusting the length of the stub

Transmission lines as Circuit Elements

Transmission line not only act as a waveguiding structures for transfering power and information from one point to other but at high frequency they act as circuit elements. At very high frequencies X_C is less and X_L is more. Since L and C are depended on frequency they are not suitable to operate at high frequencies So the transmission line itself act as the circuit element.

Consider the transmission line to be lossless

i.e, $\gamma = j\beta \quad Z_0 = R_0$

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

$$\therefore Z_{in} = R_0\left[\frac{Z_L + jR_0 \tan\beta l}{R_0 + jZ_L \tan\beta l}\right]$$

Special Cases:-

1) Open Circuited Termination: $(Z_L = \infty)$

$$\therefore Z_{in} = R_0\left[\frac{Z_L + 0}{0 + jZ_L \tan\beta l}\right] = R_0 \frac{\times Z_L}{jZ_L \tan\beta l}$$

$$= \frac{R_0}{j\tan\beta l} = -jR_0 \cot\beta l \qquad \left(\because \frac{1}{\tan\beta l} = \cot\beta l\right)$$

i.e, the input impedance of an open circuit line is purely reactive which means the line can be either inductive or capacitive.

$$Z_{in} = \frac{R_0}{j\tan\beta l}$$

if $\tan\beta l \approx \beta l$

Then $Z_{in} = \frac{R_0}{j\beta l} = \frac{\sqrt{L/C}}{j\omega\sqrt{LC}\,l}$

$$= \frac{1}{j\omega C l}$$

This is impedance of capacitance of 'Cl' farads. So when the transmission line is open circuit, the transmission line act as Capacitor.

2) Short Circuited Termination $(Z_L = 0)$

$Z_{in} = Z_0[j\tan\beta l] = Z_0 j\tan\beta l = R_0 j\tan\beta l$

βl is very small $\beta l \approx \beta l$

$Z_{in} = R_0 j\beta l$

$= \sqrt{L/C}\, j\omega\sqrt{LC}\, l = j\omega l L$

The i/p impedance include Ll, when the transmission line is short circuited it act as inductor.

3) Quater Wave line: $l = \lambda/4$; $\beta l = \pi/2$

When the length l is odd multiple of $\lambda/4$

$l = (2n-1)\lambda/4$, $n = 1, 2, 3, \ldots$

$\beta l = (2n-1)\pi/2$

$\tan\beta l = \tan(2n-1)\pi/2$

$\tan\beta l = \infty$

$$Z_{in} = R_0\left[\frac{Z_L + jR_0\tan\beta l}{R_0 + jZ_L\tan\beta l}\right] = R_0\left[\frac{\frac{Z_L}{\tan\beta l} + jR_0}{\frac{R_0}{\tan\beta l} + jZ_L}\right]$$

$$= \frac{R_0^2}{Z_L}$$

A quater wave lossless transmission line transforms the load impedance to the i/p terminals as its inverse multiplied by the square of characteristic resistance. It is also known as impedance inverter. It is also commonly referred to as quater wave transformer.

4) Halfwave transmission line :-

$$l = \lambda/2$$

$$\beta l = 2\pi\lambda/2 = \underline{\underline{\pi}} \Rightarrow n\pi \quad n=1,2,\ldots$$

When $\tan\beta l = \tan n\pi = 0$ //

$$\therefore Z_{in} = R_0 \frac{Z_L}{R_0}$$

$$\therefore \underline{\underline{Z_{in} = Z_L}}$$

A half wave lossless line transverse the load impedance to the i/p terminals without any change.

Solving Using Analytical Method

Q. A lossless transmission line with $Z_0 = 50\Omega$ is 30m long and operates at 2MHz. The line is terminated with a load of $Z_L = 60 + 40j\,\Omega$. If $u = 0.6$ on the line. Find a) Reflection coefficient b) Standing Wave Ratio c) Input impedance

Soln $Z_0 = 50\Omega$

length(ℓ) = 30m

$Z_L = 60 + 40j\,\Omega$

linear frequency (f) = 2 MHz

$u = 0.6c$

$= 0.6 \times 3 \times 10^8$

$= 1.8 \times 10^8$ m/sec

a) Reflection Coefficient $(\Gamma) = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$$= \frac{60+40j-50}{60+40j+50} = \frac{10+40j}{110+40j} = \frac{10+40j}{110+40j} \times \frac{110-40j}{110-40j}$$

$$= \frac{(10+40j)(110-40j)}{(110)^2-(40j)^2} = \frac{10(110-40j)+40j(110-40j)}{12100+1600}$$

$$= \frac{1100-400j+4400j+1600}{13700}$$

$$= \frac{2700+4000j}{13700} = \frac{2700}{13700} + \frac{4000j}{13700}$$

$$= 0.197 + 0.2919j = 0.3521 \angle 56^\circ$$

b) Standing Wave Ratio (S)

$$S = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$= \frac{1+0.3521}{1-0.3521} = \frac{1.3521}{0.6479} = \underline{\underline{2.08}}$$

c) Input Impedance (Zin)

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

$$\beta = \frac{\omega}{u} \;;\; \omega = 2\pi f = 2\pi \times 2\times10^6$$
$$= 4\pi\times10^6$$

$\tan 120^\circ = -\sqrt{3}$

$$\beta = \frac{4\pi\times10^6}{1.8\times10^8} = \underline{\underline{0.0698}}$$

$$\beta l = \frac{2\pi}{3} = \underline{\underline{120^\circ}}$$

$$Z_{in} = 50\left[\frac{(60+40j) + j50\tan120^\circ}{50 + j(60+40j)\tan 120^\circ}\right]$$

$$= 50\left[\frac{60+40j - 86.602j}{50 + j(60\tan120^\circ + 40j\tan 120^\circ)}\right]$$

$$= 50\left[\frac{60+40j - 86.602j}{50 + j(-103.923 + 69.282j)}\right]$$

$$= 50\left[\frac{60+40j - 86.602j}{50 - 103.923j + 69.282}\right]$$

$$= 50\left[\frac{60 - 46.602j}{119.282 - 103.923j}\right]$$

$$= 50\left[\frac{60 - 46.602j}{119.282 - 103.923j} \times \frac{119.282 + 103.923j}{119.282 + 103.923j}\right]$$

$$= 50\left[\frac{60(119.282 + 103.923j) - 46.602j\,(119.282 + 103.923j)}{(119.282)^2 - (103.923j)^2}\right]$$

$$= 50\left[\frac{7156.92 + 6235.38j - 5558.7797j + 4843.0196}{14228.1955 - 10799.9899}\right]$$

$$= 50\left[\frac{11999.9396 + 676.6003j}{3428.2056}\right]$$

$$= \frac{599996.98 + 33830.015j}{3428.2056}$$

$$Z_{in} = \underline{\underline{175.0177 + 9.8681j}}$$

d) Load Admittance $= \frac{1}{Z_L}$

$$= \frac{1}{175.0177 + 9.8681j} = \frac{1}{175.0177 + 9.8681j} \times \frac{175.0177 - 9.8681j}{175.0177 - 9.868j}$$

$$= \frac{175.0177 - 9.8681j}{(175.0177)^2 - (9.8681j)^2} = \frac{175.0177 - 9.8681j}{30631.1953 + 97.3774}$$

$$= \frac{175.0177 - 9.8681j}{30,728.5727} = \underline{\underline{0.0056 - 0.00032j}}$$

Smith Chart

Plotting of Impedance

Example 1 :

Given : $Z_L = 50+50j\,\Omega$

$Z_0 = 50\,\Omega$

We have to Normalize the impedance.

$$\bar{Z_L} = \frac{Z_L}{Z_0} = \frac{50+50j}{50} = \frac{50}{50} + \frac{50}{50}j$$

$$\Rightarrow 1+j \quad (r+jx)$$

r (Resistance) $\quad x$ (Reactance)

$$\Rightarrow \boxed{r=1 \;\&\; x=1}$$

Example 2 :

Given $Z_L = 100+j100\,\Omega$

$Z_0 = 50\,\Omega$

We have to Normalize the impedance

$$\bar{Z_L} = \frac{Z_L}{Z_0} = \frac{100+j100}{50} = \frac{100}{50} + j\frac{100}{50}\,\Omega$$

$$= \underline{2+2j\,\Omega}$$

$$\Rightarrow \boxed{r=2 \;\&\; x=2}$$

Example 3 :

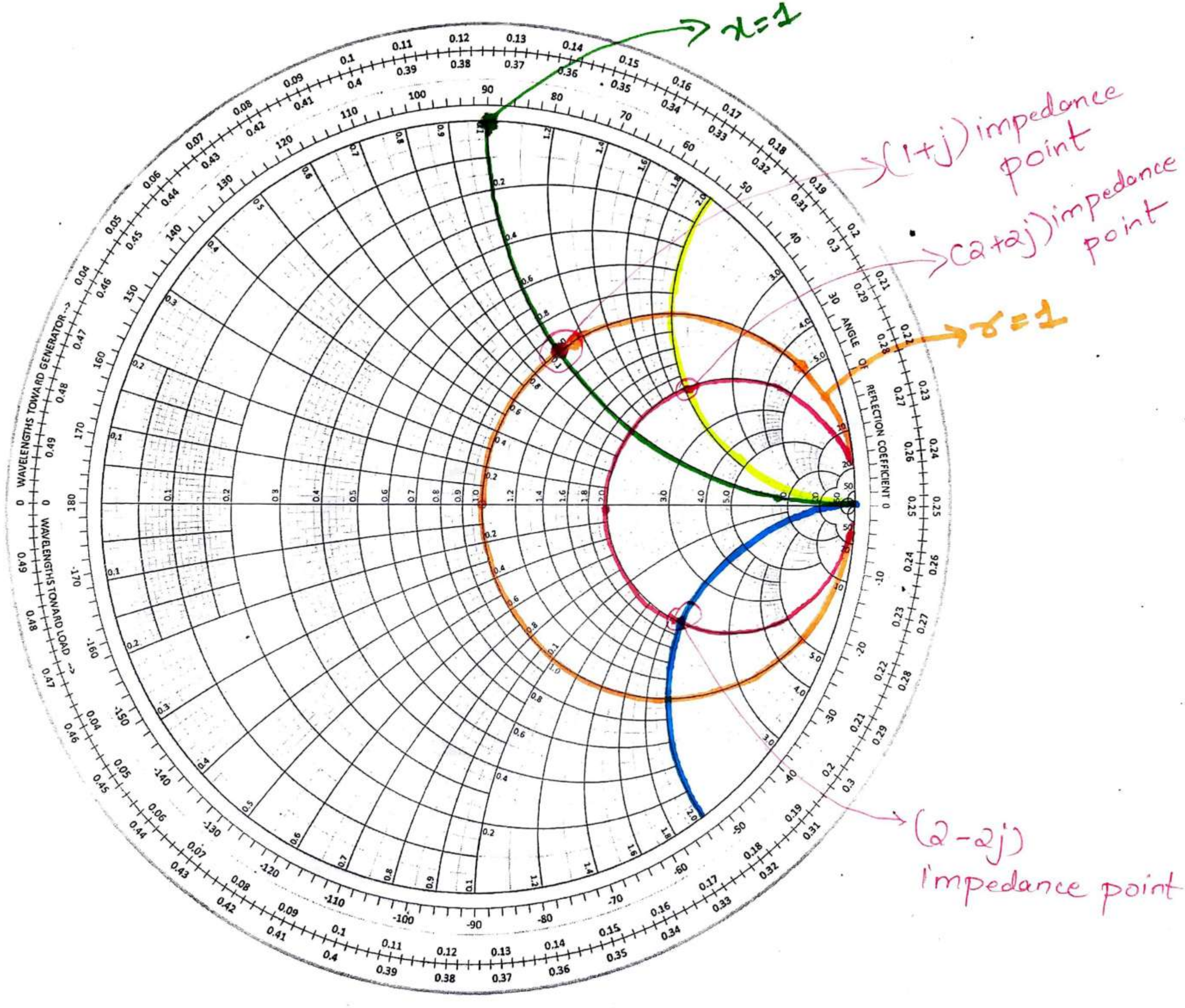
Given: $Z_L = 100-j100\,\Omega$

$Z_0 = 50\,\Omega$

We have to Normalize the impedance

$$\bar{Z_L} = \frac{100-j100\,\Omega}{50} = \frac{100}{50} - j\frac{100}{50}\,\Omega$$

$$= \underline{2-j2\,\Omega} \quad \Rightarrow \boxed{r=2 \;\&\; x=-2}$$



x=1
(1+j) impedance point
(2+2j) impedance point
r=1
(2-2j) impedance point
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT

Q A Lossless transmission line with $Z_0 = 50\Omega$, the line terminated with a load Z_L $50+50j\,\Omega$ find the 1) Reflection Coefficient
2) Standing Wave Ratio

Soln:- Normalized Impedance

$$\bar{Z}_L = \frac{Z_L}{Z_0} = \frac{50+50j\,\Omega}{50} = \frac{50}{50} + \frac{50}{50}j\,\Omega$$

$$= \underline{1+j\,\Omega}$$

1) Reflection coefficient

Using scale Measure OA & OB

$$|\Gamma_L| = \frac{OA}{OB} = \frac{3.7}{8} = 0.4625$$

Angle obtained from Smith Chart is $\angle 64^\circ$

$$\therefore \boxed{\Gamma = 0.4625 \angle 64^\circ}$$

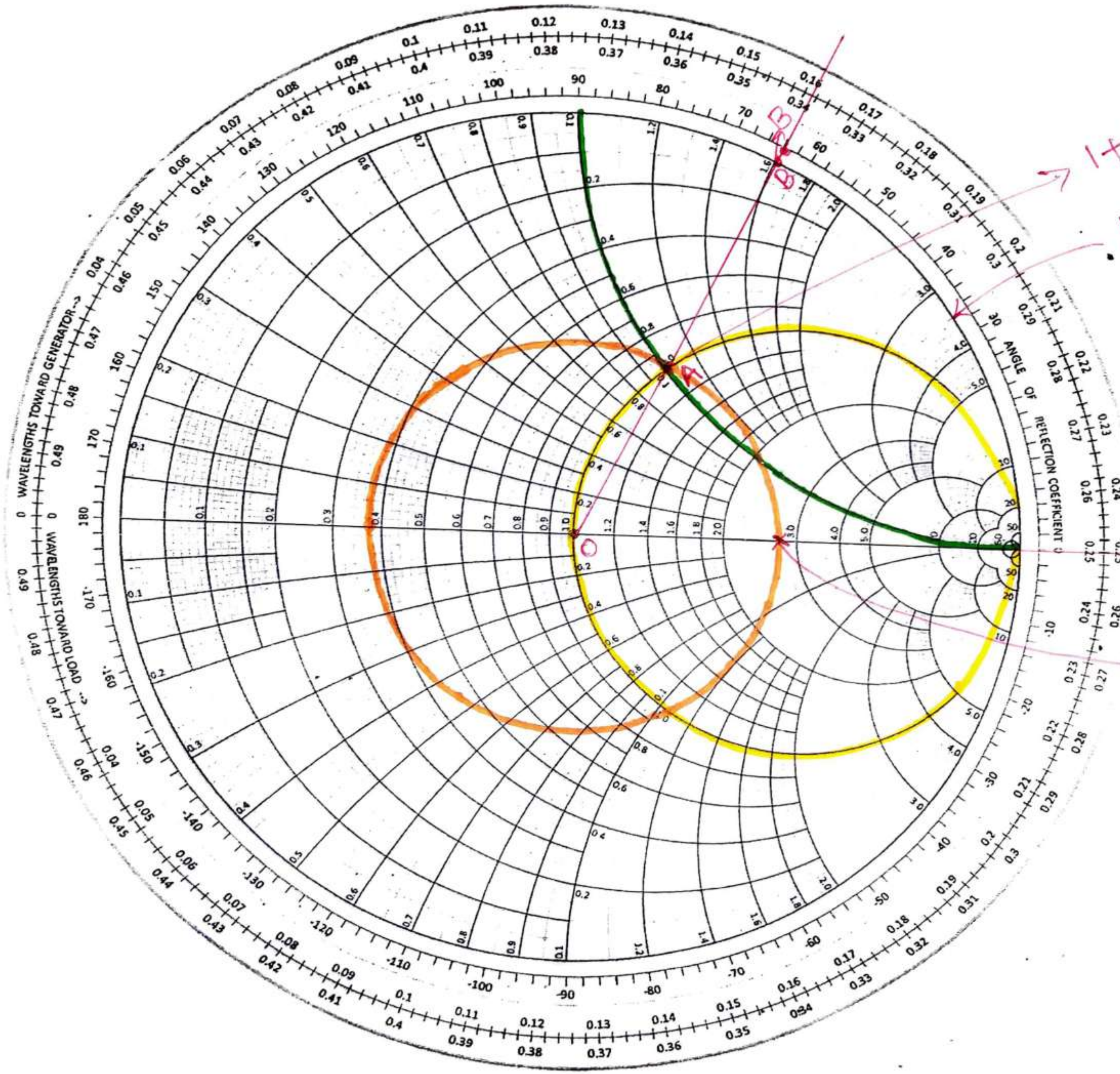
2) Standing Wave Ratio

Analytical Method $S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.4625}{1-0.4625} = 2.7209$

Using Smith chart

1) With Centre O & Radius OA. Draw a circle of VSWR
2) The value of VSWR (S) is where the VSWR circle meets Γ_r-axis to the right of origin.

VSWR Value Measured from Smith Chart is 2.8



$1+j\,\Omega$

$|\Gamma| = 1$

using scale measure OA & OB

$|\Gamma_L| = \frac{OA}{OB} = \frac{3.7}{8}$

$= 0.4625$

Γ_r axis

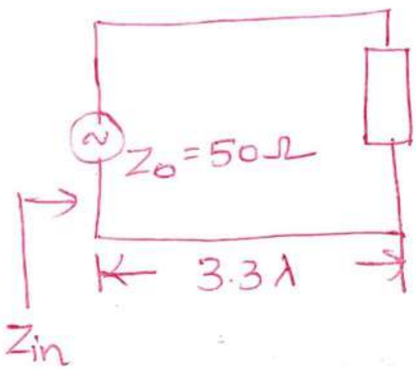
VSWR = 2.8
(S)

Q A 50 Ω transmission line is terminated to load of $25+30j$. The length of the transmission line is 3.3λ. Find 1) Reflection Coefficient

2) VSWR

3) Input Impedance

4) Input Admittance



$Z_L = 25+50j$

$\bar{Z}_L = \frac{Z_L}{Z_0}$

$\bar{Z}_L = \frac{25+50j}{50} = \frac{25}{50} + \frac{50j}{50}$

$\bar{Z}_L = 0.5+j$

1) Reflection Coefficient:

$|\Gamma| = \frac{OA}{OB} = \frac{5}{8} = 0.625$

$\angle 83^\circ$

$\therefore$ Reflection coefficient $= 0.625\angle 83^\circ$

2) VSWR:-

VSWR value from Smith chart is 4.2

3) Input Impedance

Moving distance towards the generator for 3.3λ we reach Z_{in}. One Circle of VSWR is $\lambda/2$, so after covering 6 circles we get 3λ, if we move 0.3λ further we reach Z_{in}.

$$= 0.135\,\lambda + 0.3\lambda$$

$$= \underline{0.435\lambda}$$

Mark the above value on the wavelength towards generator scale. Join the point to Origin (1,0). The point on which VSWR circle cuts line is the Input impedance point. from Smith chart Normalized input Impedance

Normalized i/p impedance

$$Z_{in} = 0.28 - 0.44j$$

$$= (0.28 - 0.44j)\,50$$

$$Z_{in} = \underline{14 - 22j}$$

4) i/p Admittance:- The line of zin impedance point lieing is extended through Origin (1,0) to cut the VSWR circle. The point of extended zin impedance line that cuts the VSWR circle is i/p admittance value.

Normalized input Admittance from Smith Chart

$$Y_{in} = 1 + 1.6j$$

$$= (1 + 1.6j)\,50$$

$$= \underline{50 + 80j}$$

Normalized Impedance point ($\bar{Z}_L$)

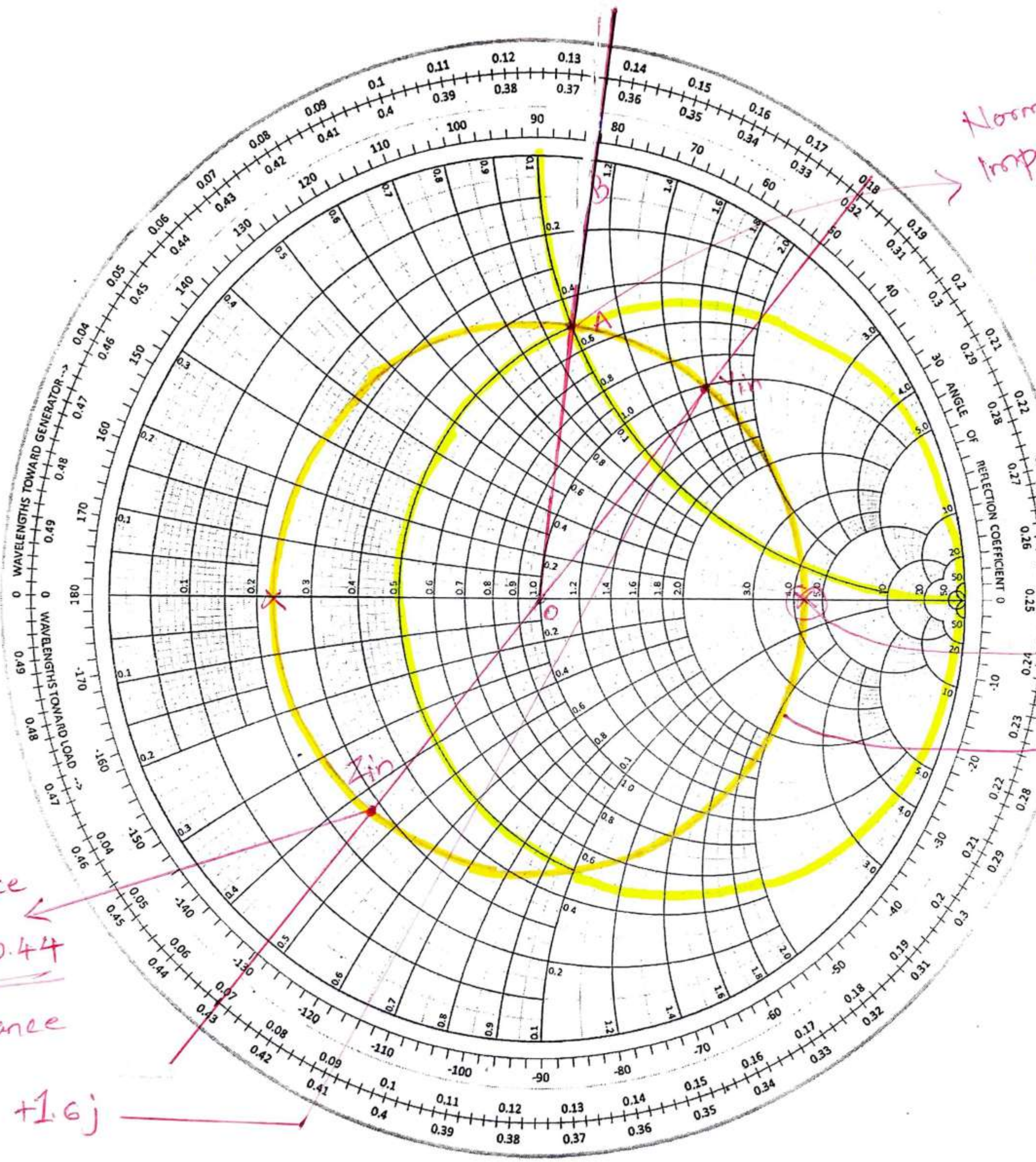
Reflection coefficient

$|\Gamma| = \frac{OA}{OB} = \frac{5}{8}$

$= \underline{0.625}$

$= \angle 83^\circ$

$0.625 \angle 83^\circ$



VSWR value $= \underline{4.2}$

VSWR circle

Normalized Input Impedance

$Z_{in} = 0.28 - j0.44$

Normalized Input Admittance

$Y_{in} = 1 \quad +1.6j$

Stub Matching Using Smith Chart

(I.N.P)

Q A 300Ω transmission line is connected to a load impedance of 450−j600Ω at 10MHz. Find the position and length of a short circuited stub required to match the line using smith chart.

Soln $Z_R = 450 - j600\,\Omega$

$Z_0 = 300\,\Omega$

Step 1: Normalized load impedance

$$\bar{Z}_L = \frac{Z_L}{Z_0} = \frac{450 - j600}{300} = \frac{450}{300} - \frac{600}{300}j$$

$$= 1.5 - j2\,\Omega$$

Step 2: Normalized load Admittance

$$Y_L = \frac{1}{Z_L} = \frac{1}{1.5 - j2} = 0.24 + j0.32\,\Omega$$

⇒ locate Normalized load admittance value on Smith Chart. As Point A.

⇒ Choose the Resistance (1) of Γ_r axis as Origin (Point O)

⇒ Draw a VSWR circle with radius OA.

⇒ Draw the Resistance 1Ω circle

⇒ The VSWR circle cuts the Resistance 1Ω circle at two points. Extend the point ~~on the~~ towards the generator. It cuts at Point C on the wavelength towards Generator scale. Extend the OA line also to cut at point B on the wavelength towards Generator scale.

⇒ The position of stub is obtained as

$$l_s = (0.18\lambda - 0.052\lambda) = \underline{\underline{0.128\lambda}}$$

⇒ Now Mark the susceptance point on E. Extend the line from O to E cut at wavelengths towards generator scale. Measure the wavelengths at short circuited end (It is 0.25λ) & wavelengths at point E (It is 0.34λ).

The length of short circuited stub (l_t) is

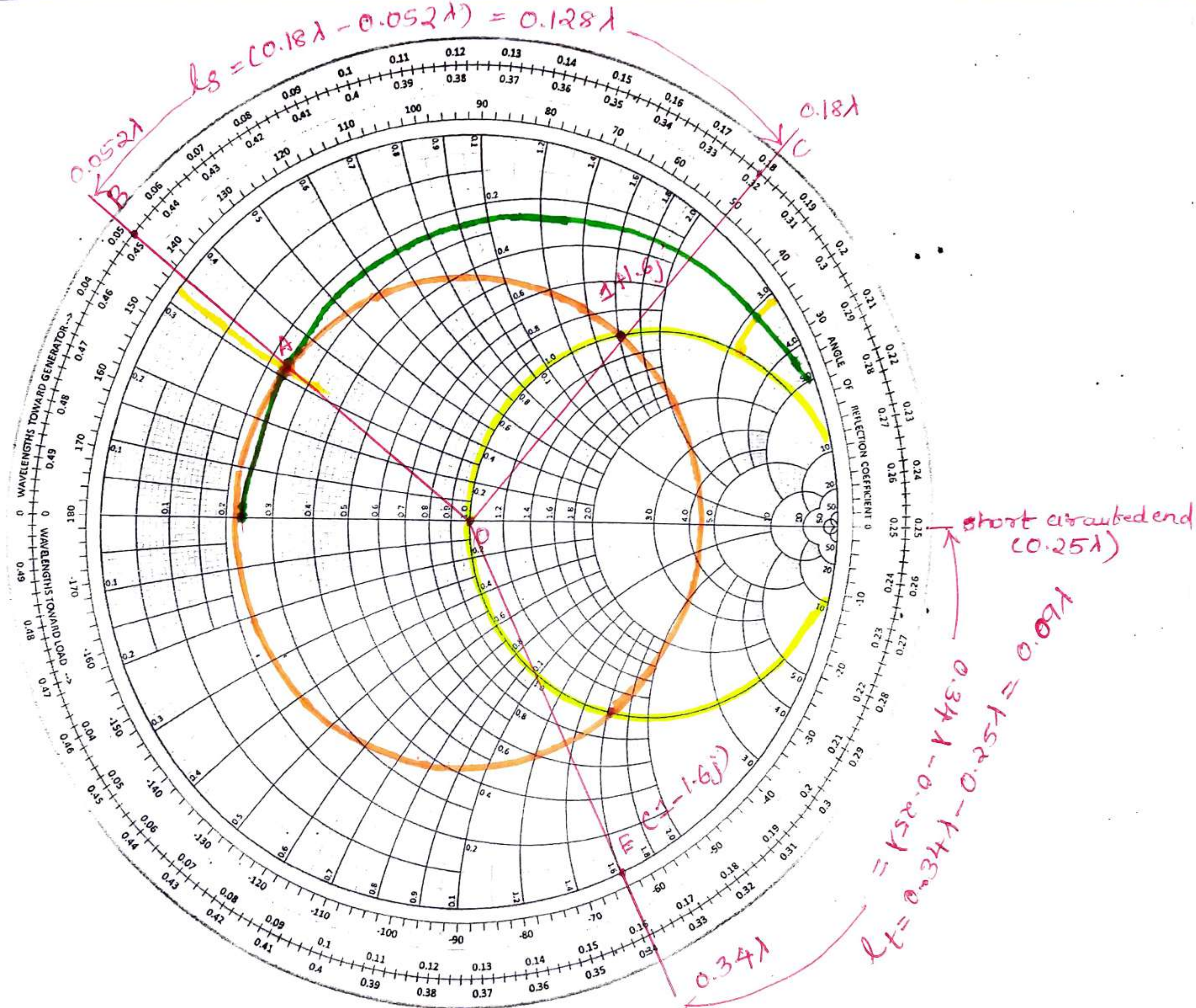
$$l_t = 0.34\lambda - 0.25\lambda = \underline{\underline{0.09\lambda}}$$

$$Y_L = \frac{1}{1.5-2j} = \frac{1}{1.5-2j} \times \frac{1.5+2j}{1.5+2j}$$

$$= \frac{1.5+2j}{(1.5)^2-(2j)^2} = \frac{1.5+2j}{2.25-(4\times-1)} = \frac{1.5+2j}{2.25+4}$$

$$= \frac{1.5+2j}{6.25} = \frac{1.5}{6.25} + j\left(\frac{2}{6.25}\right)$$

$$= \underline{0.24 + j0.32\ \Omega}$$



ls = (0.18λ − 0.052λ) = 0.128λ
0.052λ
B
0.18λ
C
A
1+1.6j
O
(1−1.6j)
E
short circuited end (0.25λ)
0.34λ
lt = 0.34λ − 0.25λ = 0.09λ
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT

Q An Antenna with impedance $90 - j25\,\Omega$ is to be matched to a $50\,\Omega$ line with a shorted stub.

Determine a) the stub length

b) Distance between stub and antenna.

Soln

$$Z_L = 90 - j25\,\Omega$$

$$Z_0 = 50\,\Omega$$

Step 1:- $\bar{Z_L} = \frac{Z_L}{Z_0} = \frac{90 - j25}{50} = \frac{90}{50} - \frac{25}{50}j$

$$= 1.8 - j0.5$$

Step 2: $Y_L = \frac{1}{Z_L} = \frac{1}{1.8 - j0.5} = \frac{1}{1.8 - j0.5} \times \frac{1.8 + j0.5}{1.8 + j0.5}$

$$= \frac{1.8 + j0.5}{(1.8)^2 - (j0.5)^2} = \frac{1.8 + j0.5}{(1.8)^2 - (j^2 0.5^2)}$$

$$= \frac{1.8 + j0.5}{3.24 - (-1 \times 0.25)} = \frac{1.8 + j0.5}{3.24 - (-0.25)}$$

$$= \frac{1.8 + j0.5}{3.24 + 0.25} = \frac{1.8 + j0.5}{3.49}$$

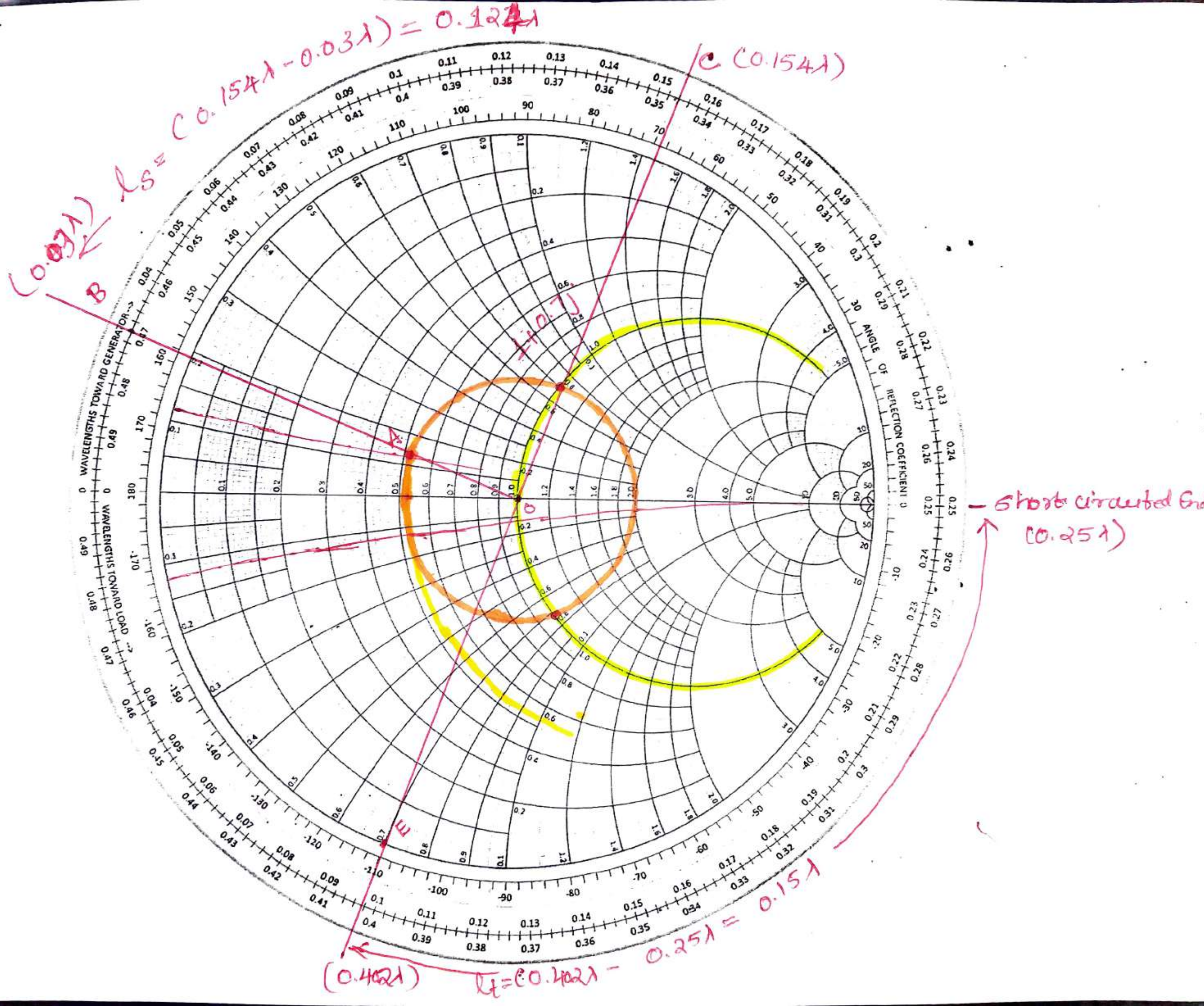
$$= \frac{1.8}{3.49} + j\left(\frac{0.5}{3.49}\right) = \underline{\underline{0.51 + j0.14}}$$

$\Rightarrow$ Distance between stub and antenna

$$l_s = 0.154\lambda - 0.03\lambda = \underline{\underline{0.124\lambda}}$$

$\Rightarrow$ The stub length $l_t = 0.402\lambda - 0.25\lambda$

$$= \underline{\underline{0.15\lambda}}$$



(0.03λ) l_S = (0.154λ − 0.03λ) = 0.124λ
C (0.154λ)
B
A
O
E
1+j0.7
Short circuited End (0.25λ)
(0.402λ)
l_t = (0.402λ − 0.25λ = 0.15λ
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT

Q. A 300Ω transmission line is connected to a load impedance of 450-j600Ω at 10 MHz. Find the portion and length of a short circuited stub required to match the line using analytical Method.

Soln:- $Z_0 = 300\Omega$, $Z_L = 450 - j600\Omega$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{450 - j600 - 300}{450 - j600 + 300} = \frac{150 - j600}{750 - j600}$$

$$= \frac{150 + j600}{750 - j600} \times \frac{750 + j600}{750 + j600} = \frac{(150 + j600)(750 + j600)}{(750)^2 - (600j)^2}$$

$$= \frac{150(750 + j600) + j600(750 + j600)}{562500 + 360000}$$

-0.2682 + 0.5853

-0.2791 + 0.6091i

0.6438 ∠114.6185

$$= \frac{112500 + 90{,}000j + 450000j - 360000}{922500} = \frac{-247500 + 540000j}{922500}$$

$$= \frac{-247500 + 540000j}{922500} = 0.2682 + 0.5853j$$

$$= 0.6439 \angle 114.6185^\circ$$

Location of Stub:-

$$l_s = \frac{\lambda}{4\pi}[\phi + \pi - \cos^{-1}|\Gamma|]$$

ϕ = angle of reflection coefficient

$|\Gamma|$ = Magnitude of Reflection Coefficient

λ = wavelength

$$l_s = \frac{\lambda}{720^\circ}\left[114.61^\circ + 180^\circ - 49.92^\circ\right]$$

$$= \frac{\lambda}{720^\circ}\left[244.69\right]$$

$$\boxed{l_s = 0.3398\lambda}$$

$$l_s' = \frac{\lambda}{4\pi}\left[\phi + \pi + \cos^{-1}|\Gamma|\right]$$

$$= \frac{\lambda}{720^\circ}\left[114.61^\circ + 180^\circ + 49.92^\circ\right]$$

$$= \frac{\lambda}{720^\circ}\left[344.53\right]$$

$$\boxed{l_s' = 0.4785\lambda}$$

<u>Length of Stub:-</u>

$$l_t = \frac{\lambda}{2\pi}\tan^{-1}\left[\frac{\sqrt{1-|\Gamma|^2}}{2|\Gamma|}\right]$$

$$l_t = \frac{\lambda}{360^\circ}\tan^{-1}\left[\frac{\sqrt{1-|0.6439|^2}}{2|0.6439|}\right] = 0.085\lambda$$

$$l_t' = \frac{\lambda}{360^\circ}\tan^{-1}\left(\frac{\sqrt{1-|0.6439|^2}}{-2|0.6439|}\right) = 0.414\lambda$$

$$\boxed{l_s = 0.3398\lambda} \qquad \boxed{l_t = 0.085\lambda}$$

$$l_s' = 0.4785\lambda \qquad l_t' = 0.414\lambda$$

The location of stub must be nearer to load. So, l_s value is selected & its corresponding l_t value is considered.